Predicting the time of occurrence of decompression sickness

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WEATHERSBY, P. K., S. S. SURVANSHI, L. D. HOMER, E. PARKER, AND E. D. THALMANN. *Predicting the time of occurrence of decompression sickness.* J. Appl. Physiol. 72(4): 1541–1548, 1992.—Probabilistic models and maximum likelihood estimation have been used to predict the occurrence of decompression sickness (DCS). We indicate a means of extending the maximum likelihood parameter estimation procedure to make use of knowledge of the time at which DCS occurs. Two models were compared in fitting a data set of nearly 1,000 exposures, in which >50 cases of DCS have known times of symptom onset. The additional information provided by the time at which DCS occurred gave us better estimates of model parameters. It was also possible to discriminate between good models, which predict both the occurrence of DCS and the time at which symptoms occur, and poorer models, which may predict only the overall occurrence. The refined models may be useful in new applications for customizing decompression strategies during complex dives involving various times at several different depths. Conditional probabilities of DCS for such dives may be reckoned as the dive is taking place and the decompression strategy adjusted to circumstance. Some of the mechanistic implications and the assumptions needed for safe application of decompression strategies on the basis of conditional probabilities are discussed.

bends; mathematical models; risk management; inert gas exchange

EXPERIENCE has shown that decompression sickness (DCS) becomes more common, but seldom becomes a certainty, when certain limits are exceeded. Rapid ascents from long stays at great depths are more apt to give rise to DCS than slower ascents from short stays at more shallow depths. However, sometimes the risky dive and decompression may be undertaken with no untoward results, and occasionally symptoms of DCS are seen in usually safe dives. Accordingly, probabilistic models have been used to predict the probability of DCS in various circumstances (10, 13, 14, 16, 18, 19).

In the probabilistic models used so far, the probability of the occurrence of DCS for the entire dive was calculated without regard for the time at which the symptoms occurred. We now have data for which the time of onset of symptoms of DCS is known at least approximately. We wish to introduce this additional information in a likelihood estimation procedure, with the expectation that this refinement will provide sounder estimates of the unknown parameters of the model. In addition, we hope that some insights may be gained into plausible mechanisms leading to symptoms when information about the time of the symptoms is used. The models we use have functional forms suggested by notations commonly used in survival or failure time analysis (3). At any *time T*, during or after the dive, the probability $P(s)_T$ for an individual to be free of DCS symptoms is related to the probability of his having suffered DCS by that time

$$P(s)_T = 1.0 - P(DCS)_T \quad (1)$$

$P(s)_T$ is sometimes called the survivor function. If no DCS has occurred, then we define the probability of suffering DCS during a short ensuing time interval, dt, as $r \times dt$, where r is the risk function or hazard function (3). Because $r \times dt$ is a probability, r cannot be negative. Freedom from DCS symptoms until *time T* requires surviving (integrating) all DCS risk incurred up to that time. The survival function is the probability of not experiencing DCS before *time T*

$$P(s)_T = \exp\left(-\int_0^T r dt\right) \quad (2)$$

The instantaneous risk is determined by the mechanisms giving rise to the terminal event of the probabilistic process. For the decay of a radioactive isotope, for example, r is a constant that is characteristic of the particular isotope. Patient survival after surgery often is characterized by an r that is large immediately after surgery (immediate postoperative mortality), declines gradually, and then follows a course that will vary with the nature of the risks at those later times. Aging corresponds to a function r that increases with time. So if the greatest risk of DCS occurs immediately after a decompression step, r should be at its highest level at that time and then decline with time until the next decompression step. Such a risk function might be expected if DCS were triggered immediately after the occurrence of bubble nucleation, because bubble nucleation depends on instantaneous supersaturation even more strongly than a linear function (15). If, on the other hand, it takes a long time for the risk to develop after a decompression step, then r might rise, reach a peak, and then decline. If DCS depended on bubble growth to a certain size, r might have such a shape (11). In the long run, we expect to find more occurrences of DCS at times when r is large than at times when it is small. If r provides a satisfactory summary of the hazard process, then DCS should not occur at all when r is zero.

The probability of not developing DCS sometime during or after the dive is expressed as the integration of the risk over the entire history of the dive and recovery.

Therefore different definitions of r can produce the same integrated risk and thus the same prediction of overall safety. The records used in earlier modeling of the probability of DCS showed only that DCS occurred or that it did not occur, so we used a value of T large enough to encompass all occurrences of DCS, typically 24 h after the end of the dive. Because this integral may be the same for different shapes of r, our previous models were not sensitive to the shape of r and could not offer reliable estimates of the time at which DCS was likely to occur.

Now we have more information about the time of DCS onset. We know a time, T_1, when the diver was definitely free of DCS symptoms, and a later time, T_2, when he definitely did have symptoms. (The finite time interval between T_1 and T_2 acknowledges the uncertainty in establishing the exact onset time of DCS). The observation then has two parts: no DCS until T_1 and then occurrence of DCS during the T_1–T_2 time interval. We then can calculate the probability of both events as the joint probability of surviving symptom free until T_1 and then experiencing DCS during the interval T_1–T_2

$$P(\mathrm{s}_{T_1}, \mathrm{DCS}_{T_2}) = P(\mathrm{s})_{T_1} P(\mathrm{DCS})_{T_1,T_2}$$

$$= \exp\left(-\int_0^{T_1} r\,dt\right)\left[1.0 - \exp\left(-\int_{T_1}^{T_2} r\,dt\right)\right] \quad (3)$$

Note that the probability of DCS in a finite T_1–T_2 interval is always smaller than the probability of DCS occurring at all ($T_1 = 0$, T_2 = long time). This decreased probability offers a more severe test of candidate models for r, especially for the shape of r in the T_1–T_2 interval. By using *Eq. 3* and the data on time of onset that has become available to us, we hope to learn more about the shape of r. As our knowledge of the shape of r improves, it will be possible to consider new applications of *Eq. 3*. One application might be to obtain customized decompression strategies after complex diving procedures involving sojourns of various lengths at many different depths.

MODELING

We will use two different models. One will postulate that r will be highest immediately after arrival at a decompression step; the other will allow for some delay between a decompression step and the evolution of the greatest risk of DCS associated with that step.

Model 1. In *model 1*, the instantaneous risk or hazard, r_1, is proportional to the sum of the risks associated with each of two tissues. The washout curve for each tissue is described by simple exponential decay ("MEPC model" in Ref. 10, *"model 3"* in Ref. 18, *"model 2"* in Ref. 19.

The relative supersaturation of two tissues is used to define the risk r_1

$$r_1 = r_{1A} + r_{1B} \quad (4)$$

where

$$r_{1A} = AA_1 \frac{\mathrm{Pti}_A - \mathrm{Pam}}{\mathrm{Pam}}, \text{ with } r_{1A} \geq 0$$

$$r_{1B} = AB_1 \frac{\mathrm{Pti}_B - \mathrm{Pam}}{\mathrm{Pam}}, \text{ with } r_{1B} \geq 0$$

where Pam is ambient pressure. The tissue pressures Pti refer only to nitrogen. Oxygen, carbon dioxide, and water vapor are ignored (13). Parameters AA_1 and AB_1 are scale factors for the two tissues. For example, if in a single tissue Pti–Pam was 0.2 ATA when Pam was 1 ATA and this condition was maintained for 100 min with a scale factor of 0.003 min^{-1}, a P(DCS) of ~6% would result [1 − exp (−0.003 × 0.2 × 100)]. Both r_{1A} and r_{1B} are constrained to be zero or positive, inasmuch as $P(\mathrm{s})_T$ must be a nonincreasing function of time. r_1 will drop to zero when calculated gas excretion decreases the tissue nitrogen partial pressures to below Pam.

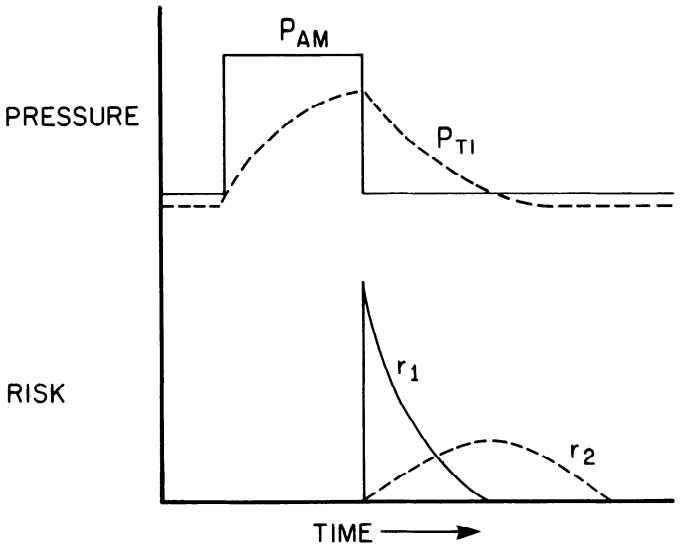

FIG. 1. Model behavior on a simple dive within a single tissue compartment characterized by a single time constant. Area under r_1 and r_2 curves is proportional to the total probability of decompression sickness [P(DCS)] for this dive. By definition, risk (r) cannot be negative.

Model 2. Model 2 uses a similar set of parameters, but the risk is obtained from the integral of the relative supersaturation

$$r_2 = r_{2A} \quad r_{2B} \quad (5)$$

where

$$r_{2A} = AA_2 \int_0^t \frac{\mathrm{Pti}_A - \mathrm{Pam}}{\mathrm{Pam}}\,\mathrm{ds}, \text{ with } r_{2A} \geq 0$$

$$r_{2B} = AB_2 \int_0^t \frac{\mathrm{Pti}_A - \mathrm{Pam}}{\mathrm{Pam}}\,\mathrm{ds}, \text{ with } r_{2B} \geq 0$$

Both positive and negative values of the supersaturation are integrated, but because the risk, r_2, is not allowed to be negative, the values of both r_{2A} and r_{2B} are again constrained to be positive or zero. The scale factors AA_2 and AB_2 (in units of min^{-2}) are expected to be considerably smaller because of the second integration. r_1 peaks just after a decompression step, whereas r_2 reaches its highest value later. Figure 1 compares the behavior of the two models (simplified to a single tissue) on a simple dive with no decompression stops. It is seen that r_1 is greatest immediately after the pressure reduction and declines monotonically thereafter. By contrast, r_2 increases until the supersaturation vanishes and then decreases thereafter.

Computations. The computational details for tracking Pti over a complicated pressure exposure and performing the appropriate integrations are tedious but straightfor-

ward. Some additional details have been published (18). Gas exchange takes place in two hypothetical well-mixed tissue compartments with time constants τ_{1A}, τ_{1B}, τ_{2A}, and τ_{2B} corresponding to *models 1* and *2* and *tissues A* and *B*. The recorded dive profile is approximated by many pressure-time ramps in calculation of the entry and exit of nitrogen from the tissue.

The four parameters for each model (2 time constants and 2 scale factors) must be estimated from data. For the estimation, we need the calculated probability of each outcome in the data. These can be obtained from *Eqs. 1–3*. If the outcome was safe, it was safe over all time intervals during and long enough after the pressure exposure for r to decay to zero. Usually 1 day after the dive is sufficient

$$\text{if no DCS, } P(\text{OUTCOME}) = \exp\left(-\int_0^{+24\text{ h}} r\,dt\right) \quad (6)$$

If the dive resulted in DCS, we need to distinguish whether we care only about the probability of occurrence or also about the time interval in which symptoms appear

Occurrence Only

$$\text{if DCS, } P(\text{OUTCOME}) = 1.0 - \exp\left(-\int_0^{+24\text{ h}} r\,dt\right) \quad (7)$$

Time of-DCS (use *Eq. 3*)

$$\text{if DCS, } P(\text{OUTCOME}) = \exp\left(-\int_0^{T_1} r\,dt\right)\left\{1.0 - \exp\left(-\int_{T_1}^{T_2} r\,dt\right)\right\}$$

Equation 7 is a special case of *Eq. 3* in which T_1 is set at zero and T_2 is >1 day later.

Using initial guesses for the scale factors (A's) and time constants (τ's), we calculate the log-likelihood function (LL) as the sum of the logs of P(OUTCOME) for each exposure in our data. The values of the unknown parameters that maximize the LL are chosen as the best possible fit of a model to the data; hence we use the term maximum likelihood (4). This fitting constitutes a calibration procedure for the model. We use a modified (1) Marquardt (7) nonlinear estimation program to vary the parameters estimated until the LL is maximized. We also use the slopes of the LL surface in the vicinity of the maximum to estimate the covariance matrix and hence obtain estimates of the precision of estimated parameters (4). A Marquardt search with these models seems to be rather efficient, but many different starting values of parameters are typically required to be convinced of convergence to a global (vs. local) maximum LL.

The actual value of a maximum LL is no more informative than the sum of squared errors from a least-squares fit. However, just as ratios of sums of squared errors may be used in an analysis of variance, the ratio of likelihoods may be used to test hypotheses of interest. Likelihood ratio tests may be used to compare different models if one of the models is obtainable as a special case of the more general model with constrained parameters (4, 16). A simplified model is one that considers risk to be equivalent at all times

$$\begin{aligned} r &= 0 \text{ before decompression begins} \\ r &= \text{constant after decompression begins} \end{aligned} \quad (8)$$

This we will refer to as our null model when making comparisons with LL. Single tissue versions of *Eqs. 4* and *5* were also explored but did not fit well at all.

DATA

Evaluation of the models requires very well-documented dives and outcomes. Data of 921 man-dives were obtained from unusually detailed computerized records of air diving trials at the US Navy Experimental Diving Unit (Panama City, FL) in 1984 and the Canadian Defence and Civil Institute of Environmental Medicine (Toronto) from 1978 to 1988. Description of the trials is available in several reports (5, 6, 8, 9). Some statistical analyses of portions of these data have already been published, in which only the occurrence (not the time) of DCS is used (19). Both single (only one dive per day) and repetitive dive combinations are represented (Table 1). Only immersed trial subjects are included. In general they were exercising and usually a little cold. The pressure profiles were all reviewed. Inconsistencies were resolved by checking original records and logs. The dive profiles were then approximated for this analysis as ≤32 pressure-time nodes, which kept the maximum deviation between recorded and approximated profiles <1.2 fsw for <33 s. Table 1 includes four cases of marginal symptoms: generally mild fleeting pain, resolving spontaneously without treatment but definitely associated with the dive. For this report, marginal cases are treated as an outcome of one-half safe, one-half DCS. The presence of these few ambiguous cases does not seriously affect our conclusions. In other instances, the manner in which marginal cases are dealt with has had a greater effect on conclusions (13, 16).

Choice of times for the T_1–T_2 interval presented difficulties. The onset of DCS is seldom a sharply defined

TABLE 1. *Characteristics of diving*

Single dives	
Depth	50–265 fsw (avg = 126 fsw)
Bottom time	3–244 min (avg = 60 min)
Decompression time	2–290 min (avg = 77 min)
Outcome	
Trials	727
DCS	38
Marginal	4
Repetitive dives	
Number of dives	2, 3, or 4
Depth	60–177 fsw
Bottom time	17–66 min
Decompression time	2–246 min
Surface interval	60–180 min
Outcome	
Trials	194
DCS	14
Marginal	0
Total	
Man-dives	921
DCS cases	52 (5.6%)
Marginal cases	4 (0.4%)

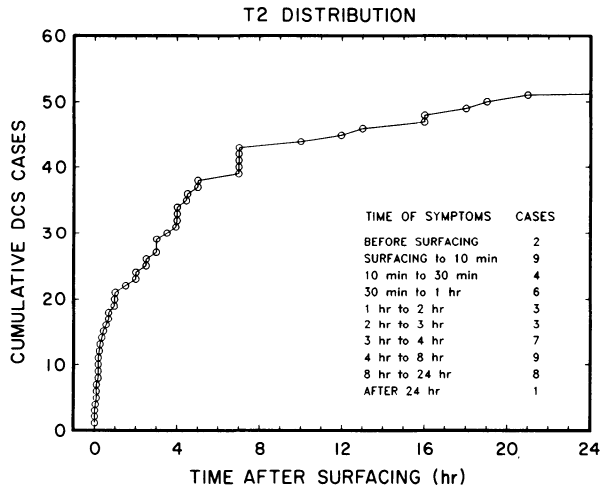

FIG. 2. Cumulative number of cases of DCS with increasing time after end of dive. In repetitive dives, time after surfacing is defined as the end of pressure exposure immediately preceding symptom onset.

event. In many cases, there is a period of minutes to hours over which the diver notices a discomfort, or soreness, that later leads to a complaint sufficient to require medical evaluation and then recompression therapy. In other cases the diver may fall asleep feeling healthy and then awake with a pain hours later. The ambiguous time tends to be longer as the time of diagnosis becomes later after the dive. The records indicate that divers were nearly always quizzed about the time the symptom first became really noticeable, and their definite answers were selected as T_2 – the time when DCS had definitely occurred. In a few cases we had to choose the time of starting recompression therapy. Figure 2 shows the distribution of T_2 in our data. The symptom reports are widely spread, from some before the diver emerged from the water to others occurring >1 day later. Nearly half occurred ≥3 h after the dive. The 52 cases of DCS arose from 35 different dive profiles of a total of 921 man-dives. The lack of many DCS cases after any single dive profile prevented any attempt to directly judge an appropriate shape for the risk function, r.

The choice of T_1, the latest time at which DCS had definitely not occurred, might be made in several ways. One conservative choice is obvious: the beginning of decompression. However, that would remove most time resolution from the data and allow risk accumulated immediately after the dive to be used to predict DCS cases that occurred much later. Another choice could be the time of the last medical examination eliciting no complaints, as was tabulated for a trial with several such examinations after the dive (13). However, these data were not obtained under conditions of regularly spaced medical examinations. As a compromise judgment, the rules in Table 2 were adopted to choose a T_1.

When more than a single dive was made, the assignment rules were applied to the latest dive. Of the rules, the one with the most impact is the maximum 2-h period between the dive and T_1. That time roughly corresponds to a standard practice for those experimental dives in which test subjects were under medical surveillance after the exposure and, if symptom free, released to home or

TABLE 2. *Timing rules*

Time of Definite Symptoms (T2)	Latest Definitely Safe Time (T1)
≥3 h after surfacing	2 h after surfacing
1–3 h after surfacing	30 min after surfacing
20 min to 1 h after surfacing	10 min after surfacing
<20 min after surfacing	Time leaving final stop depth
Before surfacing	Time leaving second previous stop depth

recreation after ~2 h. Some of the consequences of T_1–T_2 assignments are discussed below.

RESULTS

Evaluation of the models is summarized in Table 3. The column labeled "Occurrence Only" uses the overall P(DCS) for the dive and subsequent 24 h for the likelihood function (*Eq. 7*), whereas the column labeled "Time of DCS" uses the probability of developing symptoms between T_1 and T_2 (*Eq. 3*). The magnitude of the likelihood function cannot be compared across columns because they are measures of different events or outcomes.

The Occurrence Only results show that *models 1* and *2* are each a much better description of the data than the corresponding null model. Likelihood ratio tests (4, 16) show statistically significant improvement ($P < 0.001$). This indicates that the inclusion of information into r about the history of the dive definitely improves the ability to predict the probability of DCS. The maximum likelihoods for the two models are very similar and should be considered equally successful. Even the parameter estimates are similar, with the data requiring time constants (τ's) near 40 and 700 min. (For those more accustomed to half times, these are ~30 and 500 min.) As expected, scale factors are smaller in *model 2* because of the additional integration step. Parameter precision for both models is similar, with coefficients of variation (SE/parameter value) in the range of 20–40% for time constants and 30–90% for scale factors.

Column 2 (time of DCS) in Table 3 has no entries for

TABLE 3. *Parameter estimates*

Type of Data	Occurrence Only	Time of DCS
	Null model	
r, min^{-1}	3.91×10^{-5}	3.99×10^{-5}
LL	−203.83	−343.55
	Model 1	
τ_{1A}, min	39.8±20	
AA_1, min^{-1}	1.5±0.3 × 10^{-3}	
τ_{1B}, min	732±210	
AB_1, min^{-1}	8.0±6.8 × 10^{-3}	
LL	−194.95	
	Model 2	
τ_{2A}, min	37.5±13.4	59.5±7.4
AA_2, min^{-2}	1.0±0.3 × 10^{-5}	8.0±1.4 × 10^{-6}
τ_{2B}, min	624±230	871±84
AB_2, min^{-2}	1.3±1.1 × 10^{-5}	3.8±2.5 × 10^{-5}
LL	−194.66	−292.04

Values in *models 1* and *2* are means ± SE. r, constant risk factor; τ, time constant for each model (*1, 2*) and tissue (*A, B*); *A*, scale factor for each model (*1, 2*) and tissue (*A, B*); LL, log-likelihood function.

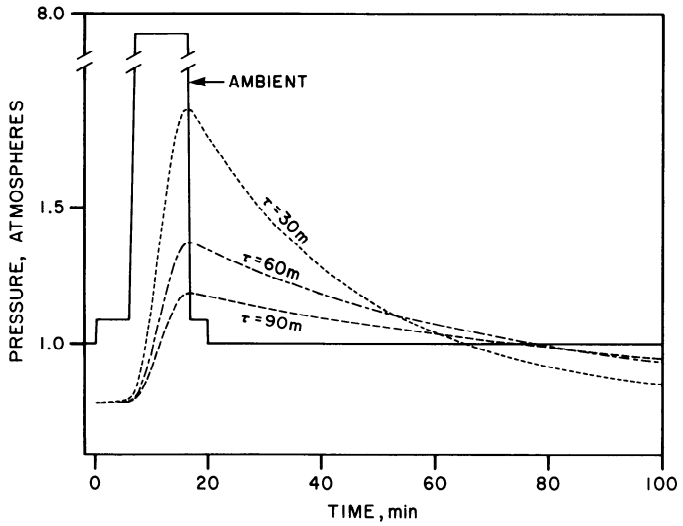

FIG. 3. Exponential gas exchange kinetics on *dive DD1936A*. Solid line, ambient pressure; other lines, washin and washout of inert gas in 3 possible compartments. Rate constants either much larger or much smaller than ~60 min develop a less persistent, or no, supersaturation.

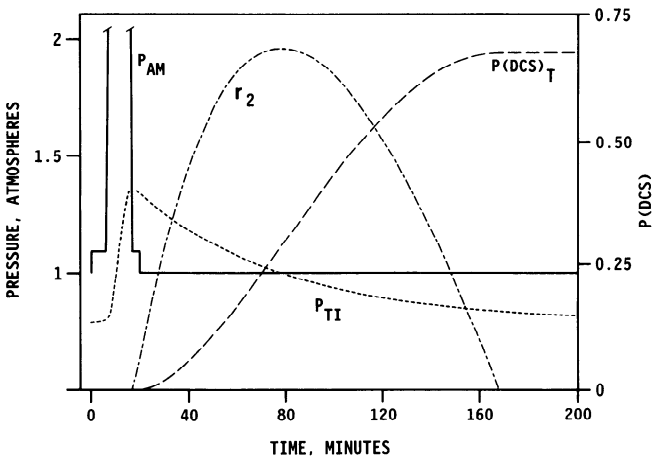

FIG. 4. *Model 2* behavior on *dive DD1936A*. Tissue nitrogen pressure from 59.5-min time constant is shown similar to Fig. 2 (874-min time constant tissue never developed a supersaturation). Scale for r_2 is arbitrary.

model 1 because this model fails to fit the data. This is because no value for a time constant exists that allows for a nonzero risk as late as 2 h after two different dives in which DCS occurred. *Model 1* fails also with one of the marginal cases, with onset >2 h after the dive. According to any parameter estimates we could find, these three cases of DCS should simply not have happened at the time they did. The reason *model 1* was unsuccessful in accounting for the late cases of DCS is illustrated in Fig. 3. For this exposure, a time constant τ_{1A} or τ_{1B} of 60 min causes r_1 to remain positive for ~55 min after the end of the dive, which is ~77 min on the time coordinates of Fig. 3. Faster time constants, like the 30-min example plotted, build up a higher Pti during the dive, but tissue nitrogen washes out to <1 ATA partial pressure in <55 min. Longer time constants (e.g., 90 min) build up tissue nitrogen to a lesser extent so that r_1 again does not persist for 55 min. Because this was such a short dive, even longer time constants do not build up the tissue partial pressures high enough that a positive risk persists as long as 2 h. Therefore no time constant can maintain a positive r_1 in this example >55 min after the dive. One diver reported symptoms after this dive at 10 min after the dive and another at ~12 h after completion. *Model 1* finds this outcome to be impossible.

We had no trouble in fitting the data with the symptom times in *model 2*. Its maximum LL is much better than the constant risk hypothesis (the null model). Time constants and scale factors are rather close to those estimated without the timing information, in most cases within 1 SE. Parameter precision is increased, especially in the case of the time constants for which SEs drop from approximately one-third to one-eighth of the estimated time constants or less. For the previous problem dive, Fig. 4 shows that the 59.7-min time constant maintained a positive r_2 for 140 min after surfacing, with about one-tenth of the total risk incurred >2 h after leaving the water.

Some more detailed information on model performance in predicting time of DCS occurrence is presented in Table 4. Five different (outcome) time categories relative to the surfacing time of the dive were constructed. All the 40 (38 actual and 4 marginal) single dive DCS case outcomes were allocated to those categories by apportioning their T_1–T_2 intervals. Then the total predicted P(DCS) was summed over each time category for the 727 single man-dives in the data. Table 4 shows that the null model fails to agree with the data by placing too high a predicted risk late after the dive. On the other hand, *model 1* fails by placing too much predicted risk near the time of surfacing. *Model 2* (evaluated with time of DCS onset data) agrees much better, usually within one or two cases per time category. The χ^2 test statistics in Table 3 measure the rareness that random variation would lead to the tabulated levels of disagreement. The null model and *model 1* predictions can be positively rejected ($P < 0.001$) as in disagreement with the data, whereas the *model 2* predictions cannot be rejected ($P > 0.2$). Repetition of this exercise, including the repetitive dives (not shown), leads to a similar conclusion, although interpretation is clouded by arbitrary choice of "surfacing time" when multiple choices from the multiple dives are possible.

DISCUSSION

Models 1 and *2* were equally successful in predicting the occurrence of DCS despite quite different risk formulations. This lack of sensitivity to the shape of the risk

TABLE 4. *Predicted and observed DCS onset time intervals*

Time Category	Observed	Predicted		
		Null	*Model 1*, occurrence	*Model 2*, time of DCS
Before surfacing	6.3	3.9	13.6	5.8
Surfacing to +30 min	4.9	0.9	11.6	2.9
Surfacing +30 min to +2 h	8.5	2.6	8.5	12.5
Surfacing +2 h to +4 h	10.6	3.4	3.7	11.3
Surfacing +4 h to +1 day	9.3	33.5	0.4	6.5
χ^2		68.1	197.6	3.0

TABLE 5. *Repetitive diving rules*

	Model 1, occurrence	*Model 2*, occurrence	*Model 2*, time of DCS conditional
First dive, 50 fsw	49 min	71 min	65 min
Second dive, 50 fsw, after 20 min on surface	0 min	8 min	26 min
Second dive, 50 fsw, after 240 min on surface	0 min	0 min	64 min

Entries are times at stated depth allowed for P(DCS) to = 2% without decompression stops, i.e., the longest "safe" durations.

function also helps explain why differently shaped nitrogen washout curves can achieve success. The single-exponential gas exchange kinetics used here, double-exponential descriptions of compartment exchange (18, 19), and the nonlinear kinetics used elsewhere (10) can all fit the same DCS occurrence data well, even though none closely follows the kinetics found in actual gas exchange measurements (17). The use of symptom onset time will allow more sensitive comparisons among competing gas exchange theories.

Model 1 failed by predicting that three cases of DCS could not have occurred as late as they did occur. Analysis of the data with the two models gives us statistical confirmation that DCS symptoms are not heavily clustered immediately after decompression steps but some symptoms seem to occur even after most of the gas has left the tissue. Such a result makes it unlikely that the symptoms are an immediate consequence of bubble nucleation (15) and suggests instead that symptoms arise only after the growth or persistence of a bubble for some reasonably long period of time.

Model failure is critically dependent on the specific choice of T_1 and T_2. If we had assigned a T_1 of much less than 2 h after the dive in Figs. 3 and 4, *model 1* would have predicted a low but nonzero probability for the occurrence of DCS in the enlarged interval. On the other hand, if we had allowed T_1 to be as late as +12 h, *model 2* would also have failed on several dives because risk would be zero that late. The sensitivity of these models to T_1 and T_2 indicates that by keeping careful records of the time of occurrence of DCS, future data may be used to refine our understanding of mechanisms of DCS. Any proposed mechanisms will need to predict not only the occurrence but also the time of occurrence of DCS. For example, models predicting the occurrence of DCS long after surfacing might require gas washout kinetics slower than simple exponential tissue washout (2, 9–11, 18).

APPLICATIONS

Practical use of models that can successfully predict the actual time of occurrence of DCS should be placed in the context of models that can predict occurrence sometime during the 24 h after a dive. The latter can be used to rationally construct decompression tables with a known level of risk for a single preplanned dive, e.g., at a 1 or 5% chance of DCS (14). These tables could be useful but would be a bit constraining. What if a person wanted to perform a second, or even third, dive later? Procedures could be developed to preplan two- and three-dive sequences (so-called repetitive diving) in a similar fashion; however, the decompression from the first dive in the sequence would almost inevitably need to be more conservative than if it were the only dive, because some of the total 5 or 1% risk would have to be allocated to the second (and subsequent) exposures. In addition to the procedures being difficult to implement (many dive combinations to tabulate), the model and parameters used would need serious evaluation against data from repetitive diving combinations.

Many diving situations would benefit from increased flexibility. Suppose you have performed one dive under conditions that bring you close to a selected limit of P(DCS), e.g., 1 or 5%. After several hours you do not exhibit symptoms of DCS and are considering another dive. You have passed through a period when DCS might have occurred, but it did not. With occurrence-only model formulations and calibrations, prudence would dictate that you do not fully believe the degree to which a hazardous interval has passed until the full integration period (e.g., 24 h) has elapsed. However, if the shape of r is better understood by occurrence-time fitting, belief in shorter time hazard accumulation can be strengthened. In fact, if the chosen r is a good approximation of DCS

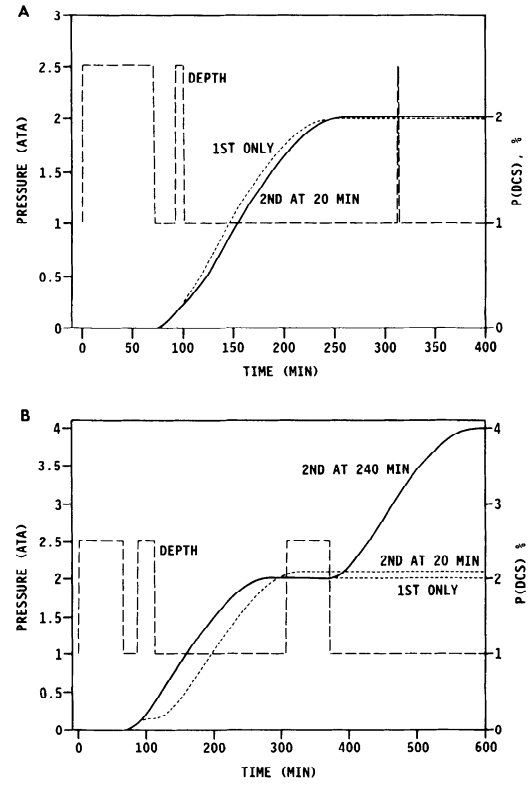

FIG. 5. Conduct of repetitive dives by use of *model 2* without (*A*) and with (*B*) use of conditional probability. In *A*, a single dive to 50 fsw for 71 min leads eventually to a total P(DCS) of 2% (dotted curve and scale at *right*). If diver returned to 50 fsw after 20 min on surface, only 8 min could be spent at depth to keep total P(DCS) from exceeding 2% (solid probability trace). A 2nd dive sought after 4 h on surface would not be allowed because full 2% P(DCS) has already been achieved. Same scenario in *B*, with parameters estimated from time of DCS data and decisions based on conditional probability. Here a 2nd dive after 20 min on surface is allowed for 26 min, with a 2% increase in P(DCS) allowed after commencing 2nd dive. If 2nd dive were performed after 4 h on surface, tissue pressure and r would be almost fully decayed, and 64 min at 50 fsw would be allowed to achieve a conditional P(DCS) of 2%.

mechanisms, the P(DCS) accumulated after 1 h on the surface can be "forgotten" if that 1 h does not produce symptoms.

This idea of fully believing the shape of r can be expressed as conditional probability: what is the future chance of suffering DCS on the condition that I am free of symptoms now? The question is formally equivalent to *Eq. 3* in setting $P(s)_{T_1} = 1.0$ and putting T_2 into the far future, say 1 day later. Implementation of conditional probability is a rational strategy to deal with repetitive diving.

An example of conditional probability is given in Table 5 and Fig. 5 with the models evaluated in Table 3. Rules are presented for a diver wishing a dive of the longest possible duration to 50 fsw requiring no decompression stops and allowing 2% chance of developing DCS after the dive. Depending on the model invoked, the longest "safe" duration is 49–71 min. Afterwards, he may wish to perform a second dive without decompression stops to 50 fsw after waiting on the surface for 20 min or 4 h. Without using conditional probability, Table 5 shows that he is greatly limited in allowed time or completely prohibited from the second dive. In other words, he cannot make both the first and second dives without exceeding the target DCS risk of 2% for the combination of dives. With *model 1* it can be shown that no combination of two dives to the same or deeper depth and immediate ascent has a lower P(DCS) than the first dive alone. Even with *model 2*, the allowed time on the second dive is curtailed, more so the longer the diver waits to start the second dive. Figure 5*A* shows how the two-dive combinations add up to the target 2% chance of DCS.

However, at the start of the second dive, the diver may be allowed to run a risk on his second dive similar to that of his first; he is not as interested in his total risk exposure for the combination. If he is free of symptoms from his first dive, he can accept a new 2% risk of DCS for the second. Data in the last column in Table 4 and Fig. 5*B* are given on the basis of that condition. From the start of the second dive, whenever it occurs, a new 2% chance of DCS ensues. With *model 2* parameters evaluated with time of DCS and under conditional probability, he is restricted in allowed time of the second dive, but the restriction gradually disappears. As both Pti and r diminish, the diver's status gradually and smoothly returns to his original state.

If the shape of r is well known, it might be used to calculate conditional probabilities of proposed decompression strategies developed and adjusted during the dive itself. This would be particularly desirable in situations in which a dive history involves many depths at various time intervals. Our present tables are really most reliable when the dive is not too complicated. Examples of complex dive patterns occur in military operations and in recreational diving. Any ad hoc decompression strategy using conditional probability is made on the assumption that the only history of the past that is required is accurately summarized by a knowledge of the inert gas remaining in the tissue and the current values of r and that the risk incurred in the future does not depend in any other way on the events of the past. If, for example, DCS depends simply on a gas bubble reaching a critical size (2, 11), and a second dive that reduces and perhaps eliminates the bubble in question is begun before that critical size is reached, then perhaps the assumption of independence is well founded and the future risk when decompression begins is calculable with the same risk function used on the first dive. Suppose, on the other hand, that DCS is caused by complement activation on the tissue-gas interface of the bubble (12). In this case, perhaps the risk of the second dive will not be characterized only by the gas remaining in the tissue but also by the degree of complement activation and consumption. Second or complex dives with interspersed partial decompression might then carry more or less risk than one could anticipate from a knowledge of gas burden and a risk function determined primarily from simple dives. Collection of data on repeat dives with accurately known T_1 and T_2 values and narrow T_2–T_1 intervals should help us refine the estimates of r and decide whether it is dependent only on the balance of gas remaining in tissue or on other factors as well.

The model and parameters presented here are not yet sufficiently developed for general diving use. More data on the time of occurrence of DCS must be obtained. At least some of these data should involve complex dives with interspersed partial decompression, so that the reliability of using conditional probability to plan decompression strategies can be evaluated. The analysis presented here provides a logical framework for model development and evaluation leading to real-time control of DCS risk in a complex diving situation.

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REFERENCES

1. BAILEY, R. C., AND L. D. HOMER. *An Analogy Permitting Maximum Likelihood Estimation by a Simple Modification of General Least Squares Algorithms.* Bethesda, MD: Naval Med. Res. Inst., 1977. (Tech. Rep. 77-55)
2. HILLS, B. A. *Decompression Sickness. The Biophysical Basis of Prevention and Treatment.* Chichester, UK: Wiley, 1977, vol. 1, p. 100–104.
3. KALBFLEISCH, J. D., AND R. L. PRENTICE. *The Statistical Analysis of Failure Time Data.* New York: Wiley, 1980, chapt. 1, p. 1–10.
4. KENDALL, M. G., AND A. STUART. *The Advanced Theory of Statistics* (4th ed.). London: Haffner, 1979, vol. 2, p. 38–180.
5. LAUCKNER, G. R., R. Y. NISHI, AND B. C. EATOCK. *Evaluation of the DCIEM 1983 Decompression Model for Compressed Air Diving (Series A-F).* North York, Ontario: Defence Civil Inst. Environ. Med., 1984. (DCIEM Rep. 84-R-72)
6. LAUCKNER, G. R., R. Y. NISHI, AND B. C. EATOCK. *Evaluation of the DCIEM 1983 Decompression Model for Compressed Air Diving (Series L-Q).* North York, Ontario: Defence Civil Inst. Environ. Med., 1985. (DCIEM Rep. 85-R-18)
7. MARQUARDT, D. W. An algorithm for least-squares estimation of nonlinear parameters. *J. Soc. Ind. Appl. Math.* 11: 431–441, 1963.
8. NISHI, R. Y., B. C. EATOCK, I. P. BUCKINGHAM, AND B. A. RIDGEWELL. *Assessment of Decompression Profiles by Ultrasonic Monitor-*

ing. Phase III. No-Decompression Dives. North York, Ontario: Defence Civil Inst. Environ. Med., 1982. (DCIEM Rep. 82-R-38)
9. THALMANN, E. D. *Air-N_2O_2 Decompression Computer Algorithm Development.* Panama City, FL: US Navy, 1986. (Exp. Diving Unit Rep. 8-85)
10. TIKUISIS, P., R. Y. NISHI, AND P. K. WEATHERSBY. Use of the likelihood method in the analysis of chamber air dives. *Undersea Biomed. Res.* 15: 310–313, 1988.
11. VAN LIEW, H. D., AND M. P. HLASTALA. Influence of bubble size and blood perfusion on absorption of gas bubbles in tissue. *Respir. Physiol.* 7: 111–119, 1969.
12. WARD, C. A, D. MCCULLOUGH, AND W. D. FRASER. Relation between complement activation and susceptibility to decompression sickness. *J. Appl. Physiol.* 62: 1160–1166, 1987.
13. WEATHERSBY, P. K., B. L. HART, E. T. FLYNN, AND W. F. WALKER. Role of oxygen in the production of human decompression sickness. *J. Appl. Physiol.* 63: 2380–2387, 1987.
14. WEATHERSBY, P. K., J. R. HAYS, S. S. SURVANSHI, L. D. HOMER, B. L. HART, E. T. FLYNN, AND M. E. BRADLEY. *Statistically Based Decompression Tables. II. Equal Risk Air Diving Decompression Schedules.* Bethesda, MD: Naval Med. Res. Inst., 1985. (Tech. Rep. 85–17)
15. WEATHERSBY, P. K., L. D. HOMER, AND E. T. FLYNN. Homogenous nucleation of gas bubbles in vivo. *J. Appl. Physiol.* 53: 940–946, 1982.
16. WEATHERSBY, P. K., L. D. HOMER, AND E. T. FLYNN. On the likelihood of decompression sickness. *J. Appl. Physiol.* 57: 815–824, 1984.
17. WEATHERSBY, P. K., K. G. MENDENHALL, E. E. P. BARNARD, L. D. HOMER, S. SURVANSHI, AND F. VIERAS. The distribution of xenon gas exchange rates in dogs. *J. Appl. Physiol.* 50: 1325–1336, 1981.
18. WEATHERSBY, P. K., S. S. SURVANSHI, L. D. HOMER, B. L. HART, R. Y. NISHI, E. T. FLYNN, AND M. E. BRADLEY. *Statistically Based Decompression Tables. I. Analysis of Standard Air Dives: 1950–1970.* Bethesda, MD: Naval Med. Res. Inst., 1985. (Tech. Rep. 85–16)
19. WEATHERSBY, P. K., S. S. SURVANSHI, AND R. Y. NISHI. Relative decompression risk of dry and wet chamber air dives. *Undersea Biomed. Res.* 17: 333–352, 1990.