Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 51-29 (2018) 86–91

Preliminary results of a dynamic modelling approach for underwater multi-hull vehicles ⋆

Roberta Ingrosso, **Daniela De Palma**, **Giovanni Indiveri**, and **Giulio Avanzini**.

Dipartimento Ingegneria Innovazione, University of Salento - ISME Node - Lecce, Italy (e-mail: name.lastname@unisalento.it).

Abstract: A dynamic modelling approach is presented to compute the lumped parameter hydrodynamic coefficients of an underwater vehicle conceived as a multi-body underwater system. The vehicle-base is composed by heterogeneous robots and bodies (both actuated or not actuated), rigidly connected giving rise to a multi-body system called "cluster" in the paper. In order to model the nonlinear dynamics of the cluster, a modular approach has been proposed based on a proper composition of the dynamic models of the individual elements.

Keywords: Marine systems, autonomous vehicles, multi-body vehicles, dynamic modelling.

1. INTRODUCTION

Raw materials are of paramount importance to produce most goods employed in everyday life. In recent years, there has been a growing interest in moving to the deep sea for the research of new deposits of raw materials, however exploration technologies are expensive and environmentally unfriendly.

In order to reduce the cost of mineral exploration at sea currently performed by Remotely Operated Vehicles (ROV) and dedicated Surface Support Vessels (SSV) with crew, there is a need to develop an autonomous, reliable, and cost effective technology to map vast terrains in terms of mineral and raw material contents. Furthermore, there is a need to identify the most rich mineral sites in an efficient manner and with minimum impact to the environment.

The ROBUST project Simetti et al. (2017), funded by the EU commission under the Horizon 2020 programme, aims to tackle the aforementioned issue by developing sea bed in-situ material identification through the fusion of two technologies, namely laser-based element-analysing capability merged with Autonomous Underwater Vehicle (AUV) technologies for sea bed 3D mapping. The underwater robotic laser process is the Laser Induced Breakdown Spectroscopy (LIBS), used for identification of materials on the sea bed.

The ROBUST underwater robotic system is required to perform inspection operations with high accuracy and with good reactivity properties for operational efficiency. Therefore, a very accurate control at both the Dynamic Control Layer (DCL) and the upper Kinematic Control Layer (KCL) is needed. The two control layers, and in particular DCL should be consequently based on an accurate model of the overall system, for guaranteeing the adequate level of control accuracy.

The skeleton of the ROBUST robotic platform, sketched in figure 1, has been conceived with a modular approach, by composition of four basic units: three torpedo-shaped AUVs connected with a rigid frame and an internal structure containing sensing, processing, and communication equipment (ROBUST payload) necessary for the execution of the reference mission.

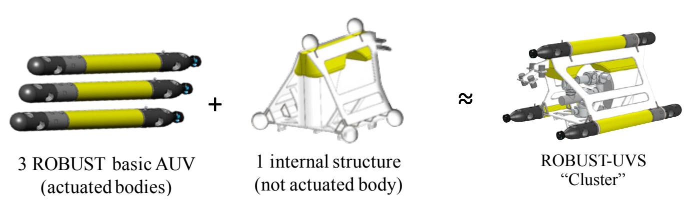

Fig. 1. ROBUST underwater vehicle system.

It should be noted that the ROBUST basic AUV is similar to the Folaga AUV Alvarez et al. (2009), Caffaz et al. (2010) having a known model. Indeed the main objective of the research described in the paper is to derive a model for the ROBUST Underwater Vehicle System (ROBUST UVS), i.e. "cluster", building on the knowledge of the models of each single basic AUV.

Starting from the modelling of generic underwater vehicles, as in Fossen (2011) and Antonelli (2014), the challenge for a multi-body cluster is to be able to describe the nonlinear dynamics of the cluster from the dynamics of individual elements.

Related results in the literature include papers by Zhang and Wang (2007), Ke et al. (2013, 2014), Park and Kim (2015), Abreu et al. (2016) that have studied multi-body dynamics methods for modelling underwater vehicles. Kamman and Huston (1985, 2001) address the study

⋆ This work was partially supported by the European Union's Horizon 2020 research and innovation programme under the project ROBUST: Robot subsea exploration technologies, grant agreement N°690416 (call H2020-SC5-2015-one-stage).

10.1016/j.ifacol.2018.09.474

of multi-body dynamics of tether cables for underwater applications.

A method for dynamic modelling of multi-body systems, where the generalized forces that contribute to dynamics are determined by Kane's approach, is presented in Ke et al. (2013, 2014). The use of generalized forces in Kane dynamic equations offers advantages over Newton-Euler and Lagrange methods for the computation of multi-body dynamics, because the interaction and constraint forces among bodies is eliminated. The same method is used in Abreu et al. (2016) where a dynamic simulation model for a coupled streamer-vehicle system is proposed for the cooperative and navigation control problem for fleets of streamer-vehicle systems.

Nielsen et al. (2016b,a) investigates the dynamic modelling of reconfigurable underwater systems building on the Udwadia-Kalaba Equation (see Udwadia and Schutte (2011)): the equations of motion for a system comprised of N rigidly connected robots are developed using quasi-velocities to derive the constraints imposed by rigid connections.

The rest of the paper is organized as follows: Section 2 introduces the notation and describes the main tools for the transformation of the generalized forces and moment. Section 3 addresses the problem formulation and the description of the equation of motion for the cluster. In Section 4 the cluster model is derived building on the knowledge of the single body models. Numerical results are reported in Section 5. Finally, concluding remarks are reported in Section 6.

2. PRELIMINARIES AND NOTATION

In order to derive the model for the ROBUST UVS, the following are defined (also refer to Figure 2):

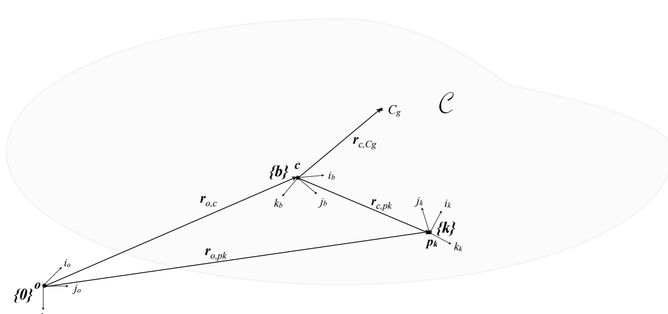

Fig. 2. Adopted frames and notation.

- $\{0\}$: inertial NED (North-East-Down) earth-fixed reference frame with origin in $o \in \mathbb{R}^3$;
- $\{b\}$: cluster-fixed reference frame with origin in $c \in \mathbb{R}^3$. Point c is chosen as pole for forces and moments;
- $\{k\}$: k th body-fixed reference frame ($k = 1, \ldots, N$) with origin in a point $p_k \in \mathbb{R}^3$ of the k th body. Point p_k is chosen as pole for forces and moments.

Moreover, the following notation will be adopted for vectors in the coordinate systems $\{0\}$, $\{b\}$, $\{k\}$:

$^b\mathbf{r}_{c,p_k}$ = position vector from c to p_k expressed in frame $\{0\}$;
$^0\mathbf{v}_{p_k/o}$ = linear velocity of the point p_k with respect to o expressed in $\{0\}$;
$^b\boldsymbol{\omega}_{b/0}$ = angular velocity of $\{b\}$ with respect to $\{0\}$ expressed in $\{b\}$;
$^b\boldsymbol{\nu}_{c/o}$ = generalized velocity of c with respect to o expressed in $\{b\}$;
$^k_{p_k}\mathbf{f}$ = force with line of action through p_k expressed in $\{k\}$;
$^b_c\mathbf{m}$ = moment about the point c expressed in $\{b\}$;
$^0\boldsymbol{\eta}_b$ = Euler angles between $\{b\}$ and $\{0\}$;
$^b\dot{\boldsymbol{\nu}}_{c/o} = \frac{d}{dt}{}^b\boldsymbol{\nu}_{c/o}$ time derivative of $^b\boldsymbol{\nu}_{c/o}$;
C_g = cluster center of gravity.

The symbol $S(\cdot) \in \mathbb{R}^{3\times3}$ denotes the skew symmetric matrix associated to the cross product $\mathbf{a} \times \mathbf{b} = S(\mathbf{a})\mathbf{b}$ for any $\mathbf{a}, \mathbf{b} \in \mathbb{R}^{3\times3}$.

From Figure 2 it follows that

$$^0\mathbf{r}_{o,p_k} = {}^0\mathbf{r}_{o,c} + {}^0R_b{}^b\mathbf{r}_{c,p_k}, \tag{1}$$

where $^0R_b \in SO(3)$ is the rotation matrix between frame $\{b\}$ and $\{0\}$. Time differentiation of (1) gives the velocity of p_k (origin of k th frame $\{k\}$) with respect to o (origin of frame $\{0\}$) expressed in $\{0\}$, that is:

$$^0\mathbf{v}_{p_k/o} = \frac{d}{dt}{}^0\mathbf{r}_{o,p_k} = {}^0\mathbf{v}_{c/o} + {}^0\boldsymbol{\omega}_{b/0} \times {}^0\mathbf{r}_{c,p_k}. \tag{2}$$

Yet, from time differentiation of (2) it follows that:

$$^0\mathbf{a}_{p_k/o} = \frac{d}{dt}{}^0\mathbf{v}_{c/o} + \left(\frac{d}{dt}{}^0\boldsymbol{\omega}_{b/0}\right) \times {}^0\mathbf{r}_{c,p_k} + + {}^0\boldsymbol{\omega}_{b/0} \times ({}^0\boldsymbol{\omega}_{b/0} \times {}^0\mathbf{r}_{c,p_k}). \tag{3}$$

In the following, we compute how to transform generalized forces and moments between different reference frames.

2.1 *Generalized velocity vectors in different reference frames*

The generalized velocity $^k\boldsymbol{\nu}_{p_k/o}$ of p_k of the k th body with respect to o expressed in $\{k\}$, is denoted as:

$$^k\boldsymbol{\nu}_{p_k/o} := \begin{bmatrix} ^k\mathbf{v}_{p_k/o} \\ ^k\boldsymbol{\omega}_{k/0} \end{bmatrix}. \tag{4}$$

In order to derive the transformation between $^k\boldsymbol{\nu}_{p_k/o}$ and $^b\boldsymbol{\nu}_{c/o}$ we first need to project the vectors in the same reference $\{b\}$. Defining the following matrix $^b\bar{R}_k \in SO(6)$:

$$^b\bar{R}_k := \begin{bmatrix} ^bR_k & 0_{3\times3} \\ 0_{3\times3} & ^bR_k \end{bmatrix} \in \mathbb{R}^{6\times6}, \tag{5}$$

being ^bR_k the rotation matrix from $\{k\}$ to $\{b\}$, the generalized velocity of p_k will be:

$$^b\boldsymbol{\nu}_{p_k/o} = \begin{bmatrix} ^b\mathbf{v}_{p_k/o} \\ ^b\boldsymbol{\omega}_{k/0} \end{bmatrix} = {}^b\bar{R}_k{}^k\boldsymbol{\nu}_{p_k/o}. \tag{6}$$

Note that, since all points of a rigid body have the same angular velocity, the assumption of a rigid cluster implies that $^b\boldsymbol{\omega}_{k/0} = {}^b\boldsymbol{\omega}_{b/0}$.

Now, the transformation between $^b\boldsymbol{\nu}_{p_k/o}$ and $^b\boldsymbol{\nu}_{c/o}$ can be expressed as follows:

$$^b\boldsymbol{\nu}_{p_k/o} = T({}^b\mathbf{r}_{c,p_k}){}^b\boldsymbol{\nu}_{c/o}, \tag{7}$$

where $T({}^b\mathbf{r}_{c,p_k})$ is given by:

$$T({}^b\mathbf{r}_{c,p_k}) = \begin{bmatrix} I_{3\times3} & -S({}^b\mathbf{r}_{c,p_k}) \\ 0_{3\times3} & I_{3\times3} \end{bmatrix}. \quad (8)$$

Summarizing

$${}^k\boldsymbol{\nu}_{p_k/o} = {}^k\bar{R}_b T({}^b\mathbf{r}_{c,p_k}){}^b\boldsymbol{\nu}_{c/o}. \quad (9)$$

Similarly it can be shown that time differentiating (6) leads to the following:

$${}^k\dot{\boldsymbol{\nu}}_{p_k/o} = {}^k\bar{R}_b T({}^b\mathbf{r}_{c,p_k}){}^b\dot{\boldsymbol{\nu}}_{c/o}. \quad (10)$$

2.2 Generalized forces in different reference frames

Letting ${}^k_{p_k}\boldsymbol{\tau}_k$ be the generalized forces vector acting on the kth body about p_k expressed in $\{k\}$:

$${}^k_{p_k}\boldsymbol{\tau}_k = \begin{bmatrix} {}^k_{p_k}\mathbf{f}_k \\ {}^k_{p_k}\mathbf{m}_k \end{bmatrix}. \quad (11)$$

The generalized forces can be expressed in the cluster-fixed reference frame $\{b\}$ using the matrix ${}^b\bar{R}_k$, previously defined:

$${}^b_{p_k}\boldsymbol{\tau}_k = {}^b\bar{R}_k \; {}^k_{p_k}\boldsymbol{\tau}_k. \quad (12)$$

The transformation of the generalized forces between the two points p_k and c in the frame $\{b\}$ can be derived as follows:

$${}^b_c\boldsymbol{\tau}_k = T^\top({}^b\mathbf{r}_{c,p_k}) \; {}^b_{p_k}\boldsymbol{\tau}_k. \quad (13)$$

Hence, the generalized forces about the point c expressed in $\{b\}$, that is ${}^b_c\boldsymbol{\tau}_k$, can be written as function of the generalized forces about the point p_k expressed in $\{k\}$, ${}^b_{p_k}\boldsymbol{\tau}_k$ through the following transformation:

$${}^b_c\boldsymbol{\tau}_k = T^\top({}^b\mathbf{r}_{c,p_k}) \; {}^b\bar{R}_k \; {}^k_{p_k}\boldsymbol{\tau}_k. \quad (14)$$

3. CLUSTER EQUATIONS OF MOTION

The goal of this section is to describe the kinematics and dynamics of the multi-body system.

3.1 Cluster Kinematic equations

The 6 DOF kinematic equations of the cluster can be expressed as follows:

$${}^0\dot{\mathbf{r}}_{o,c} = {}^0R_b{}^b\mathbf{v}_{c/o} \quad (15)$$

$${}^b\dot{R}_0 = -S({}^b\boldsymbol{\omega}_{b/0}){}^bR_0 \quad (16)$$

where ${}^0\mathbf{r}_{o,c} = [x, y, z]^\top$ denotes the North-East-Down position of the cluster in frame $\{0\}$, ${}^b\mathbf{v}_{c/o} = [u, v, w]^\top$ and ${}^b\boldsymbol{\omega}_{b/0} = [p, q, r]^\top$ are the cluster-fixed linear and angular velocity vectors, respectively. Equation (15) describes the translational motion and (16) describes the rotational motion (i.e. kinematics of the rotation matrix).

It is worth highligthing that the standard kinematic equations for the rotational motion used, for example, in Fossen (2011), make use of the Euler angles as a parametrization of $SO(3)$. It is known that any minimal rotation matrix parametrization is bound to be singular.

The formulation proposed in (16) have the advantage to be derived without needing any specific parametrization of $SO(3)$, hence avoiding all the issues related to minimal representation singularities. Of course, if needed, the Euler angles $({}^0\boldsymbol{\Phi}_b)$ could always be derived from the rotation matrix ${}^bR_0$ as reported in Siciliano and Khatib (2008).

3.2 Cluster Dynamic equations

As shown in Fossen (2011, 2012), the standard lumped parameter model used in most robotics applications is given by:

$$M_{rb}{}^b\dot{\boldsymbol{\nu}}_{c/o} + C_{rb}({}^b\boldsymbol{\nu}_{c/o}){}^b\boldsymbol{\nu}_{c/o} = {}^b_c\boldsymbol{\tau}_{rb}. \quad (17)$$

Equation (17) represents the Newton-Euler dynamic equation of motion expressed in an arbitrary cluster-fixed coordinate frame where ${}^b\boldsymbol{\nu}_{c/o} := [{}^b\mathbf{v}^\top_{c/o} \; {}^b\boldsymbol{\omega}^\top_{b/0}]^\top$ is the linear and angular velocity vector projected in the cluster frame, ${}^b_c\boldsymbol{\tau}_{rb}$ is a generalized vector of external forces and moments, M_{rb} is the rigid-body inertia matrix, and C_{rb} is the rigid-body Coriolis and centripetal matrix.

The generalized vector of external forces and moments ${}^b_c\boldsymbol{\tau}_{rb}$ is given by:

$${}^b_c\boldsymbol{\tau}_{rb} = {}^b_c\boldsymbol{\tau}_{dp} + {}^b_c\boldsymbol{\tau}_{drag} + {}^b_c\boldsymbol{\tau}_{rf} + {}^b_c\boldsymbol{\tau}_E + {}^b_c\boldsymbol{\tau} + {}^b_c\boldsymbol{\tau}_L, \quad (18)$$

where

- ${}^b_c\boldsymbol{\tau}_{dp}$ is the vector of dynamic pressure forces and moments on a rigid body;
- ${}^b_c\boldsymbol{\tau}_{drag}$ is the vector of viscous drag effects forces and moments on a rigid body;
- ${}^b_c\boldsymbol{\tau}_{rf}$ is the vector of restoring (gravitational and buoyancy) forces and moments on a rigid body;
- ${}^b_c\boldsymbol{\tau}_E = {}^b_c\boldsymbol{\tau}_{wave} + {}^b_c\boldsymbol{\tau}_{wind}$ is the vector of enviromental forces and moments on a rigid body (it will be considered negligible in the following);
- ${}^b_c\boldsymbol{\tau}$ is the vector of propulsion forces and moments;
- ${}^b_c\boldsymbol{\tau}_L$ is the vector of lifting forces and moments.

4. CLUSTER MODEL BUILDING ON THE SINGLE BODY ONE

In this section we will derive a model for the cluster building on the knowledge of the single body one. As already highlighted in the previous section, the inertia, lifting, damping, restoring and propulsion (if any) forces of all bodies can be expressed in a common reference frame $\{b\}$ making use of the cluster velocity $\boldsymbol{\nu}_{c/o}$ only. This allows to specify the generalized vector of external forces for the cluster in equation (17) as the sum of the individual contributions of the N rigidly connected heterogeneous robots/bodies, i.e.

$$M_{rb}{}^b\dot{\boldsymbol{\nu}}_{c/o} + C_{rb}({}^b\boldsymbol{\nu}_{c/o}){}^b\boldsymbol{\nu}_{c/o} =$$
$$= \sum_{k=1}^{N}({}^b_c\boldsymbol{\tau}_{dp_k} + {}^b_c\boldsymbol{\tau}_{drag_k} + {}^b_c\boldsymbol{\tau}_{rf_k} + {}^b_c\boldsymbol{\tau}_k + {}^b_c\boldsymbol{\tau}_{L_k}). \quad (19)$$

The sum on the right hand side of equation (19) includes all the generalized forces and moments (projected in $\{b\}$) acting on the individual bodies composing the cluster. The contribution given by the propulsion forces and moments ${}^b_c\boldsymbol{\tau} = \sum_{k=1}^{N} {}^b_c\boldsymbol{\tau}_k$ can be rewritten as:

$$ {}^b_c\boldsymbol{\tau} = B_{cluster}\, U, \tag{20}$$

with

$$B_{cluster} = [B_1^* \cdots B_N^*], \tag{21}$$

$$B_k^* = T^\top({}^b\mathbf{r}_{c,p_k})\, {}^b\bar{R}_k\, B_k, \tag{22}$$

and U the juxtaposition of the thruster input vectors: $U = [\mathbf{u}_1 \ldots \mathbf{u}_N]^\top$. As a result, the effects of the control input of each actuated robot are directly mapped on the cluster motion.

Building on the transformations described in the previous subsections 2.1 and 2.2, the terms on the right hand side of equation (19) can be expressed as a function of the cluster generalized velocity and acceleration in place of the individual body ones. The detailed computation of how each and every term is transformed is here omitted for the sake of brevity. The overall result of such approach is nevertheless reported in equation (23),

$$\begin{aligned} &[M_{rb} + M_{A\,cluster}]\,{}^b\dot{\boldsymbol{\nu}}_{c/o} + \\ &[C_{rb}({}^b\boldsymbol{\nu}_{c/o}) + C_{A\,cluster}({}^b\boldsymbol{\nu}_{c/o})]\,{}^b\boldsymbol{\nu}_{c/o} + \\ &+ (D_{l\,cluster} + D_{q\,cluster}({}^b\boldsymbol{\nu}_{c/o}))\,{}^b\boldsymbol{\nu}_{c/o} + \\ &+ \tau_{rf\,cluster} + \tau_{L\,cluster} = B_{cluster}\, U. \end{aligned} \tag{23}$$

The terms $M_{A\,cluster}$, $C_{A\,cluster}$, $D_{l\,cluster}$, $D_{q\,cluster}$, $\tau_{rf\,cluster}$, $\tau_{L\,cluster}$, and $B_{cluster}$, are reported in table 1.

The advantage of this approach with respect to the alternative Udwadia-Kalaba formulation Udwadia and Schutte (2011) is the derivation of a dynamic-hydrodynamic model for the whole multi-body system without considering explicitly the constraints imposed by the rigid connections. The proposed approach allows expressing directly the motion of the cluster using the six degrees of freedom (dof) equation (23), rather than using a vector of quasi-velocities $\in \mathbb{R}^{6N}$ as in Nielsen et al. (2016b,a). Indeed, the overall system has just 6 dof in the 3D space. Moreover, the use of the cluster allocation matrix $B_{cluster}$ in equation (21) has the benefit of mapping directly the control input of each actuated robot on the cluster. This is an important feature as it allows to facilitate the design of dynamic controllers for the overall system.

5. SIMULATIONS

A numerical simulator (in Matlab) has been developed implementing the described modelling approach for the ROBUST UVS. Tables 2 and 3 report a comparison between the added masses and linear drag coefficients of the AUVs, the central ROBUST payload module structure composing the cluster, and the cluster itself.

Moreover the derived model is numerically integrated to simulate the motion of the cluster along a specific trajectory as depicted in figure 3. As an example of how the hydrodynamic forces and moments are being composed, the surge, sway, and yaw drag forces acting on one of the AUVs, the payload module structure, and the cluster are compared in figure 4.

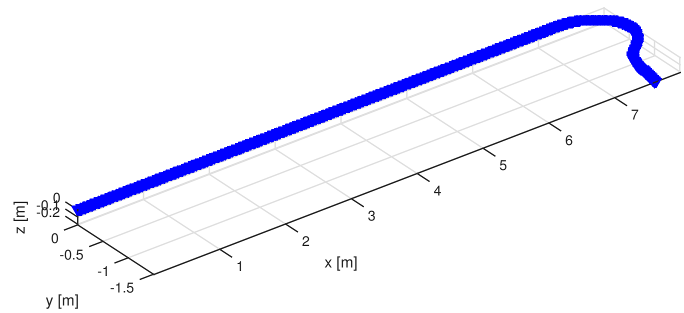

Fig. 3. Cluster trajectory.

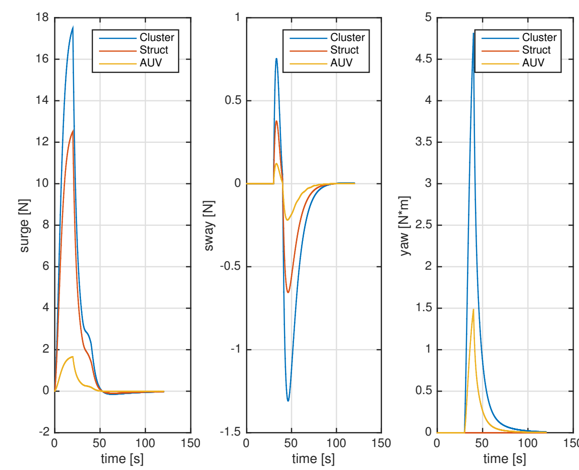

Fig. 4. Surge, sway drag forces, and yaw moment during the execution of the trajectory in Fig. 3.

6. CONCLUSION

A modelling approach has been implemented to compute the lumped parameter hydrodynamic coefficients of a multi-body underwater vehicle system. The approach has been applied to the ROBUST robotic platform. The proposed approach appears to be particularly well suited for motion control and navigation filter design as it allows to derive a standard 6 dof dynamic model exploiting the knowledge of the components of the cluster. Alternative multi-body modelling approaches may result in high dimensional models that cannot be easily employed for control and navigation design.

A simple numerical simulation of the complete ROBUST system using the developed modelling approach is reported.

7. ACKNOWLEDGEMENTS

The authors are grateful to the H2020 ROBUST (ID: 690416) project partner Graal Tech S.r.l. for having provided the ROBUST cluster model details and images.

REFERENCES

Abreu, P., Morishita, H., Pascoal, A., Ribeiro, J., and Silva, H. (2016). Marine vehicles with streamers for geotechnical surveys: Modeling, positioning, and control. In *Proceedings of the 10th IFAC Conference on*

Table 1. Inertia, hydrodynamic, hydrostatic and propulsion parameters for the single body and the cluster system.

	$\mathbf{k^{th}}$ **body** $_{\mathbf{k=1,...N}}$	**Cluster**
Added mass terms	M_{A_k}	$M_{A\,cluster} = \sum_{k=1}^{N} (T^\top(^b\mathbf{r}_{c,p_k})\ \bar{M}_{A_k}\ T(^b\mathbf{r}_{c,p_k}))$, $\bar{M}_{A_k} = {}^b\bar{R}_k M_{A_k} {}^k\bar{R}_b$
	$C_{A_k}(^k\boldsymbol{\nu}_{p_k/o})$	$C_{A\,cluster}(^b\boldsymbol{\nu}_{c/o}) = \sum_{k=1}^{N} T^\top(^b\mathbf{r}_{c,p_k})\ \bar{C}_{A_k}(^b\boldsymbol{\nu}_{c/o}) T(^b\mathbf{r}_{c,p_k})$, $\bar{C}_{A_k}(^b\boldsymbol{\nu}_{c/o}) = \begin{bmatrix} S(^b\boldsymbol{\omega}_{b/0}) & 0_{3\times3} \\ S(^b\mathbf{v}_{c/o} + {}^b\boldsymbol{\omega}_{b/0} \times {}^b\mathbf{r}_{c,p_k}) & S(^b\boldsymbol{\omega}_{b/0}) \end{bmatrix} \bar{M}_{A_k}$
Damping terms	${}^k_{p_k}D_{k_l}$	$D_{l\,cluster} = \sum_{k=1}^{N} (T^\top(^b\mathbf{r}_{c,p_k}) {}^b\bar{R}_k\ {}^k_{p_k}D_{k_l}\ {}^k\bar{R}_b {}^b T(^b\mathbf{r}_{c,p_k}))$
	${}^k_{p_k}D_{k_q}(^k\boldsymbol{\nu}_{p_k/o})$	$D_{q\,cluster}(^b\boldsymbol{\nu}_{c/o}) = \sum_{k=1}^{N} (T^\top(^b\mathbf{r}_{c,p_k}) {}^b\bar{R}_k\ {}^k_{p_k}D_{k_q}(^b\boldsymbol{\nu}_{c/o}) {}^k\bar{R}_b {}^b T(^b\mathbf{r}_{c,p_k}))$
Lifting terms	${}^k_{p_k}\boldsymbol{\tau}_{L_k}$	$\tau_{L\,cluster} = \sum_{k=1}^{N} (T^\top(^b\mathbf{r}_{c,p_k})\ {}^b\bar{R}_k\ {}^k_{p_k}\boldsymbol{\tau}_{L_k})$
Restoring terms	${}^k_{p_k}\boldsymbol{\tau}_{rf_k}$	$\tau_{rf\,cluster} = \sum_{k=1}^{N} (T^\top(^b\mathbf{r}_{c,p_k})\ {}^b\bar{R}_k\ {}^k_{p_k}\boldsymbol{\tau}_{rf_k})$
Propulsion terms	B_k	$B_{cluster} = \begin{bmatrix} B_1^* ... B_N^* \end{bmatrix}$

Table 2. Added Mass coefficients ROBUST single-body vs ROBUST cluster

	Units	**AUV**	**Struct**	**Cluster**
$X_{\dot{u}}$	$[Kg]$	0.47	215.2	216.62
$Y_{\dot{v}}$	$[Kg]$	22.7	777.2	845.3
$Z_{\dot{w}}$	$[Kg]$	22.7	896.7	964.81
$K_{\dot{p}}$	$[Kg\ m^2]$	0.1	48.9	67.6
$M_{\dot{q}}$	$[Kg\ m^2]$	3.64	42.6	53.7
$N_{\dot{r}}$	$[Kg\ m^2]$	3.64	45.1	56.21

Table 3. Linear drag coefficients ROBUST single-body vs ROBUST cluster

	Units	**AUV**	**Struct**	**Cluster**
X_u	$[Kg/s]$	1.08	8.13	11.37
Y_v	$[Kg/s]$	10.21	30.7	61.3
Z_w	$[Kg/s]$	10.21	34.65	65.28
K_p	$[Kg\ m^2/s]$	$3 \cdot 10^{-3}$	*neglected*	8.27
M_q	$[Kg\ m^2/s]$	1.06	*neglected*	3.62
N_r	$[Kg\ m^2/s]$	1.06	*neglected*	3.62

Control Applications in Marine Systems, CAMS 2016, volume 49, 458 – 464. doi:http://dx.doi.org/10.1016/j.ifacol.2016.10.448. URL http://www.sciencedirect.com/science/article/pii/S2405896316320365.

Alvarez, A., Caffaz, A., Caiti, A., Casalino, G., Gualdesi, L., Turetta, A., and Viviani, R. (2009). Folaga: A low-cost autonomous underwater vehicle combining glider and AUV capabilities. *Ocean Engineering*, 36, 24–38. doi:10.1016/j.oceaneng.2008.08.014. URL http://dx.doi.org/10.1016/j.oceaneng.2008.08.014.

Antonelli, G. (2014). *Underwater Robots. Springer Tracts in Advanced Robotics.*, chapter Modelling of Underwater Robots., 23–63. Cham.

Caffaz, A., Caiti, A., Casalino, G., and Turetta, A. (2010). The hybrid glider/AUV folaga. *Robotics & Automation Magazine, IEEE*, 17(1), 31–44. doi:10.1109/MRA.2010.935791. URL http://dx.doi.org/10.1109/MRA.2010.935791.

Fossen, T.I. (2011). *Handbook of Marine Craft Hydrodynamics and Motion Control.* John Wiley & Sons, Ltd. doi:10.1002/9781119994138. URL http://dx.doi.org/10.1002/9781119994138.

Fossen, T.I. (2012). How to incorporate wind, waves and ocean currents in the marine craft equations of motion. In *9th IFAC Conference on Manoeuvring and Control of Marine Craft*, volume 45, 126 – 131. doi: http://dx.doi.org/10.3182/20120919-3-IT-2046.00022. URL http://www.sciencedirect.com/science/article/pii/S1474667016312162.

Kamman, J.W. and Huston, R.L. (1985). Modelling of submerged cable dynamics. *Computers & Structures*, 20(1), 623 – 629. doi:http://dx.doi.org/10.1016/0045-7949(85)90110-5. URL http://www.sciencedirect.com/science/article/pii/0045794985901105. Special Issue: Advances and Trends in Structures and Dynamics.

Kamman, J.W. and Huston, R.L. (2001). Multibody dynamics modeling of variable length cable systems. *Multibody System Dynamics*, 5(3), 211–221. doi:10.1023/A:1011489801339. URL https://doi.org/10.1023/A:1011489801339.

Ke, Y., Xu-yang, W., Tong, G., and Chao, W. (2014). A dynamic model of an underwater quadruped walking robot using kane's method. *Journal of Shanghai Jiaotong University (Science)*, 19(2), 160–168. doi:10.1007/s12204-014-1485-7. URL https://doi.org/10.1007/s12204-014-1485-7.

Ke, Y., Xuyang, W., Tong, G., and Chao, W. (2013). A dynamic model of rov with a robotic manipulator using kane's method. In *2013 Fifth International Conference on Measuring Technology and Mechatronics Automation*, 9–12. doi:10.1109/ICMTMA.2013.13. URL http://dx.doi.org/10.1109/ICMTMA.2013.13.

Nielsen, M.C., Eidsvik, O.A., Blanke, M., and Schjølberg, I. (2016a). Validation of multi-body modelling methodology for reconfigurable underwater robots. In *OCEANS 2016 MTS/IEEE Monterey*, 1–8. doi:10.1109/OCEANS.2016.7761240. URL http://dx.doi.org/10.1109/OCEANS.2016.7761240.

Nielsen, M.C., Blanke, M., and Schjølberg, I. (2016b). Efficient modelling methodology for reconfigurable underwater robots. In *Proceedings of the 10th IFAC Conference on Control Applications in Marine Systems, CAMS*

2016, volume 49, 74–80. doi:10.1016/j.ifacol.2016.10.324. URL http://dx.doi.org/10.1016/j.ifacol.2016.10.324.

Park, J. and Kim, N. (2015). Dynamics modeling of a semi-submersible autonomous underwater vehicle with a towfish towed by a cable. *International Journal of Naval Architecture and Ocean Engineering*, 7(2), 409–425. doi:http://dx.doi.org/10.1515/ijnaoe-2015-0029. URL http://www.sciencedirect.com/science/article/pii/S2092678216300899.

Siciliano, B. and Khatib, O. (2008). *Handbook of Robotics*. Springer-Verlag Berlin Heidelberg. doi:10.1007/978-3-540-30301-5. URL http://dx.doi.org/10.1007/978-3-540-30301-5.

Simetti, E., Wanderlingh, F., Casalino, G., Indiveri, G., and Antonelli, G. (2017). Robust project: Control framework for deep sea mining exploration. In *OCEANS 2017*, 1–5. Anchorage.

Udwadia, F.E. and Schutte, A.D. (2011). A unified approach to rigid body rotational dynamics and control. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 468(2138), 395–414. doi:10.1098/rspa.2011.0233. URL http://rspa.royalsocietypublishing.org/content/468/2138/395.

Zhang, H. and Wang, S. (2007). Modelling and analysis of an autonomous underwater vehicle via multibody system dynamics. In *Proceeding of the the 12th IFToMM World Congress*, 1–6. Besancon, France.