Stability Design for Concrete Mattresses

Jol Godbold and Nikki Sackmann Atteris Pty Ltd Perth, Western Australia, Australia

W/Prof. Liang Cheng University of Western Australia Perth, Western Australia, Australia

ABSTRACT

Concrete mattresses are widely used within the offshore industry for scour control, stabilisation and protection. There is currently no uniform industry methodology for concrete mattress stability design for any application of concrete mattresses. This paper reviews design works previously carried out for a project in the North West Shelf of Australia and proposes an industry design methodology. This methodology encompasses the application of recognised pipeline codes and standards, and the requirements and results of 2D and 3D Computational Fluid Dynamics (CFD) modelling as well as physical model testing.

KEY WORDS: Methodology; stability; concrete mattresses; physical model testing; computational fluid dynamics (CFD).

NOMENCLATURE

- A_s Significant acceleration (m/s²)
- C_D Drag force coefficient, given as 0.7 (-)
- C_L Lift force coefficient, given as 0.9 (-)
- C_M Inertia force coefficient, given as 3.29 (-)
- C_{Y}^{*} Vertical peak load coefficient (-)
- C_Z^* Horizontal peak load coefficient (-)
- *D* Outside diameter of the pipeline (m)
- F_D . Drag force acting on the structure (N)
- F_{I} Inertia force acting on the structure (N)
- F_L Lift force acting on the structure (N)
- F_R Passive soil resistance (N)
- F_{V}^{*} Vertical force acting on the structure (N)
- F_{z}^{*} Horizontal force acting on the structure (N)
- $r_{tot,v}$ Vertical load reduction factor (-)
- $r_{tot,z}$ Horizontal load reduction factor (-)
- U_c Current velocity (m/s)
- U_s Significant near-bottom velocity (m/s)
- w_s Submerged weight of the structure (N)
- θ Phase angle of the hydrodynamic force in the wave cycle
- μ Coefficient of friction (-)
- γ_{SC} Safety factor (-)

INTRODUCTION

Concrete mattresses are widely used in the offshore industry. Mattresses can be used to protect subsea structures, pipelines, umbilicals and cables. Mattresses can also be employed for secondary stabilisation of such infrastructure against any hydrodynamic loading, assist in preventing seabed scour around subsea structures, or be used as supports for the construction of crossings and the like.

There is no industry standard stability design methodology, Standard or Code of Practice which suitably addresses all the diverse marine applications of concrete mattresses. Research has been conducted into the failure mechanisms of concrete mattresses used for slope stabilisation (Leidersdorf, Gadd and McDougal 1988 and Dunlap, S 2001), however these studies focused only on shoreline applications and not completely submerged concrete mattresses. For the purposes of this paper additional vertical forces due to waves in shallow water have been excluded. This paper looks at the stability design process for completely submerged concrete mattresses. The design procedure includes steps to be taken where confidence in the mattresses stability is essential through discussion of a case study where mattresses have been used as scour protection for a platform foundation.

INPUT DATA GATHERING

To perform a concrete mattress stability design a number of inputs are required. The following are the typical inputs used:

- Metocean data.
- Seabed geotechnical and bathymetry information (including details of any proposed on bottom structures).
- Functional requirements and design constraints.

Application and final placement of a concrete mattress also impacts the design. Concrete mattresses are not able to be adequately designed in isolation of their location without the use of high safety factors. Therefore, an understanding of the location and the factors likely to impact the design of a concrete mattress is important. Additionally, the collection of as much input data as possible shall aid concrete mattress stability design.

Metocean Data

Use of limited Metocean data for design will potentially require the use of high safety factors to ensure mattress stability. Site specific, detailed metocean data of the local environment where the concrete mattresses are to be placed will allow for design optimisation. Directionality of the hydrodynamic load conditions is important to the design. Testing has demonstrated that concrete mattresses largely fail due to leading edge lift (Lagasse, P 2007), therefore directional metocean data is preferable for complete optimisation, otherwise more conservative omnidirectional data can be used. The typical information required for design is the significant wave height, peak wave period, water depth, wave orbital velocity and the current velocity at 1 m ASB for the appropriate Return Period (RP) storm events. During a high RP cyclonic event, where wave periods are increased, there is a potential to cause mattress failure during a single wave, therefore accurate data is required.

Seabed Conditions

Geotechnical, and bathymetry information is an important input to concrete mattress design as this can have an impact on embedment, sliding and amplification of any hydrodynamic forces e.g. uplift under a mattress edge or accelerations around a structure. Two factors which can influence the stability of a concrete mattress are:

Embedment: Concrete mattress embedment can occur when a mattress is placed on loosely compacted soils or an area subject to seabed erosion / deposition as illustrated in Figure 1. The effect of embedment is the increased stability, given that the mattress corners are no longer exposed and there is a significant increase in sliding failure resistance. In some situations where seabed mobility is not a concern any initial embedment of the concrete mattress may provide additional stability. This initial embedment may be caused by settlement of the concrete mattress or consolidation of the soil. However, work done (Palmer and King, 2008) has shown that cyclones can greatly change the seabed profile. Therefore careful consideration of the local environment is required before additional stabilisation due to embedment can be applied to the design. It should be noted that scour could also occur under the mattress leading edge in isolation and this may result in an increase in uplift forces with a subsequent increased risk of lifting failure. The effects of embedment for pipelines have only recently been included in design practices, albeit to a limited extent. Physical model testing performed in the University of Western Australia's O-Tube facility has shown that even small amounts of embedment can greatly increase the stability of a pipeline (Jas, O'Brien, Fricke, Gillen, Cheng, White and Palmer 2012).



Figure 1 - Mattress embedment

• Orientation: Concrete mattresses placed on slopes or over structures are subjected to different forces to those laid flat on the seabed, Figure 2. The risk of mattress sliding (specifically for slope stabilities) and the likelihood of edge uplift is increased. This may pose significant for certain designs e.g. where a mattress is used for scour protection at the toe of a structure / top of a slope. Research has been conducted into mattress behaviour on slopes (Leidersdorf, Gadd and McDougal 1988 and Dunlap, S 2001), however; much of this research is specific to designs in breakwater areas and is not suitable for a range of subsea applications.



Figure 2 - Mattress placement on slopes

Functional Stability Requirements and Design Constraints

Each application will have a range of requirements / constraints which will need to be considered in the concrete mattress stability design. These include whether absolute stability is required for the mattress' design life, or some movement is permitted. In subsea applications typical considerations are the minimum RP cyclonic event in which the concrete mattresses are required to be stable and the size and weight restrictions imposed by the concrete mattress installation equipment. In addition, the design application of the concrete mattress, for example:

• Supports: Concrete mattresses are often used under pipelines, umbilicals and cables, Figure 3, for example as part of pipeline crossings or to provide scour protection.



Figure 3 – Mattress underneath a pipeline

Protection and/or stabilisation: Concrete mattresses are often placed above pipelines, umbilicals and cable as part of the secondary stabilisation and protection design of the pipelines as shown in Figure 4. The orientation of the mattresses results in a change in the hydraulic flow around the mattress which modifies the inherent stability characteristics. These effects differ, depending on the size of the mattresses as well as the orientation and the mattress location over pipelines, umbilicals and cables. For example, a mattress placed with its edge too close to the pipeline may have a lower resistance to the edge of that mattress flipping. Additionally; mattresses placed over an object may receive additional forces due to movement of that object e.g. an unstable pipeline with mattresses placed at discrete locations along its length.



Figure 4 – Mattress over a pipeline

Proximity to other structures: Adjacent objects, shown in Figure 5, may cause reduced or increased hydrodynamic velocities and shear stresses. Shielding from adjacent structures, such as other concrete mattresses and / or armour rock can positively impact the stability of the concrete mattress. When a number of mattresses are placed together they are thought to act as one larger structure, therefore the internal mattresses do not experience the same loading as those of the perimeter / leading edge. Arrangements such as this must subsequently consider both local and global failure mechanisms. Adjacent man made or natural structures, such as platforms or large seabed features, may potentially cause shear stress and water particle velocity amplifications. Mattresses which may be stable under given hydrodynamic load conditions become unstable when placed around / adjacent to a structure. This is particularly important to consider when the concrete mattresses are placed around a structure for scour protection or stabilisation of that structure. Figure 5 demonstrates shear stress amplifications; calculated using Computational Fluid Dynamics (CFD) around an offshore platform.



Figure 5 – Example of shear stress amplifications

Additional design considerations which may affect the stability of the concrete mattresses are:

- The density, shape and dimensions of the concrete mattresses. In areas where high hydrodynamic conditions are expected or modelling has shown the mattresses are likely to be unstable, increased concrete density may offer additional stabilisation. It is important to note the installation limitations of the project, increased concrete density and dimensions will increase stability, but it will also increase the weight of the mattress. Increased weight may result in installation issues as it may increase the installation equipment requirements.
- The use of edge lift straps. Testing has demonstrated that concrete mattresses largely fail via edge lift, therefore by strapping rows of edge blocks to the next row inside, the mattress stability is increased. Edge lift straps do not increase the overall weight of the mattress significantly and therefore are a good solution when there are weight constraints for installation equipment.

DESIGN METHODOLOGIES

This paper reviews two different methodologies currently used to analyse the stability of mattresses and examines their strengths and weaknesses. These two methodologies are adaptations of *On-Bottom Stability Design of Submarine Pipelines* DNV-RP-E305 and *On-Bottom Stability Design of Submarine Pipelines* DNV-RP-F109 and are used to obtain an initial concrete mattress size based on stability. It should be noted that the methods and equations are for pipelines rather than concrete mattresses and may subsequently give inaccurate results. Therefore the current design method is to apply a large safety factor which may give conservative results. However, as no dedicated concrete mattress design code exists some conservatism in the results must be accepted.

DNV-RP-E305 and DNV-RP-F109 both present a method for determining the expected hydrodynamic forces on a subsea pipeline. Both DNV-RP-E305 and DNV-RP-F109 use Morison-type equations which are applicable methods for determining hydrodynamic forces on concrete mattresses with some modification to the hydrodynamic coefficients. This is not without precedence. These Morison-type equations are frequently altered to suit different subsea structures, Lan, Guo, Liu, Song and Yuan (2010) have used this method to study a horizontal slab and pile arrangement. DNV-RP-E305 has been superseded by DNV-RP-F109; however DNV-RP-E305 Morison-type equations are still used within on-bottom stability design of structures (Bryndum, Jacobsen and Tsahalis 1992).

DNV-RP-E305 and DNV-RP-F109 do not provide guidance for concrete mattresses, resulting in the following assumptions to be made in order to assess stability:

- Area the forces are acting over (plan face for vertical loads and elevation face for horizontal loads).
- Directional effects of the metocean conditions on the concrete mattress.
- Modified lift, drag and inertia coefficients for non-pipeline structures.
- Modified factors of safety, as these typically have been designed for pipelines.

Both DNV-RP-E305 and DNV-RP-F109 use Morison-type equations, EQ's 1-3 and 6-7 respectively, to determine the forces acting on a pipeline. Then, using the submerged weight and friction forces to determine stability, EQ's 4-5 and 8-9.

$$F_L = \frac{1}{2} \rho_w D C_L (U_s \cos \theta + U_c)^2 \tag{1}$$

$$F_D = \frac{1}{2} \rho_w DC_D | U_s \cos \theta + U_c | (U_s \cos \theta + U_c)$$
(2)

$$F_I = \frac{(\pi D^2)}{4} \rho_w C_M A_s \sin \theta \tag{3}$$

$$\frac{\mu(w_s - F_L)}{F_D + F_I} > 1 \tag{4}$$

$$\frac{w_s}{F_L} > 1 \tag{5}$$

$$F_Y^* = r_{tot,y} \cdot \frac{1}{2} \cdot \rho_W \cdot D \cdot C_Y^* \cdot (U^* + V^*)^2$$
(6)

$$F_{Z}^{*} = r_{tot,Z} \cdot \frac{1}{2} \cdot \rho_{W} \cdot D \cdot C_{Z}^{*} \cdot (U^{*} + V^{*})^{2}$$
(7)

$$\gamma_{SC} \frac{F_Y + \mu F_Z}{\mu(w_S) + F_R} \le 1.0 \tag{8}$$

$$\gamma_{SC} \frac{F_Z}{w_s} \le 1.0 \tag{9}$$

The difference between DNV-RP-E305 and DNV-RP-F109 is the hydrodynamic coefficients:

 DNV-RP-E305 provides the lift, drag and inertia coefficients for a pipeline on the seabed. These coefficients have been calculated from extensive physical model testing of a pipeline. For concrete mattress design, the coefficients presented in DNV-RP-E305 are not considered applicable. However, opinion varies greatly on what are applicable coefficients, with no one organisation having done extensive testing. In concrete mattress design the ambiguity surrounding the coefficients is usually compensated for by the addition of high safety factors. This leads to increased concrete mattress weight and dimensions, potentially increasing installation and equipment costs. The DNV-RP-E305 method has the advantage that it allows for an easy preliminary assessment of concrete mattress stability (from a hydrodynamic load calculation perspective). Thus, an approximate mattress size can be determined, assuming good judgement when determining the coefficients.

DNV-RP-F109 does not provide the horizontal and vertical coefficients; instead it provides equations to calculate them. These calculations are dependent on the hydrodynamic conditions at the location. This makes the DNV-RP-F109 method reliant on accurate metocean data, more complex and generally unsuitable as an easy preliminary assessment tool.

Through the use of physical model testing conducted on concrete mattresses it has been determined the main failure mechanism is edge lift. During the testing program, the hydrodynamic loads were increased until the model mattress failed. Failure occurred when the leading edge of the mattress lifted, allowing water to pass underneath it. Once the leading edge lifted sufficiently, it curled back and the mattress rolled upon itself and down the test section of the testing facility. Therefore the Morison –type equations require edge lifting failure to be incorporated into concrete mattress design. This is usually achieved by calculating the overturning moment of the leading edge or corner. Care should be taken when calculating the overturning moment on the leading edge or corner to ensure the forces are applied correctly.

CASE STUDY

Problem

Concrete mattresses are a relatively low cost item. Therefore most concrete mattress designs typically apply high safety factors to overcome the shortcomings of the design. In addition, concrete mattresses can be replaced or repositioned after an event, if any damage or failure has occurred. Typically this approach is usual in applications with low consequence following failure, therefore the equations presented in DNV-RP-E305 and DNV-RP-F109, with a high safety factor, are generally considered acceptable for concrete mattress design. In some cases the concrete mattress design is required to be more accurate. For example, the case study concrete mattresses are used as scour control of an oil and gas platform's foundation. In this case deterioration of the under lying foundation material, due to exposure from concrete mattress failure, may cause a catastrophic failure of the platform foundations. Additionally, due to operational constraints around the platform, it is not feasible to regularly inspect and replace mattresses after a cyclonic event. The platform is located in an area subject to tropical cyclones; therefore confidence in the concrete mattress' ability to remain in situ during cyclone events was imperative. In addition, due to the proximity to the platform and available installation equipment, a concrete mattress weight restriction has been imposed. Therefore the method of using DNV-RP-E305 and DNV-RP-F109 with high safety factors was not applicable and a more detailed design was required.

Solution

Initially the concrete mattresses had been sized based on the design method presented in DNV-RP-E305, which allowed for a first pass stability assessment. These mattresses were then checked against the stability assessment provided in DNV-RP-F109.

Given that the application was in close proximity to the platform and required certainty in the hydrodynamic loads; CFD modelling was conducted to assess any increase in shear stress amplifications around the platform. CFD allows the modelling of flow patterns around obstructions, such as platforms, pipelines, uneven seabed's and the like. This assists in gaining a better understanding of the excepted local hydrodynamic load conditions. CFD analysis estimates shear stress and water particle velocity amplification which can be used in the design. The CFD results can be used within recognised stability software's, such as Deltares System's PROBED, or to increase the applicability of non-mattress specific industry codes, standards and recommended practices.

The University of Western Australia (UWA) has developed a CFD package, which has been utilised in numerous stability studies, SCOUR-3D. The SCOUR-3D package is a finite element program and simulates the turbulent flow field around any structure by solving the Reynolds-Averaged Navier-Stokes (RANS) equations with a k- ω turbulence model (Zhao, Cheng and Zhou, 2009). The parallel computing capability of SCOUR-3D allows large scale simulations to be conducted in a relatively short period of time. SCOUR-3D has been validated for use in a wide range of applications, these are documented within Cheng and Zhao (2010), Zhao, Cheng and Zang (2010) and Zhao, Cheng and An (2010). Figures 6 - 8 present some typical outputs of SCOUR-3D analysis around a platform.



Figure 6 - Typical streamlines around a platform



Figure 7 - Typical vortex flow around a platform



Figure 8 - Velocity amplifications around a platform

The results of the SCOUR 3D CFD modelling revealed large areas around the platform which have increased amplifications, particularly at the corners of the platform structure.

After further DNV-RP-E305 and DNV-RP-F109 analysis had been conducted, including the maximum shear stress amplifications, it was determined the concrete mattresses were not considered to be stable under design cyclonic conditions.

However, given that the analysis included a number of assumptions and the concrete mattresses are required for the structural integrity of the platform's foundation, Physical Model Testing (PMT) was to be considered as the only way to validate the design.

PMT, assuming access to a facility capable of simulating the required hydrodynamic load conditions, would provide valuable data on concrete mattresses stability performance.

In considering PMT there needs to be a cost benefit analysis over the selection of 2D and 3D physical model testing strategies and how to select a suitable facility. This is essential to ensure the PMT program can give meaningful results.

The most common PMT testing facilities are wave and current flumes, Figure 9. These allow for scale testing to be performed and the hydrodynamic effects on a structure to be assessed. Xia, Sun, Liu, Li and Wan (2010). Bryndum, Jacobsen and Tsahalis (1992) performed extensive physical model testing to determine the hydrodynamic forces acting on a pipeline using Morison-typed equations in a wave and current flume.



Figure 9 – Wave and current flume

The O-Tube, Figure 10, at the University of Western Australia (UWA) is a sophisticated hydrodynamic testing facility which allows for testing

near-seabed conditions during cyclonic events. The O-Tube operates using an electric powered propeller to circulate or oscillate 60 m^3 of water through a closed loop system. It has several advantages over other facilities including:

- Simulates high oscillatory and steady currents.
- Simulates the mobility of seabed material
- Simulates the interaction between seabed material and the test piece.
- In the case of mattress testing it was also possible to use a large scale of 1:10, reducing inaccuracies inevitable due to small scaling.



Figure 10 – O-Tube

The O-Tube facility has been utilised for a number of projects, allowing for the verification of designs when analytical / theoretical methods have been unable to provide conclusive results, (Jas, O'Brien, Fricke, Gillen, Cheng, White and Palmer 2012). The O-Tube allows for on bottom conditions to be modelled while still maintaining a reasonable scale. In addition, the O-Tube is able to model up to and including 10,000 year RP cyclonic oscillatory conditions.

The case study has conducted a comprehensive PMT programme. The programme has been completed at the UWA's O-Tube. The testing programme has been conducted to validate the concrete mattress design and provide increased confidence that the mattresses will remain in place during the required cyclonic event. Given the position and purpose of the concrete mattresses PMT was considered important. Using the theoretical calculations the concrete mattresses were expected to be unstable within the areas of increased shear force amplifications around the platform. Through the use of PMT the expected unstable areas around the platform were decreased and able to be mitigated using additional scour protection measures. The PMT has provided valuable insight into the failure mechanism of concrete mattresses when subjected to extreme wave loading. A major advantage of testing at this specific facility is the ability to test at a scale of 1:10 for a typical concrete mattress.

From the PMT it was observed that once the leading corner of the concrete mattress was lifted enough water would flow under the mattresses. This caused the concrete mattress to roll onto itself and then down the O-Tube, resulting in catastrophic failure. From this testing it can be concluded that edge lift is the main failure mechanism for these concrete mattresses. The PMT programme validated the concrete mattress design and increased confidence in their application.

CONCRETE MATTRESS STABILITY DESIGN PROCESS

As a result of the work performed designing the concrete mattresses for the platform scour protection, a concrete mattress design procedure has been developed. This procedure, Figure 11, is presented as an iterative flow chart and is suitable for simple and complicated concrete mattress designs; in relation to the severity of the consequence of failure.



Figure 11 - Design flow chart

As shown in Figure 11, the process of designing concrete mattresses is iterative; the initial design can be completed using the basic information available. Then, as more information becomes available, for example surveys, CFD modelling, PMT and additional metocean information, the mattress design is further refined. The number of iterations a concrete mattress design requires is dependent on the outcomes of design investigations and whether this level of assessment is suitable for the project requirements.

FURTHER WORK

The physical model testing performed at the UWA O-Tube only had a limited scope because it was project specific. Information on concrete mattress design methods is limited and no formal code / guideline exists. Should a design code / guideline be produced the following validation work is recommended:

- Testing concrete mattress under a range of hydrodynamic conditions, allowing the results to be verified and identify the drivers for failure, i.e. wave height, wave period, current and the like.
- Testing the concrete mattress at a number of orientations, allowing the directionality effects to be quantified.

- Testing concrete mattresses with varied densities and dimensions, allowing the effects of changing these factors to be quantified.
- Testing the concrete mattress with different seabed conditions.
- Testing the concrete mattresses around an obstruction, allowing the shear stress amplifications to be modelled.
- Testing the concrete mattresses to determine lift, drag and inertia coefficients, providing increased confidence in the calculations.

The project specific O-Tube testing which has been performed did not look at the interactions between the concrete mattresses and the seabed. As detailed within Jas, O'Brien, Fricke, Gillen, Cheng, White and Palmer (2012), the mattress – seabed interaction may affect the stability of the concrete mattresses and requires further work (as previously noted embedment will influence stability performance). Physical model testing to gain an understanding of how they interact could be filtered into future designs.

Additionally, concrete mattress stacking and overlapping have not been addressed within this paper. When concrete mattresses are used for pipeline crossings they can be stacked on one another to meet the required height. Therefore the forces acting on the top mattress will be applied differently than those acting on the bottom mattress. If the top mattresses are not laid accurately there is a risk of mattress overhang, which may result in decreased mattress stability and potential damage to an asset. Physical model testing on mattress stacks is recommended to understand the interaction between mattresses and how the forces differ between them.

REFERENCES

Bryndum, M, Jacobsen, V and Tsahalis, D (1992). "Hydrodynamic Forces on Pipelines: Model Tests," *Journal of Offshore Mechanics and Arctic Engineering*, Vol. 114, pp. 231-241.

Cheng, L, Zhao, M (2010). "Numerical Model for Three-Dimensional Scour Below A Pipeline in Steady Currents," 5th International Conference on Scour and Erosion, USA, ASCE. Pp. 482-490.

Delft Hydraulics (2006). "PROBED: Design tool for bed protections subjected to currents and waves."

Det Norske Veritas (1998). "DNV-RP-E305 On-bottom stability of submarine pipelines."

Det Norske Veritas (2010). "DNV-RP-F109 On-bottom stability of submarine pipelines."

Dunlap, S (2001). "Design Manual for Articulated Concrete Block Systems" Prepared for Harris County Flood Control District.

Jas, E, O'Brien, D, Fricke, R, Gillen, A, Cheng, L, White, D and Palmer, A (2012). "Pipeline Stability Revisited," *The Journal of Pipeline Engineering*, Vol. 12, No. 4, pp. 259-268.

Kirkvik. R (2013). "Flexible Pipe On-Seabed Stability, Upheaval Buckling and Hub Protection Methods," *Proceedings* 23rd Int Offshore and Polar Engineering Conference, Anchorage, ISOPE. Vol 2, pp-280-287.

Lagasse, P. (2007). *Countermeasures to Protect Bridge Piers from Scour*, Transportation Research Board.

Lan, Y, Guo, W, Song, Q and Yuan, J (2010). "Experimental Study on Hydrodynamic Loads on a Horizontal Slab and Pile," *Proceedings* 9th *Pacific / Asia Offshore Mechanics Symposium*, Korea, ISOPE. pp. 319-324.

Leidersdorf, C, Gadd, P and McDoughal, W (1988). "Articulated Concrete Mat Slope Protection," *Proceedings* 21st *International Conference on Coastal Engineering*, Spain, ASCE. Chapter 177, pp. 2400-2415.

Palmer, A, King, R (2008). *Subsea Pipeline Engineering 2nd Edition*, PennWell Corporation.

Xia, Z, Sun, D, Liu, Y, Li, Y and Wan, Q (2010). "Experimental Study on Phase Difference between Total Horizontal and Vertical Forces Acting on Perforated Caissons Located on a Rubble Mound Foundation," *Proceedings 9th Pacific / Asia Offshore Mechanics Symposium*, Korea, ISOPE. pp. 306-312. Zhao, M, Cheng, L, Zhou, T (2009). "Direct Numerical Simulation of Three-Dimensional Flow Past a Pawed Circular Cylinder of Infinite Length," *Journal of Fluids and Structures*, Vol 25, pp. 831-847.

Zhao, M, Cheng, L, Zang, Z (2010). "Experimental and Numerical Investigation of Local Scour Around A Submerged Vertical Circular Cylinder in Steady Currents," *Coastal Engineering*, Vol. 57, pp. 709-721.

Zhao, M, Cheng, L, An, H (2010). "Three-dimensional Numerical Simulation of Flow Around A Circular Cylinder Under Combined Steady and Oscillatory Flow," *Journal of Hydrodynamics Ser. B*, Vol. 22, pp. 144-149.