Analysis and Design of a Fully Submerged Underground Water Tank Using the Principle of Beam on Elastic Foundations

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Abstract

The basic requirement in the design of reinforced water tank is to ensure it is crack free. This research studied the analysis and design of a fully submerged underground reinforced concrete water tank using the principle of beam on elastic foundations. To achieve this, a Microsoft Excel Spreadsheet Design and Analysis Program (MESDAPro) was generated for quick assessment of various moments of the tank, geometrical features and soil conditions for both full and empty conditions of the tank. It was observed that the wall moments, moment at base of wall and base slab moment decreases with increase in soil sub-grade modulus at constant capacity, height and breadth of the tank. However, wall moments, moment at base of wall and base slab moment increases with increase in height of the tank at constant value of sub-grade modulus, tank capacity and breadth. In all the examined cases, the moments obtained is higher when the tank is considered empty than when considered full.

Key words: *Fully submerged, Water tanks, Elastic foundation, Sub-grade modulus, MESDAPro.*

1.0 INTRODUCTION

Water tanks are structures require to contain, or exclude water. The need for a water tank is as old as the civilized man, providing storage of water for [drinking,](http://en.wikipedia.org/wiki/Drinking_water) food preparation, [irrigation, fire](http://en.wikipedia.org/wiki/Irrigation) [suppression, chemical manufacturing, construction](http://en.wikipedia.org/wiki/Irrigation) [of civil engineering works](http://en.wikipedia.org/wiki/Irrigation) and many other applications. The main emphasis in the structural analysis and design of a reinforced water tank should be to ensure it is crack free both as a consequence of the loading and as a result of temperature and shrinkage effect (Mosley *et al.*, 2012; Reynolds *et al.*, 2008).

The most fundamental classification of reinforced water tank is based on whether they are on ground, above ground or below ground. The tanks resting on ground are supported on the ground directly, their walls are subjected to pressure and the base is subjected to weight of water and pressure of soil. The tanks may or may not be covered at the top. The elevated tanks are supported on staging by columns or frames. Their walls are subjected to water pressure and their base has to carry the water load, the top and walls load while the staging has to carry water and the entire tank load. The staging is also designed for wind forces. The underground tanks have their walls subjected to water pressure from inside and the earth pressure from outside while the base is subjected to weight of water and soil pressure depending on the soil condition (wet or dry). These tanks may be covered at the top and could be fully or partially submerged. The tanks are anchored into the ground and should not be able to pop out during periods when ground water surrounds the tank. In addition, because they are underground, they may be subjected to severe corrosion (Ilugbo, 2012).

2.0 ANALYSIS AND DESIGN OF WATER TANK

The water savings potential of rainwater tanks fitted in multi-unit residential buildings in three Australian cities was investigated by Eroksuz and Rahman (2010). The study revealed that a larger tank size is more appropriate to maximise water savings in multi-unit buildings and that rainwater tank of appropriate size in a multi-unit building can provide significant mains water savings even in dry years. An equation to predict mean annual water

savings from having a rainwater tank in a multiunit building was also proposed in the study. The sizing of rainwater storage tanks using simplified and detailed approaches was the focus of Santos and Taveira-Pinto (2013) while communal rainwater tank systems design, life cycle costing and economies of scale was investigated by Gurung and Sharma (2014).

A study of rainwater tank evaluation and design for large roof areas in Melbourne, Australia was presented by Imteaz, *et al.* (2011a). A comprehensive Decision Support Tool for the performance analysis and design of rainwater tanks derived from a simple spreadsheet based daily water balance model was developed using daily rainfall data, contributing roof area, rainfall loss factor, available storage volume, tank overflow and irrigation water demand. The effectiveness of two underground rainwater tanks $(185 \text{ m}^3 \text{ and } 110 \text{ m}^3)$ under different climatic conditions (i.e. dry, average, and wet years) was also established.

A research by Fewkes (1999) presented the results from field testing of a rainwater storage tank; the data collected was used to verify and refine a rainwater collector sizing model. The model was used to produce a set of dimensionless design curves which enables the storage capacity required to achieve a desired performance of tanks. The reliability analysis of rainwater storage tank was the focus of Imteaz, *et al.* (2011b), while Tam *et al.* (2010) discusses cost effectiveness of water tanks. Other studies which centered on optimising the sizing and capacity of water storage tanks includes (Ghisi *et al.*, 2007; Helmreich & Horn, 2009; Khastagir & Jayasuriya, 2010; Villarreal & Dixon, 2005).

The optimization of ground-based horizontal cylindrical steel tanks of different capacities for effective proportions of their dimensions was

carried out by Magnucki *et al.* (2004). Two load cases were assumed: total internal pressure and total special pressure. The critical state and optimum proportions of basic dimensions of the tanks were obtained. The constraints for the obtained solutions were the strength and stability conditions. The outcome was compared with the formulae used in the practice for the design of tanks.

Ground and underground horizontal cylindrical tanks supported at both ends were the focus of another research by Magnucki and Stasiewicz (2003). The ground tanks were loaded with internal hydrostatic pressure and small negative pressure while the underground tanks situated in saturated soil, were loaded with external hydrostatic pressure. By solving the equations of stability of cylindrical shell, the critical states of both structures were obtained and utilised to predict the critical thicknesses of walls for similar circular cylindrical tanks of different capacities using dimensionless critical thickness and tank length.

A novel fibre-reinforced plastic (FRP) hot water storage tank was developed by Nwosu and Agbiogwu (2013). The FRP tank can efficiently retain hot water and various hydrothermal fluids and resist high ambient pressures. Thermal analysis of the tank revealed that its insulation material properties and geometric parameters influenced its performance. A numerical study on the effects of shaping and location of supports on the strength of a ground-based horizontal cylindrical tank was done by Magnucki *et al.* (2003), the strength of such a tank is not the sole problem; another problem arises from its stability. A computer program which efficiently produces the optimum design for circular concrete reservoirs and elevated water towers was also presented by Lloyd and Doyle (1981).

A computer-aided design package RCTANK was developed by Chau and Lee (1991) based on Turbo Pascal version 5.0 programming language for computer-aided design of medium-sized liquid retaining tanks of rectangular and circular shapes. The design package is capable of performing analysis of the wall, the roof slab and the base slab of the tank, and, according to the analysis results, also giving recommendations about suitable concrete thicknesses and reinforcements entailed for different structural elements.

According to Anchor (1992), the design of liquid retaining structures was based on the use of elastic design with material stresses so low that no flexural tensile cracks were developed. This led to the use of thick concrete sections with copious quantities of mild steel reinforcement. The probability of shrinkage and thermal cracking was not dealt with on satisfactory basis and nominal quantities' of reinforcement were specified in most codes of practices. The consequential development of analytical procedures involving flexural crackwidths estimation; allowing for comparisons with specified maxima, and also the thermal and shrinkage strains calculation are the two developments that introduce the limit state design to liquid-retaining structures (Oyenuga, 2005) .

A study on the analysis and design of a fully submerged underground reinforced concrete water tank using the principle of beam on elastic foundations is reported in this paper. The soil was considered as not totally rigid but acting as a bed of springs, the roof slab was analysed as four sided discontinuous slab while the walls and the base slabs were taken as an entity. Limit state philosophy is utilised for this study because it provides a more rational method for determining safety factor and enables the different failure modes of structures to be identified so that premature form of failure may be prevented.

Likewise, it ensured that failure for water structures have low permeability so as to prevent leakage through the concrete and also provide adequate durability and prevention against corrosion of the reinforcement (Sani *et al.*, 2014).

3.0 METHODOLOGY

3 .1 Beam on elastic foundation

In this study, the underground water tank walls and base slab was analyzed as a fully submerged underground water tank. The analysis was done using the theory of beam on elastic foundations (Biot, 1937) which was developed on the assumption that the reaction forces of the foundation are proportional at every point to the deflection of the beam at that point. This theory is adopted when the flexural rigidity'(EI) of a beam is taken into account, which is considered as Winkler foundation regarding the soil acting as a bed of springs.

Consider a strip of an elemental beam (Figure 1) supported along its entire length by an elastic medium and subjected to vertical forces acting in the principal plane of the symmetrical cross section. Because of this action the beam will deflect, producing continuously distributed reaction forces in the supporting medium. The forces exerted on the elemental strip are the shearing force, Q, and bending moment, M, the values of which are derived as discussed later in section 3.3

In order to avoid repetition of the tedious calculations involved in the analysis and design of an underground reinforced concrete water tank, a Microsoft Excel Spreadsheet Design and Analysis Program, named MESDAPro, was generated in this research as a tool for quick assessment of various tank capacities, different soil conditions and other parameters.

The structural layout of the underground water tank was carried out, the dimensions for the capacity were deduced by the application of the related formula for solid shapes' volume- calculations i.e. $(L \times B \times H)$; where L is the Length, B is the Breadth, and H is the Height. Water free board was provided for the possible-increase above the required capacity, and to limit or check the overflow of the tanks which practically increased the tank capacity designed.

3.2 Loading of the Water Tank

The loading of the underground structural layout was achieved based on the consideration that the walls and base slab were subjected to external soil pressures due to soil and surcharge from vehicles which may park near the structure. In addition is the ground water which may over flow at any point in time of the structure life span. For the purpose of this research, each element of the underground water tank was loaded using two critical loading

conditions: (a) underground water tank full and no ground water outside and (b) underground water tank empty and ground water outside.

The top slab uniformly distributed load (kN/m/m) was obtained by summing up its combined dead load to derive the characteristic serviceability load. Factors of safety of 1.4 and 1.6 were applied to the combined dead load and live load respectively and their sum was made to achieve the required ultimate design load for the top slab.

The loading conditions for the tank walls were based on the forces acting against the wall under these conditions:

(a) Surcharge due to vehicular load parked around the underground water tank. This was derived by multiplying the surcharge load (kN/m²) by coefficient of active earth pressure (ka) which was computed from the formula: " $\tan^2(45 - \frac{\phi}{2})$," using an angle of internal friction (ϕ) to be 30^{σ}

(b) Earth pressure which was computed by multiplying the earth density $(kN/m³)$ by wall height and the coefficient of active earth pressure (ka)

(c) Ground water pressure which was also derived by multiplying the unit weight of water (ρ) in kN/m³ and the height of the wall.

The base slab's serviceability uniformly distributed load (kN/m/m), was the sum of its dead load; the concrete self weight and finishes, and its live load; the weight of the water to be contained. The earth pressure, ground water pressure under the floor slab and the internal water pressure were the acting pressure considered for the loading conditions.

3.3 Structural Analysis of the Water Tank

The method of analysis adopted in this study was the theory of beams on elastic foundation and this was due to the fact that past analysis that has been carried out did not considered the effect of soil which is acting as a bed of spring on the tank. This method of analysis was used in deriving bending moment and the shear force at any section of the walls and base slab using equations (1-8). The

equations were based on the load type and end conditions of the beam (Figure 2) where M and Q represents the moment and shear force of the beam respectively.

(i). Uniformly distributed loading for beams fixed at both ends

M = {(- q/2β2) (sinh βL + sin βL)-1 [(sinh βx cos βx' + cos βx sinh βx' - sin βx cosh βx'-cosh βx sin βx')] }………………………………………………………….(1)

Q= { (-q/β) [(cosh βx cos βx' – cos βx cosh βx') / (sinh βL + sin βL)]}………(2)

(ii). Varying distributed loading for cantilever beam

M = [(-q o/4β3l) (cosh2βl + cos2βl)-1 {cosβx (sinh βx – sinh β (2l – x)) – cosh βx (sin βx – sin β (2l – X))} + {2βl (Sinh Β (L – X)(Sinh Βl Cos Βx – Cosh Βl Sin Βx) + Sin Β (L – X) (Cosh Βx Sin Βl – Sinh Βx Cos Βl))}]……………………………..(3)

Q= (-qo/ 2β2L) (cosh2βL + cos2βL)-1{[cosh βx (sin βx sinh β(L-x) + cos βx cosh β(L-x)) + cos βL (sinh βx sin β(L-x) – cosh βx cos β(L- X))]+[βL (cosh βx (sin βx - sin β(2L-x))+ cos βx (sinh βx – sinh β(2L-x)))]} …………………………………(4)

(iii). Uniformly distributed loading for cantilever beam

M = {(-q/2β2) (cosh2βL + cos2βL)-1[cosh βL (sin βx sinh βx'– cos βx cosh βx')+ cosh βL (sinh βx sin βx' + cosh βx cos βx')]}……………………………………….(5)

Q= (-q/β)(cosh2βL + cos2βL)-1[cosh βx sin βx' cos βL + cos βx sinh βx'cosh βL]….(6)

(iv). Equal concentrated moments at both ends

M = (M0) (sinh βL + sin βL)-1(sinh βx cos βx' + cosh βx sin βx'+ sinh βx'cos βx+ cosh βx' sin βx)………………………………………………………………………(7)

Q= (2M0β) (sinh βL + sin βL)-1(sinh βx sin βx' – sinh βx' sin βx)………………(8)

Figure 2: (a) uniformly distributed loaded beam fixed at both ends, (b) varying distributed loaded for a cantilever beam, (c) uniformly distributed loaded for a cantilever beam and (d) beam subjected to equal moment at both ends.

3.4 Modulus of sub-grade reaction

The modulus of sub grade reaction is a conceptual relationship between soil pressure and deflection that is widely used in the structural analysis of foundation members. It is the ratio of applied pressure in kNm-2 divided by the corresponding soil movement or displacement in metres (m) (Horvath, 1983a, 1983b; Terzaghi, 1955).

3.5 Principle of superposition

The principle of superposition is a powerful mathematical technique for analyzing certain types of complex problems in many areas of science and technology, and it has an important application in analysis of underground water tank. The principle of superposition states that problem solutions can be added together to obtain composite solutions. This principle applies to linear systems governed by linear differential equations. The principle of superposition means that for linear systems, the solution to a problem involving multiple inputs (or stresses) is equal to the sum of the solutions to a set of simpler individual problems that form the composite problem (Hibbeler, 2011).

3.6 Underground water tank analysis using the rigid approach

The rigid approach analysis of underground water tank is the conventional method of analysis. This is achieved by assuming that the soil modulus of sub grade is zero (0). The maximum bending moment considering tank wall to be a triangular loaded and uniformly loaded as a simple cantilever fixed at the left-hand end and base slab to be uniformly loaded for a beam fixed at both ends.

3.7 Microsoft Excel Spreadsheet Design and Analysis Program, MESDAPro

This was built as a tool for a quick design and assessment of various tank capacities and it is flexible enough to accommodate other variation such as change in soil conditions. Basically, the program (Figure 3) is limited to rectangular or square underground water tank only. The input values in the white background cells are the only values to be adjusted to suit the desired requirements. Using the developed program, the various moments of the tank, geometrical features and soil conditions for both full and empty conditions of the tank were obtained as discuss in details under section 3.2

Tank Parameters	
CASES	TANK FULL
Length	10000mm
Width	10000mm
Height	1000mm
FREE BOARD	0mm
CAPACITY	100.00cu.m
WALL LOADING	-8.99 kN/sq.m
WALL THICKNESS	250mm
WALL BREADTH	1000mm
BASE SLAB LOADING	9.81 kN/sq.m
BASE SLAB DEPTH	250 mm
fcu	25 N/mm
SURCHAGE	0 KN/sq.m
$\mathbf{\hat{y}}$	380 N/mm
E (conc)	21.7185 KN/sq.mm
TYPE OF SOIL	DENSE SAND
K	4800.00 kN/cu.m
I	13.0208E+08 mm4
β	4.539E-04 mm-1

Figure 3: A screen shot of the developed MESDAPro

4.0 RESULT AND DISCUSSION

Figure 4: Moment Versus Modulus of Sub-Grade Reaction

Figure 4 shows the tank wall, continuity and the base slab moments in kNm for both full and empty tank conditions that were analysed considering different values for modulus of Sub-grade reaction in kN/m^3 for a 102,600 litres tank capacity keeping the breadth of the tank (B) constant at 6000mm. Higher moment values were observed for lower modulus of sub-grade reaction values in all the cases considered; this suggests the soil acting as a bed of spring or having a spongy nature.

4.1 Elastic Foundation Approach versus Rigid Approach

It can be deduced in Figure 4 that the elastic foundation approach of tank analysis gave an

accurate and the actual values of bending moment for both cases of tank empty and tank full than the rigid (conventional) approach. This variation suggests that the soil is not totally rigid rather it acts as a bed of spring which is in line with the principle of elastic foundation which considers the flexural rigidity of the tank and modulus of sub grade reaction in its computation. Furthermore, the gradual increase in the values of bending moment as the value of the modulus of sub grade reaction decreases (i.e., from clayey soil to dense soil and to loose soil) suggest the conceptual relationship between soil pressure $(kN/m²)$ and displacement (m) which is widely used in the structural analysis of foundation members.

Figure 5: Moment against Height

Figure 5 shows wall moment, wall base moment and the base slab moment in kNm for both full and empty tank conditions computed for different tank height in m. A volume of 100,000 litres was adopted using a constant modulus of sub-grade of $4,800$ kN/m³. It could be deduced that for a given capacity that the wall moment, wall base moment and the base slab moment varies with the changes in the tank height. The moment was minimum for tank heights with low values and increases as the height increases which suggest a rational analysis of moment being dependent on depth. The values of moment was much when the tank was empty than when the tank was full due to the water pressure. This conforms with the result of Said and AbdulMajeed (2011) which utilised Numerical Simulation using ANSYS Finite Element Program for seismic analysis of liquid storage tanks.

5.0 CONCLUSION

A study on the analysis and design of a fully submerged underground reinforced concrete water tank using the principle of beam on elastic foundations is reported in this paper. The soil was considered as not totally rigid but acting as a bed of springs, the roof slab was analysed as four sided discontinuous slab while the walls and the base slabs were taken as an entity. Limit state philosophy is utilised for this study because it provides a more rational method for determining safety factor and enables the different failure modes of structures to be identified so that premature form of failure may be prevented. Likewise, it ensured that failure for water structures have low permeability so as to prevent leakage through the concrete and also provide adequate durability and prevention against corrosion of the reinforcement.

A Microsoft Excel Spreadsheet Design and Analysis Program (MESDAPro) was generated for quick assessment of various moments of the tank, geometrical features and soil conditions for both full and empty conditions of the tank. It was observed that the moments of wall, wall base and base slab decreases with increase in soil sub-grade modulus at constant capacity, height and breadth of the tank while they increase with increase in height

of the tank at constant value of sub-grade modulus, tank capacity and breadth. In all the examined cases, the moments obtained is higher when the tank is considered empty than when considered full.

The developed program, MESDAPro facilitates a very quick and accurate analysis and design for reinforced concrete water tanks thereby eliminating tedious and repetitive calculations often encountered by Engineers in practice.

5.1 RECOMMENDATION

- Analysis of underground water tank should take cognisance of elastic foundation approach in order to be cost effective.
- Necessary laboratory procedure should be adopted in determining Modulus of Subgrade reaction used for various analysis of underground water tank.

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