

FUNDAMENTALS of
UNDERWATER ACOUSTICS

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CONTENTS

1. SONAR EQUATION -----	1
2. ACOUSTIC WAVES -----	2
2.1. FUNDAMENTAL QUANTITIES -----	3
2.2. ACOUSTIC IMPEDANCE, INTENSITY, POWER and ENERGY -----	3
2.3. decibel CONCEPT -----	4
2.4. Pascal UNIT -----	5
3. UNDERWATER ACOUSTIC EQUATIONS -----	6
3.1. EXTRACTION of ACOUSTIC WAVE EQUATION -----	6
3.2. SOLUTION of ACOUSTIC WAVE EQUATION -----	7
3.3. EXTRACTION of HELMHOLTZ EQUATION -----	8
3.3.1. Boundary Conditions-----	9
3.3.2. Continuity Conditions-----	9
3.4. PROPAGATION LOSSES -----	10
3.4.1. Spreading Loss-----	10
3.4.2. Absorption Losses-----	12
3.4.3. The Other Losses-----	13
4. REVERBERATION -----	15
4.1. SEA SURFACE REVERBERATION -----	16
4.2. SEA BOTTOM REVERBERATION -----	16
4.3. VOLUME REVERBERATION -----	17
5. SOLUTION of HELMHOLTZ EQUATION -----	18
5.1. SOUND SPEED PROFILE (SSP) -----	19
5.2. SOLUTION METHODS OF HELMHOLTZ EQUATION -----	20
5.2.1. Ray Method-----	20
5.2.2. The Wave Method (Mode Theory)-----	26

5.3. TIME DOMAIN CALCULATIONS -----	29
5.3.1. Direct Time Domain Methods -----	29
5.3.2. Inverse Fourier Transform-----	31
6. NOISE SOURCES -----	33
6.1. THERMAL NOISE -----	33
6.2. SEA AMBIENT NOISE -----	33
6.3. SHIP (VESSEL) NOISE -----	34
6.3.1. Ship Acoustic Signatures -----	35
7. CAVITATION -----	38
7.1. CALCULATION FOR THE CAVITATION -----	39
8. TARGET STRENGTH -----	40
8.1. REDUCTION OF TARGET STRENGTH -----	41
8.1.1. Coating-----	41
8.1.2. Coating / Shaping-----	43
8.2. Active Noise Cancellation -----	43
9. REFERENCES -----	45

1. SONAR EQUATION

In order to determine sonar performance, sonar equation relates parameters of

- the sonar and its platform,
- the target and the environment.

The sonar equation is used to predict

- the performance of a known sonar design or,
- to design a sonar for a given performance.

The fundamental sonar equation is given as

$$SE = S - N - DT$$

where SE is Signal Excess, S is Signal, N is Noise and DT is Detection Threshold.

In fact, SE shows the full sonar system detection performance. DT is related to the probabilities and used signal processing algorithms. Specially, S and N are directly related to the **acoustic pressure**.

Now, let us have look the sonar equation in detail. It can be given for passive and active sonar systems respectively as

$$SE = \underbrace{(SL - PL)}_S - N - (DT - DI)$$

$$SE = \underbrace{(SL + TS - 2PL)}_S - N - (DT - DI)$$

where SL is Source Level, PL is propagation Loss, TS is Target Strength and DI is Detection Index. Here, SL, PL and TS are related to the **acoustic pressure**. DI is related to the sensor properties.

Acoustic pressure means that we have to understand about underwater acoustic wave propagation.

2. ACOUSTIC WAVES

Acoustic waves propagate mechanically vibrating the medium with mass and elasticity.

In a simple medium,

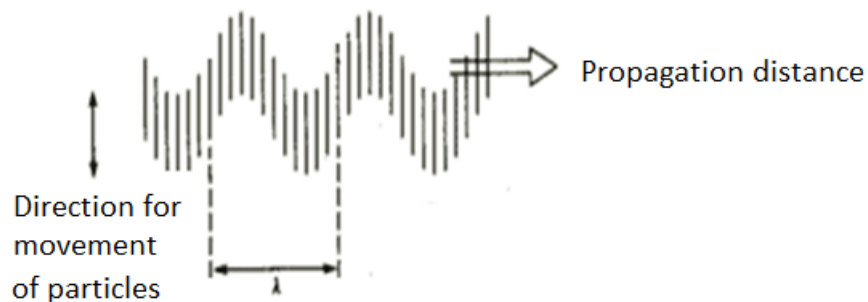
- horizontal vibration (compressional waves or P wave) of medium particles takes place only.
- vibration direction and propagation direction are in parallel (Fig. 1).



Horizontal vibration direction and propagation direction are in parallel.

In a complex medium,

- additional vertical vibration (shear wave or S wave) of medium particles take place.
- vibration direction and propagation direction are perpendicular (Fig. 2).



Vertical vibration direction and propagation direction are perpendicular.

The shear waves cannot propagate in liquids with zero (or very low) viscosity. They may propagate in liquids with high viscosity.

2.1. FUNDAMENTAL QUANTITIES

Model of the underwater wave propagation can be settled on fundamentals quantities of wave and medium. They are

- Wave Quantities

Acoustic pressure $u(\vec{r}, t)$ [Pa]

Particle velocity vector $\vec{v}(\vec{r}, t)$ [m/s]

- Medium Quantities (Parameters)

Density $\rho(\vec{r}, t)$ [kg/m³]

Elasticity $\beta(\vec{r}, t)$ [N/m²]

Viscosity $\mu(\vec{r}, t)$ [N. s/m²]

The wave quantities model the acoustical wave phenomena while the medium quantities are related to the medium material features.

2.2. ACOUSTIC IMPEDANCE, INTENSITY, POWER and ENERGY

The acoustic impedance, intensity, power and energy are also important parameters for understanding some steps of the acoustical wave phenomena. They are

- Impedance

$$Z = \frac{u}{v} = \rho \cdot c \quad \text{kg m}^{-2}\text{s}^{-1}$$

- Intensity

$$I = \frac{u^2}{Z} \quad \text{Watt/m}^2$$

- Power

$$\vec{P} = A \times \frac{u^2}{Z} \vec{e}_{propagation} \quad \text{Watt}$$

- Energy

$$Energy = \frac{1}{2} \int_v \frac{u^2}{\rho c^2} dv < \infty \quad \text{Joule}$$

Specially, in the acoustic calculations, a reference intensity is commonly used. It is calculated for $u = 10^{-6}$ and $\rho \times c = 1.5 \times 10^6$ as

$$I_{reference} = 0.67 \times 10^{-18} \quad \text{Watt/m}^2$$

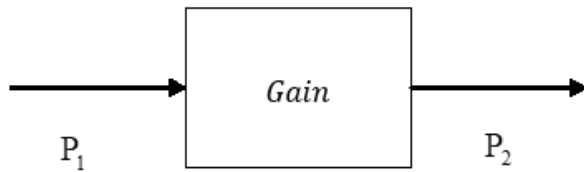
2.3. decibel CONCEPT

decibel (dB) unit is used for more easy and simple representation of very large and very small numbers. The decibel scale is a relative logarithmic scale. This is important in any engineering calculations.

dB unit can be formulated over Bell unit as

$$Bel = \log_{10} \left(\frac{X_2}{X_1} \right) \Rightarrow decibel = 10 \log_{10} \left(\frac{X_2}{X_1} \right) \text{ dB.}$$

dB unit is given as a ratio between quantities X_2 over X_1 . For example, if this quantity is power



$$decibel = 10 \log_{10} \left(\frac{P_2}{P_1} \right) > 0 \Rightarrow \text{Power Gain}$$

$$decibel = 10 \log_{10} \left(\frac{P_2}{P_1} \right) < 0 \Rightarrow \text{Power Loss}$$

For current gain and voltage gain, it is formulated as

$$\text{Current Gain} = 10 \log_{10} \left(\frac{P_2}{P_1} \right) = 10 \log_{10} \left(\frac{I_2^2 R}{I_1^2 R} \right) = 20 \log_{10} \left(\frac{I_2}{I_1} \right) \text{ dB}$$

$$\text{Voltage Gain} = 10 \log_{10} \left(\frac{P_2}{P_1} \right) = 10 \log_{10} \left(\frac{V_2^2 / R}{V_1^2 / R} \right) = 20 \log_{10} \left(\frac{V_2}{V_1} \right) \text{ dB}$$

All the quantities related to the acoustic pressure are generally given in a dB unit. For example, Source Level (SL) in the sonar equation is

$$SL = 10 \log \left(\frac{\text{source intensity at a defined distane}}{\text{reference intensity (1 } \mu\text{Pa)}} \right)$$

where the defined range is taken as 1 m from an acoustic center of the omnidirectional projector.

For the broadband signals, the sound levels are given over Spectrum Level (SpL) in 1 Hz band or Band Level (BL) in some frequency band (Δf). For example

$$SpL = 10 \log \left(\frac{I \text{ (in 1 Hz band)}}{I_{ref} \text{ (in 1 Hz band)}} \right)$$

and for a flat spectrum

$$BL = SpL + 10 \log(\Delta f).$$

2.4. Pascal UNIT

The commonly used unit of the acoustic pressure

$$1 \text{ Pa} = 1 \text{ Newton/m}^2 = 0.01 \text{ mbar}$$

A reference value for the plane waves is taken as 1 μPa for the water pressure

$$u \text{ [dB]} = 20 \log_{10} \left(\frac{u \text{ [Pa]}}{u_0 \text{ [Pa]}} \right) = 20 \log_{10} \left(\frac{u \text{ [Pa]}}{1 \mu\text{Pa}} \right) \quad [\text{dB re } 1 \mu\text{Pa}]$$

As an example

$$u = 20 \text{ dB} = 20 \text{ dB re } 1 \mu\text{Pa} = 20 \log_{10} \left(\frac{u \text{ [Pa]}}{1 \mu\text{Pa}} \right) \Rightarrow u = 10 \mu\text{Pa}$$

A reference value for the plane waves is taken as 20 μPa for the air pressure.

3. UNDERWATER ACOUSTIC EQUATIONS

The fundamental equations of underwater acoustics are obtained by linearized basic equations of fluid mechanics. Accordingly, three basic equations [Urick, 1982], namely

- State Equation,
- Continuity Equation
- Euler Equation

are underwater acoustic equations that model the underwater wave propagation.

- State Equation (SE)

$$u(\mathbf{r}, t) = \beta(\mathbf{r})s(\mathbf{r}, t)$$

- Linearized Continuity Equation (LCE)

$$\frac{\partial s(\mathbf{r}, t)}{\partial t} + \nabla \cdot \vec{\mathbf{v}}(\mathbf{r}, t) = 0 \quad \Rightarrow \quad \frac{1}{\beta(\mathbf{r})} \frac{\partial u(\mathbf{r}, t)}{\partial t} + \nabla \cdot \vec{\mathbf{v}}(\mathbf{r}, t) = 0$$

- Linearized Euler Equation (LEE)

$$\rho(\mathbf{r}) \frac{\partial \vec{\mathbf{v}}(\mathbf{r}, t)}{\partial t} + \nabla u(\mathbf{r}, t) = 0$$

where $s(\mathbf{r}, t)$ is concentration, $\beta(\mathbf{r})$ is elasticity of the medium, $\rho(\mathbf{r})$ is density of the medium.

3.1. EXTRACTION of ACOUSTIC WAVE EQUATION

Using three main acoustic equations, an acoustic wave equation can be extracted. To perform this, let us start to take divergence of the LEE

$$\nabla \cdot \left[\rho(\mathbf{r}) \frac{\partial \vec{\mathbf{v}}(\mathbf{r}, t)}{\partial t} \right] + \nabla \cdot \nabla u(\mathbf{r}, t) = 0 \quad \Rightarrow \quad \nabla \cdot \left[\rho(\mathbf{r}) \frac{\partial \vec{\mathbf{v}}(\mathbf{r}, t)}{\partial t} \right] + \Delta u(\mathbf{r}, t) = 0$$

where $\rho(\mathbf{r})$ can be evaluated by using a mathematical relation of $\nabla \cdot (\alpha(\mathbf{r}) \vec{\mathbf{A}}) = \nabla \alpha(\mathbf{r}) \cdot \vec{\mathbf{A}} + \alpha(\mathbf{r}) \nabla \cdot \vec{\mathbf{A}}$

$$\Rightarrow \Delta u(\mathbf{r}, t) + \left[\nabla \rho(\mathbf{r}) \cdot \frac{\partial \vec{\mathbf{v}}(\mathbf{r}, t)}{\partial t} + \rho(\mathbf{r}) \nabla \cdot \frac{\partial \vec{\mathbf{v}}(\mathbf{r}, t)}{\partial t} \right] = 0$$

One can use the LEE as $\partial \mathbf{v}(\mathbf{r}, t) / \partial t = -(1/\rho(\mathbf{r})) \nabla u(\mathbf{r}, t)$ and submit it to the above equation

$$\Rightarrow \Delta u(\mathbf{r}, t) + \left[-\nabla \rho(\mathbf{r}) \cdot \frac{\nabla u(\mathbf{r}, t)}{\rho(\mathbf{r})} + \rho(\mathbf{r}) \nabla \cdot \frac{\partial \vec{\mathbf{v}}(\mathbf{r}, t)}{\partial t} \right] = 0$$

One can use the LCE as $\nabla \cdot \vec{\mathbf{v}}(\mathbf{r}, t) = -(1/\beta(\mathbf{r})) \partial u(\mathbf{r}, t) / \partial t$ and submit it to the above equation

$$\Rightarrow \Delta u(\mathbf{r}, t) + \left[-\frac{\nabla \rho(\mathbf{r})}{\rho(\mathbf{r})} \cdot \nabla u(\mathbf{r}, t) - \frac{\rho(\mathbf{r})}{\beta(\mathbf{r})} \frac{\partial^2 u(\mathbf{r}, t)}{\partial t^2} \right] = 0$$

Now, let us define a propagation velocity

$$c(\mathbf{r}) = \sqrt{\frac{\beta(\mathbf{r})}{\rho(\mathbf{r})}}$$

In the sea water, $\rho = \rho(\mathbf{r})$ slowly varies. Therefore, it can be considered as a constant (practical). In this case

$$c(\mathbf{r}) = \sqrt{\frac{\beta(\mathbf{r})}{\rho(\mathbf{r})}} = \sqrt{\frac{\beta(\mathbf{r})}{\rho}}$$

and rearranging the last equation

$$\Delta u(\mathbf{r}, t) - \frac{\vec{\nabla} \rho}{\rho} \cdot \nabla u(\mathbf{r}, t) - \frac{1}{c^2(\mathbf{r})} \frac{\partial^2 u(\mathbf{r}, t)}{\partial t^2} = 0.$$

Simplifying this equation, an acoustic wave equation is found to be

$$\Delta u(\mathbf{r}, t) - \frac{1}{c^2(\mathbf{r})} \frac{\partial^2 u(\mathbf{r}, t)}{\partial t^2} = 0$$

In fact, a similar equation can also be found for $\vec{v}(\mathbf{r}, t)$. But, it is not widely used due to vector form.

3.2. SOLUTION of ACOUSTIC WAVE EQUATION

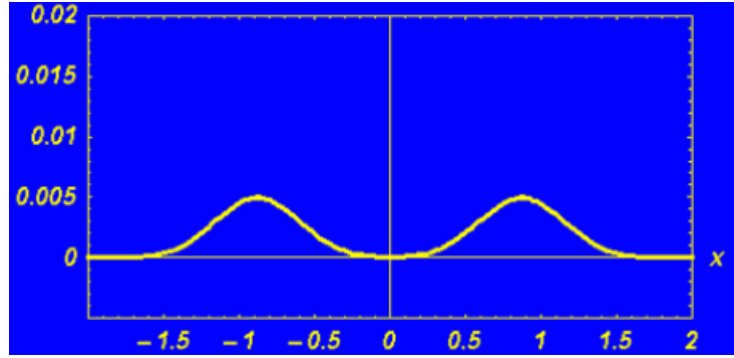
Let us consider one-dimensional acoustic wave equation for simplicity (c is constant)

$$\frac{\partial^2}{\partial x^2} u(x, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} u(x, t) = 0$$

d'Alembert solution for infinite space (travelling waves) is

$$u(x, t) = f(x - ct) + g(x + ct)$$

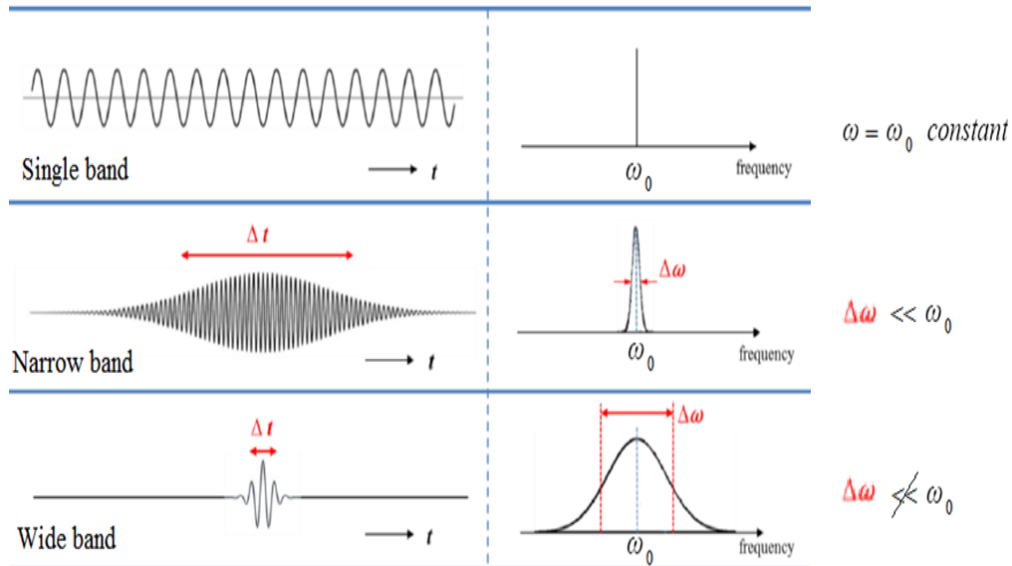
where $f(x - ct)$ shows $+x$ direction propagated waves with a velocity of c (right direction) and $g(x + ct)$ shows $-x$ direction propagated waves with a velocity of c (left direction). This behavior is shown below.



d'Alembert solution for infinite space (travelling waves).

3.3. EXTRACTION of HELMHOLTZ EQUATION

In practice, many applications are based on monochromatic (or narrow band) sources. This and the other type sources (signals) are shown below.



Single band, narrow band and wide band signals.

The monochromatic (or single frequency) sources produce the monochromatic fields as

$$u(x, t) = A \cos(\mathbf{kx} - \omega t) = \text{Re}\{A e^{i\mathbf{kx}} e^{-i\omega t}\} = \text{Re}\{u(\mathbf{x}) e^{-i\omega t}\}$$

where $u(\mathbf{x})$ is known as a **phasor** of $u(x, t)$. Now, let us substitute the phasor form into the wave equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 u(x, t)}{\partial t^2} = 0 \quad \Rightarrow \quad \frac{\partial^2}{\partial x^2} \text{Re}\{u(\mathbf{x}) e^{-i\omega t}\} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \text{Re}\{u(\mathbf{x}) e^{-i\omega t}\} = 0$$

$$\Rightarrow \quad \text{Re}\left\{\left(\frac{\partial^2}{\partial x^2} u(\mathbf{x})\right) e^{-i\omega t}\right\} - \frac{1}{c^2} \text{Re}\left\{u(\mathbf{x}) \left(\frac{\partial^2}{\partial t^2} e^{-i\omega t}\right)\right\} = 0$$

$\underbrace{\left(\frac{\partial^2}{\partial t^2} e^{-i\omega t}\right)}_{-\omega^2 e^{-i\omega t}}$

Then

$$\operatorname{Re}\left\{\left(\frac{\partial^2}{\partial x^2}u(x)\right)e^{-i\omega t}\right\} + \frac{\omega^2}{c^2}\operatorname{Re}\{u(x)e^{-i\omega t}\} = 0 \Rightarrow \boxed{\frac{\partial^2}{\partial x^2}u(x) + k^2u(x) = 0}$$

where the last one is known as **Helmholtz equation** and $k = \omega/c$ shows the wavelength.

More generally (3D case), Helmholtz equation is

$$\Delta u(\mathbf{r}) + k^2(\mathbf{r})u(\mathbf{r}) = 0$$

where $k(\mathbf{r}) = \omega/c(\mathbf{r})$ is position dependent.

In this way of the solution, the acoustic pressure $u(\mathbf{r})$ is a complex-number as

$$u(\mathbf{r}) = a + jb = |u(\mathbf{r})|e^{i\varphi}$$

where $|u(\mathbf{r})| = \sqrt{a^2 + b^2}$ and $\varphi = \arctan(b/a)$ show the amplitude and phase of the acoustic pressure.

The **main problem** of underwater acoustic wave propagation is to calculate the acoustic pressure $u(\mathbf{r})$ in underwater medium. This can be performed by solving Helmholtz equation with proper additional (boundary and continuity) conditions. These conditions are listed below:

3.3.1. Boundary Conditions

- Dirichlet (pressure release) boundary condition

$$u|_S = 0$$

- Neumann (rigid) boundary condition

$$\left.\frac{\partial u}{\partial n}\right|_S = 0$$

- Impedance boundary condition

$$\left.\frac{u(\mathbf{r})}{\partial u/\partial n}\right|_S = \xi$$

3.3.2. Continuity Conditions

- Field continuity: $u_1|_S = u_2|_S$

- Particle velocity (displacement) continuity: $\left.\frac{1}{\rho_1}\frac{\partial u_1}{\partial n}\right|_S = \left.\frac{1}{\rho_2}\frac{\partial u_2}{\partial n}\right|_S$

3.4. PROPAGATION LOSSES

As noted before, to understand the importance of $u(\mathbf{r})$ in the sense of engineering aspect, it is necessary to remember sonar equations

$$SE = (SL - PL) - N - (DT - DI) \text{ for passive transmission}$$

$$SE = (SL + TS - 2PL) - N - (DT - DI) \text{ for active transmission}$$

where PL shows the Propagation Loss.

The propagation Loss (PL) is a measure of reduction in sound intensity between source and a distant receiver and can be formulated over the receiver and the source intensities as

$$PL = -10 \log \left(\frac{I_{receiver}}{I_{source} (r = 1 \text{ m})} \right).$$

The Propagation Loss (PL) is composed of

- Spreading Loss
- Absorption Loss
- Other Losses

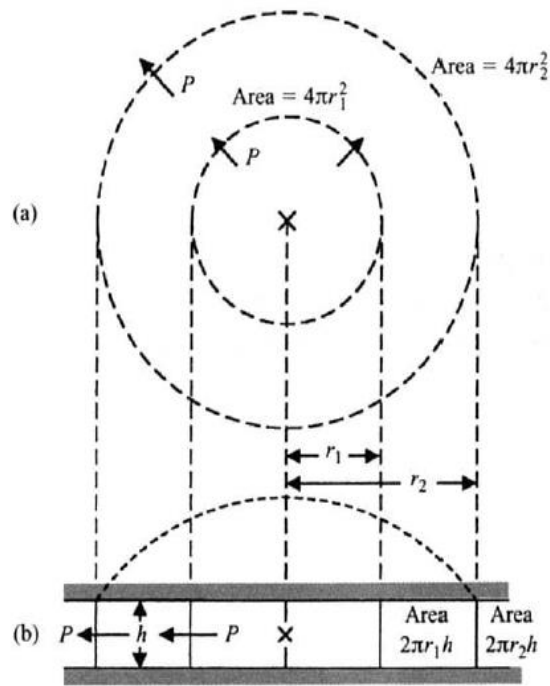
The propagation Loss (PL) is also known as a Transmission Loss (TL) and can be formulated alternatively as

$$TL = PL = -20 \log \left(\frac{u}{u_0 (r = 1 \text{ m})} \right).$$

Now, let us analyze the composition of the Propagation Loss.

3.4.1. Spreading Loss

In an unbounded and lossless medium, the acoustic source power is radiated equally in all directions as shown below. In this case, the power conservation gives us the spreading loss. Two types are the spherical spreading (3 dimension) and the cylindrical spreading laws (2 dimension).



The acoustic source power radiation and the power conservation concept in 3D and 2D.

- **The Spherical Spreading Law:** The power conservation leads to

$$P_1 = P_2 \Rightarrow 4\pi r_1^2 I_1 = 4\pi r_2^2 I_2 \Rightarrow \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

where

$$r_1 = 1 \text{ m} , r_2 = r \Rightarrow PL = -10 \log \left(\frac{I_2}{I_1} \right) = -20 \log \left(\frac{1}{r} \right) = 20 \log(r)$$

This is known as an inverse square law.

- **The Cylindrical Spreading Law:** The power conservation leads to

$$P_1 = P_2 \Rightarrow 2\pi r_1 h I_1 = 2\pi r_2 h I_2 \Rightarrow \frac{I_1}{I_2} = \frac{r_2}{r_1}$$

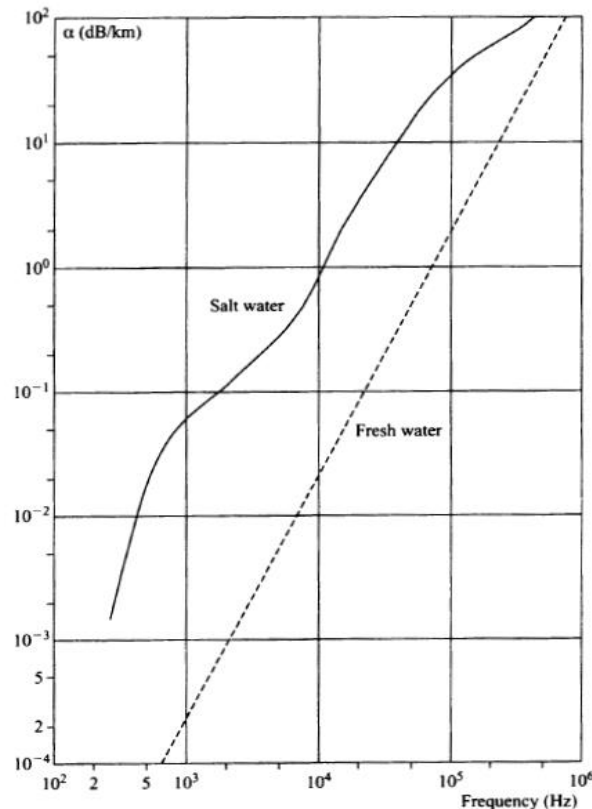
where

$$r_1 = 1 \text{ m} , r_2 = r \Rightarrow PL = -10 \log \left(\frac{I_2}{I_1} \right) = -10 \log \left(\frac{1}{r} \right) = 10 \log(r).$$

3.4.2. Absorption Losses

The absorption losses in the sea occur through two principal mechanisms of viscosity and molecular relaxation.

- **The Viscosity Loss:** They are losses due to viscosity are present in fresh water and salt water. It can be represented by an attenuation coefficient α [dB/km] as shown in below log-log graph. The strong frequency dependency is present for the salt water and the fresh water. Specially, the plot is a straight line for the fresh water.



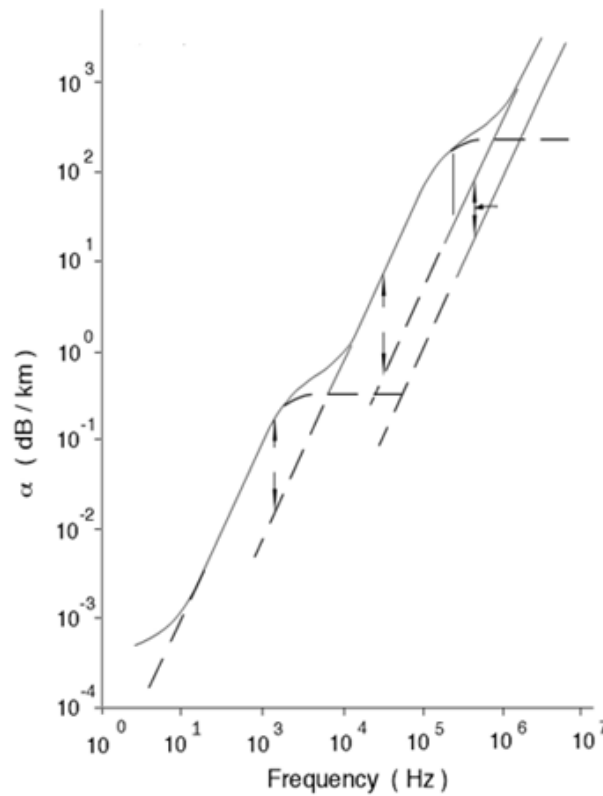
The attenuation coefficient α [dB/km] for the salt water and the fresh water.

- **The Molecular Relaxation Loss:** It is only present in the salt water. This is due to the presence of the saltwater ions (salinity). It can also be presented by attenuation coefficient α [dB/km] as an approximation for the standard sea water

$$\alpha = 0.05 \times f^{1.4} \text{ dB/km}$$

where being as an example $\alpha = 5$ dB/km for $f = 30$ kHz.

For the real sea conditions, extensive measurements of these losses are necessary. As an example of this for the frequency dependence of the molecular relaxation loss is shown below.



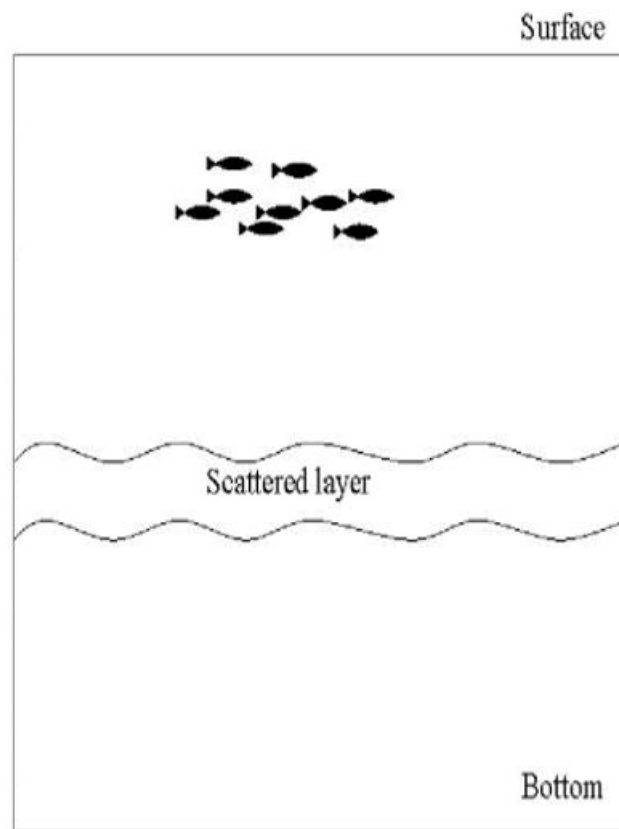
The frequency dependence of the molecular relaxation loss.

The absorption losses can be taken into account by summing with spherical spreading losses as

$$PL = 20 \log(r) + \underbrace{\alpha \times r \times 10^{-3}}_{\text{Absorption Loss}} \text{ dB}$$

3.4.3. The Other Losses

In the real sea environment, scattering from marine animals and sea layers, reflections from upper and lower sea boundaries causes in additional losses for the underwater wave propagation. Due to geometrical location dependency of these kinds of losses, it is still being expended on developing reliable propagation models which take account of this. A schematic representation of these losses is shown in below figure.



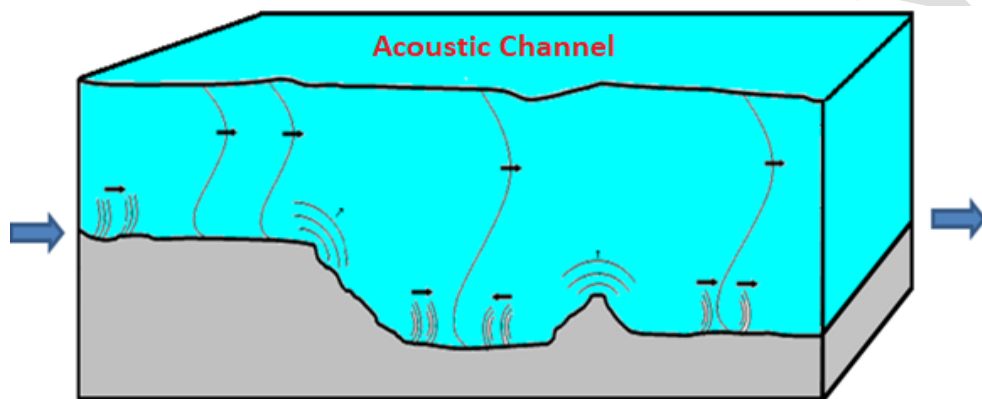
A schematic representation of the other losses.

4. REVERBERATION

In the calculation of the PL and acoustic pressure, reverberation must be taken into account. In this sense, the sources of the **scattering** in the real sea are

- marine life,
- matters distributed in the sea,
- inhomogeneous structure of the sea,
- reflections from sea surface,
- reflections from sea bottom.

A representative figure for the real sea is shown below.



A representative figure for the real sea.

Backscattering means that incident sound energy reflected back to the source. **Reverberation** is backscattering acoustic energy.

Backscattering Strength is the fundamental parameter that decides the intensity of the reverberation.

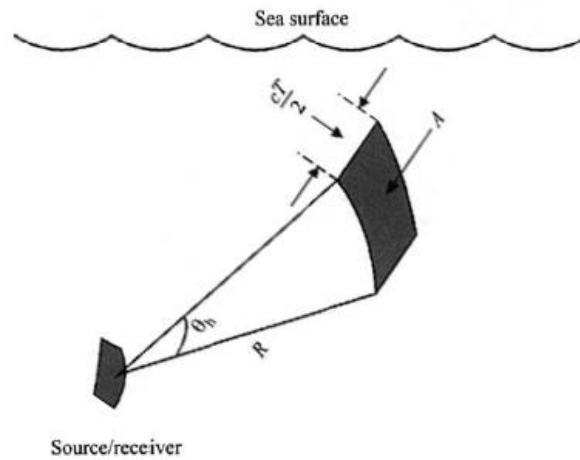
$$SS_{backscattering} = 10 \log \left(\frac{I_{scattering}}{I_{source}} \right)$$

Reverberation Type are

- Sea surface reverberation,
- Sea bottom reverberation,
- Volume reverberation.

4.1. SEA SURFACE REVERBERATION

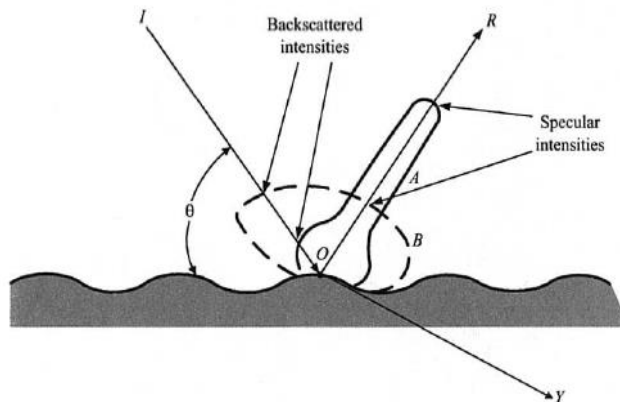
This reverberation originates from scatterers within a volume of the sea. It is shown below.



Scattering of sound from a rough sea surface boundary.

4.2. SEA BOTTOM REVERBERATION

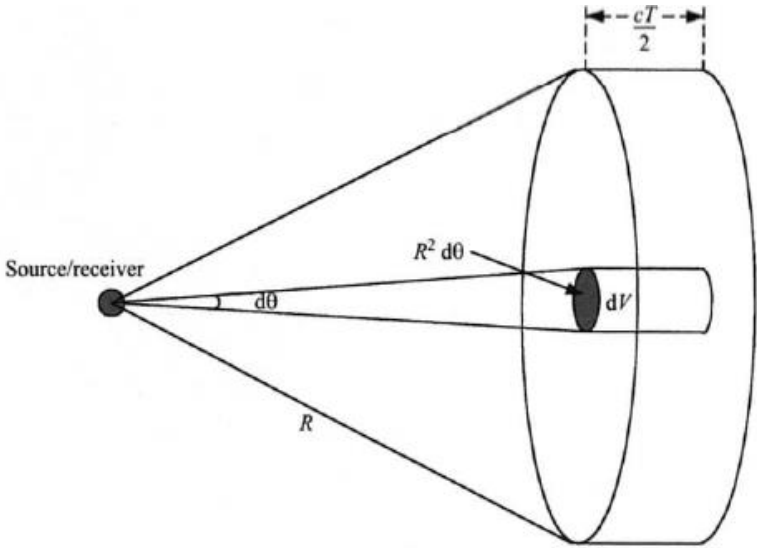
This reverberation originates from scatterers spread over an area of the surface of the sea. It is shown below.



Scattering of sound from a rough sea bottom boundary.

4.3. VOLUME REVERBERATION

This reverberation originates from scatterers within a volume of the sea. It is shown below.

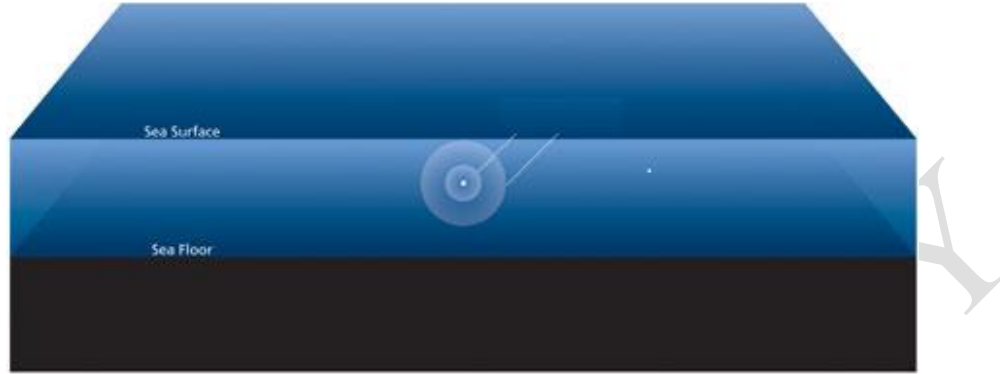


Scattering of sound from a volume of the sea.

Dr. Serkan ALIYEV

5. SOLUTION of HELMHOLTZ EQUATION

To find the acoustic pressure $u(\mathbf{r})$ in underwater medium, the problem geometry must be considered. The geometry of the underwater acoustic medium can be considered as a kind of a **waveguide** shown below.



A waveguide model for the underwater acoustic medium.

In the three-dimensional waveguide model, there are two layers as a water layer and a sediment layer. In practice, this three-dimensional model can be reduced to a two-dimensional model by considering an axial-symmetry along θ in cylindrical coordinate system. It means that the acoustic pressure will be a function of r and z as $u(\mathbf{r}) = u(r, z)$.

Now, let us evaluate the case of the axial-symmetry along θ for Helmholtz equation. In three-dimensional cylindrical coordinate system, Helmholtz equation can be written as

$$\Delta u(\mathbf{r}) + k^2(\mathbf{r})u(\mathbf{r}) = f(\mathbf{r}) \Rightarrow \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} + k^2(r, \theta, z) \right] u(r, \theta, z) = f(r, \theta, z)$$

where the axial-symmetry along θ means that $\partial(\cdot)/\partial\theta = 0$ in the above equation that is

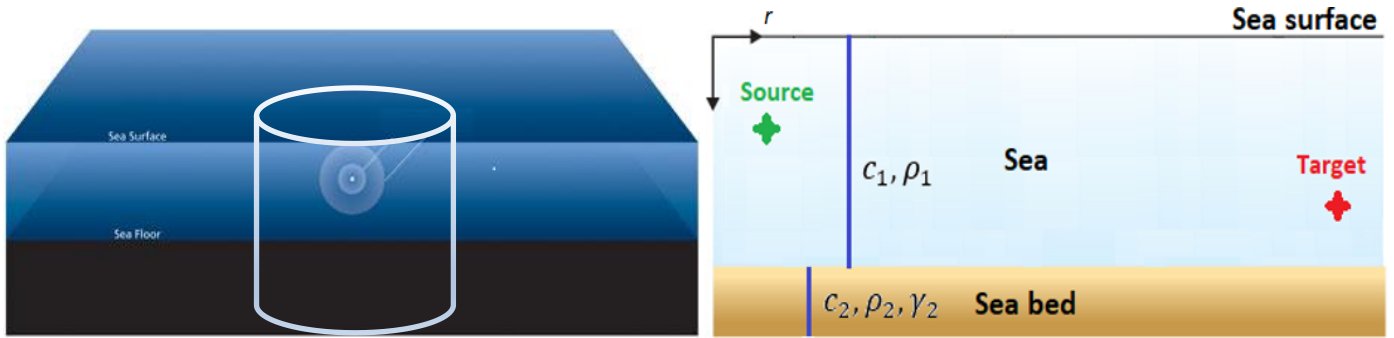
$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2} + k^2(r, z) \right] u(r, z) = f(r_0, z_0).$$

In practice, the range (r) dependence of the wavelength (k) is generally negligible in which only $k = k(z)$. In this case, Helmholtz equation becomes

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2} + k^2(z) \right] u(r, z) = f(r_0, z_0).$$

where $k(z) = \omega/c(z)$ is the z dependent wavelength.

Then, an equivalent 2D acoustic problem can be settled out as shown below.



An equivalent 2D acoustic problem for the underwater medium.

In the underwater acoustic literature, this equivalent problem is known as a **Pekeris waveguide** [Pekeris, 1948]. Although c, ρ and γ can be considered as constants at a first glance, specially, the depth dependence of the sound velocity $c = c(z)$ cannot be ignored in practice. Therefore, it will be discussed in the next section.

5.1. SOUND SPEED PROFILE (SSP)

The depth dependence of the sound velocity $c = c(z)$ is generally known as a **Sound Speed Profile (SSP)**. In fact, in the sea water, the sound velocity is also dependent on temperature and salinity as

$$c = c(T, S, z)$$

where T is temperature ($^{\circ}\text{C}$), S is salinity (ppt: parts per thousand) and z (m) is water column depth. Although a variety of empirical formulae exist for c calculation, commonly used one is given by

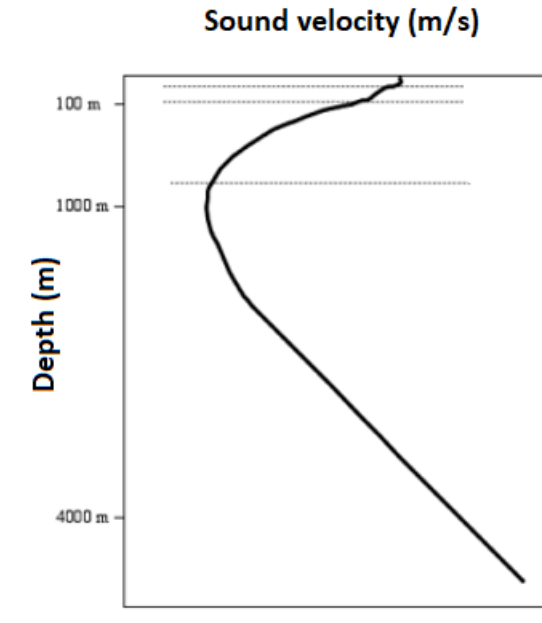
$$c = c(T, S, z) = 1492.9 + 3(T - 10) - 6 \times 10^{-3} (T - 10)^2 - 4 \times 10^{-2} (T - 18) + 1.2(S - 35) - 10^{-2} (T - 18)(S - 35) + z/61$$

As an example, $c = 1490$ m/s is found for $T = 10$ $^{\circ}\text{C}$, $S = 35$ ppt and $z = 0$ m (sea surface).

One of the other well-known depth-dependent SSP for the deep water is Munk profile formulated below

$$c = c(z) = 1500 \left[1 + 0.00737 \left[\frac{2(z - 1300)}{1300} + e^{-\frac{2(z-1300)}{1300}} - 1 \right] \right] \text{ m/sn}$$

where z (m) shows water column depth. Let us draw the z dependence of Munk profile.



The z dependence of Munk profile.

5.2. SOLUTION METHODS OF HELMHOLTZ EQUATION

Two main approaches for the solution of Helmholtz equations are Ray Method and Wave (Mode) Method.

5.2.1. Ray Method

The ray method is based on wavefronts and existence of rays which indicate where the sound from the source is propagating. It is based on a **Luneberg Kline series** expansion of $u(\mathbf{r}, \omega)$ as

$$u(\mathbf{r}, \omega) \cong e^{ik\psi(\mathbf{r})} \sum_{n=0}^{\infty} \frac{u_n(\mathbf{r})}{(-i\omega)^n} \Rightarrow u(\mathbf{r}, \omega) \cong u_0(\mathbf{r})e^{ik\psi(\mathbf{r})}$$

where the higher-order terms of this series can be neglected at high frequencies. Therefore, the solution can be constructed from the first term of the series. This method is a kind of high-frequency approximation. Therefore, it is restricted to short wavelengths.

Helmholtz equation can be arranged for $u_0(\mathbf{r})$ and $\psi(\mathbf{r})$ as

$$2\nabla u_0(\mathbf{r})\nabla\psi(\mathbf{r}) + u_0(\mathbf{r})\Delta\psi(\mathbf{r}) = 0$$

$$|\nabla\psi(\mathbf{r})|^2 = k^2$$

where the first and the second equations are known as **Transport equation** and **Eikonal equation**, respectively. Their interpretation (solution) gives a chance to trace **rays** in the underwater acoustic medium.

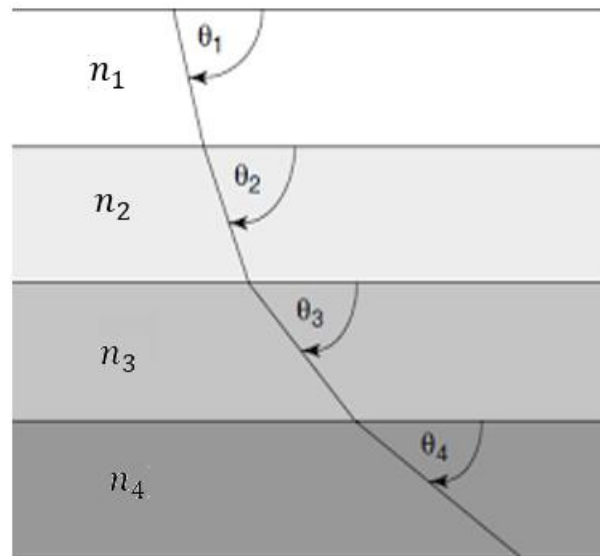
In fact, the ray method is similar to a Snell law used in geometric optics. In this sense, considering the medium index as

$$n(\mathbf{r}) = \frac{c_0}{c(\mathbf{r})}$$

and Snell law for layered medium

$$\frac{\cos\theta_1}{c_1} = \frac{\cos\theta_2}{c_2} = \dots = \frac{\cos\theta}{c}$$

the ray bending can be predicted as shown below for a four layered medium.



The representation of ray bending.

Some highlighted features of the Ray Method can be listed as

- It is valid at high frequencies.
- The rays describing the acoustic propagation are represented by curves.
- Paths of propagation are easy to interpret, but difficult to understand mathematically.
- It is easy to apply real boundary conditions.
- The propagation losses cannot be calculated everywhere in the problem space.
- It is desired for active transmissions of the high frequencies.

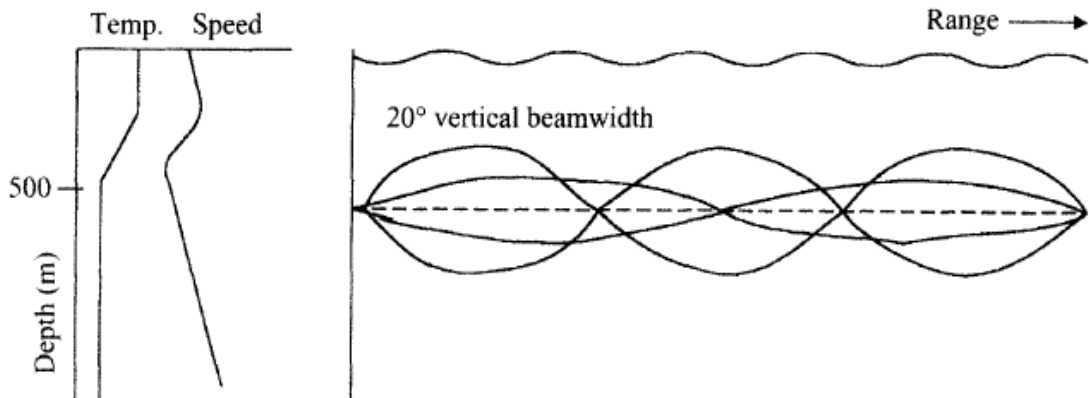
Common RAY codes are

- [LYBIN](#)
- [BELLHOP](#)

5.2.1.1. Special Propagation Paths by Ray Theory

5.2.1.1.1. Deep Sound Channel

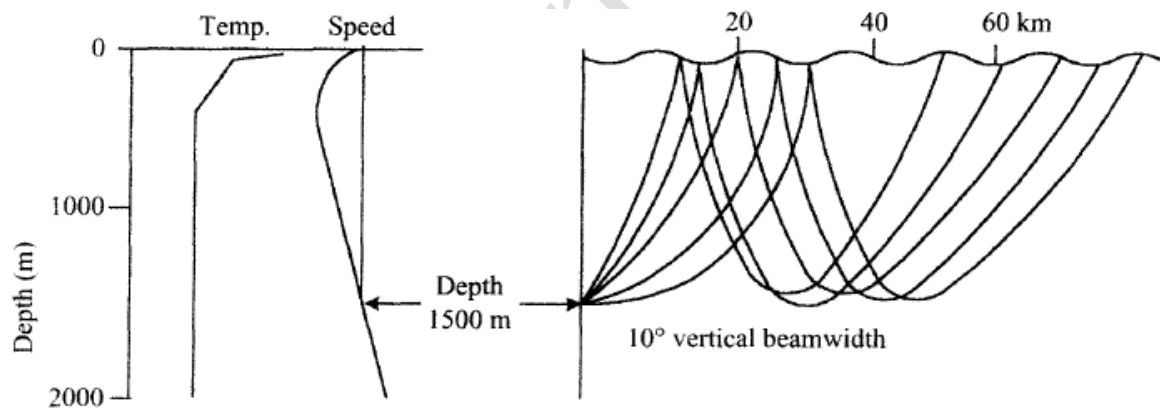
At a depth where the sound speed is minimum, the sound tends to be focused. The deep sound channel propagation occurs at this depth that is also known as **SOFAR** (Sound Fixing and Ranging) channel. In this channel, a very long-range propagation is possible. This is shown below.



The deep sound channel propagation.

5.2.1.1.2. Reliable Acoustic Path

The reliable acoustic path exists when the source is placed at a depth (critical depth) where the sound speed is equal to the sound speed at the surface. It is shown below.

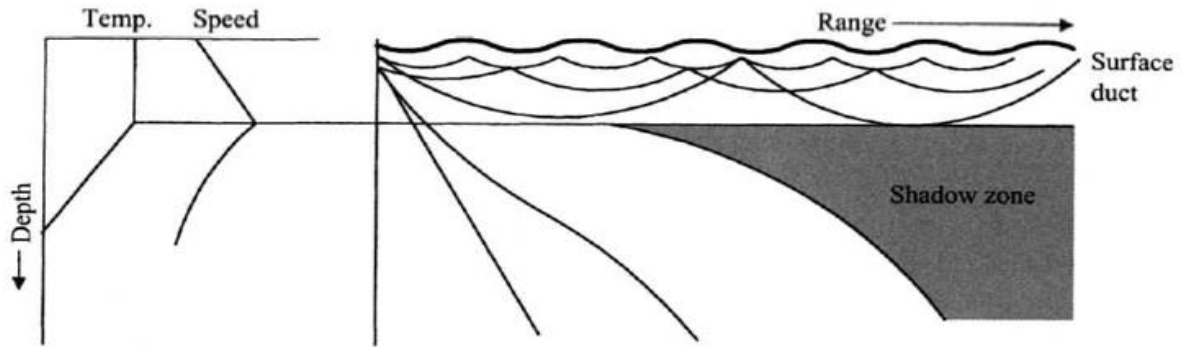


The reliable acoustic path.

This is called “**reliable acoustic path**” since it is insensitive to the surface scattering and the bottom scattering. It means that this is a direct path between the source and the receiver.

5.2.1.1.3. Surface Duct Propagation

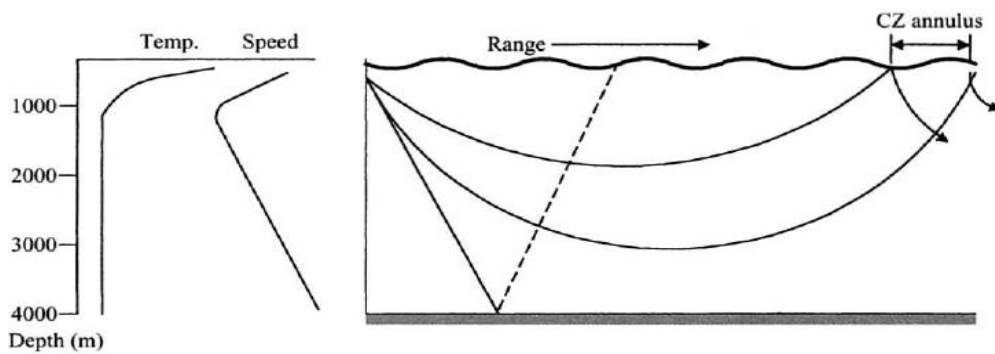
An isothermal surface layer (surface duct) occurs because of surface winds and waves. A shadow zone is observed in which it is difficult to detect the targets. This is shown below.



The surface duct propagation.

5.2.1.1.4. Convergence Zone Propagation

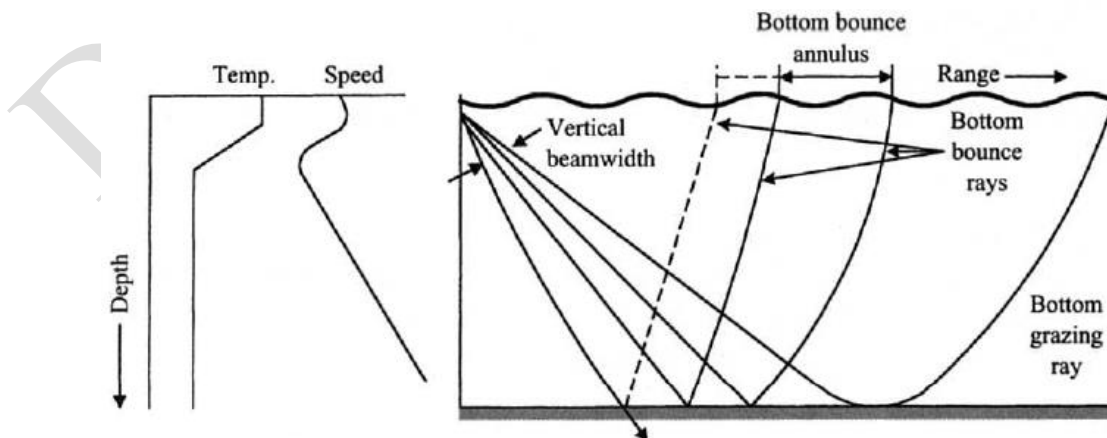
At greater depths than the surface duct propagation, pressure bends these rays and constructs Convergence Zone (CZ) annulus. The successive CZs are possible. It is shown below.



The convergence zone propagation.

5.2.1.1.5. Bottom Bounce Propagation

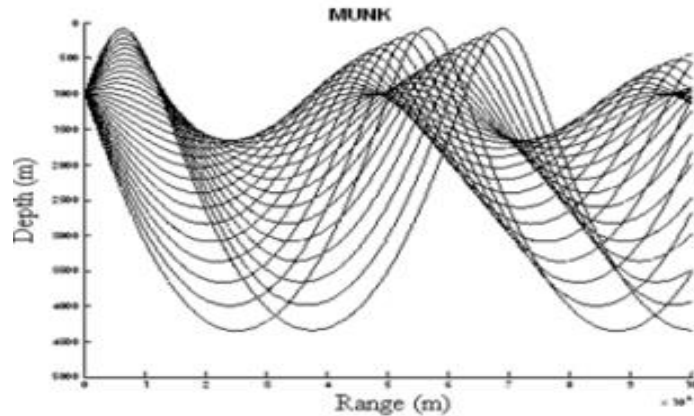
It is strongly depended on bottom material. The Bottom Bounce (BB) annulus is present like the CZ annulus. This is shown below.



The bottom bounce propagation.

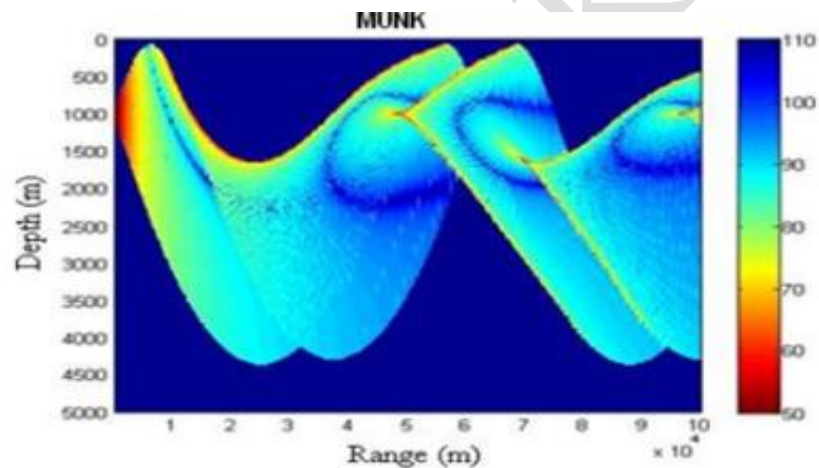
5.2.1.1.6. Munk Profile

In the case of Munk profile for the deep water, the distributions of the rays for the representation of the acoustic propagation are shown below.



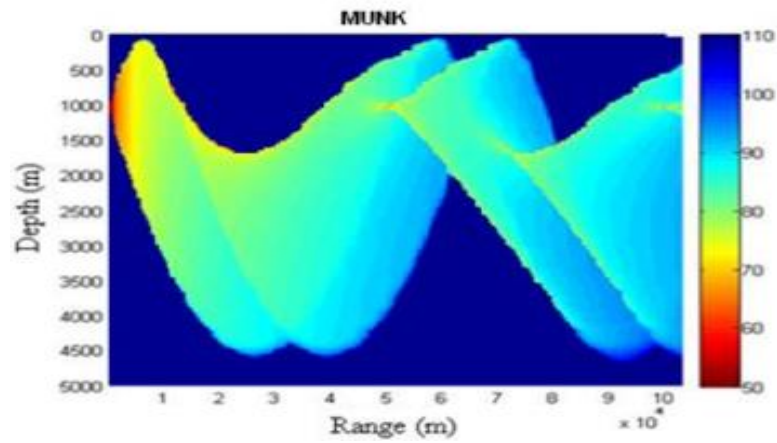
The rays of the acoustic propagation for Munk profile.

Using these rays, Transmission Loss (TL) distribution can be calculated as below.



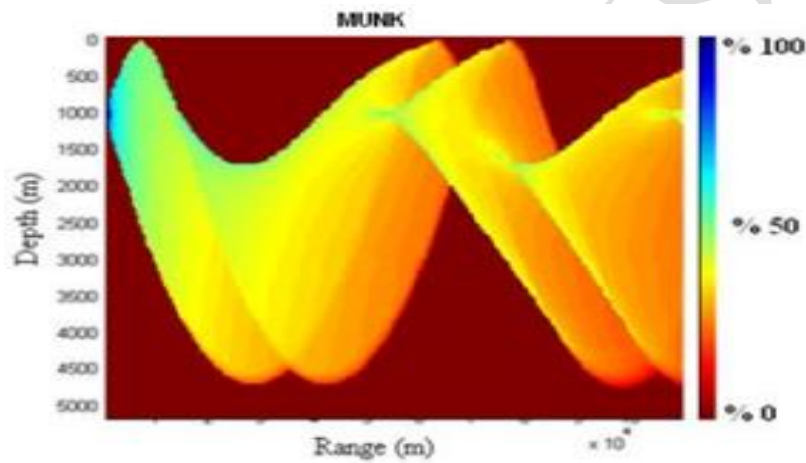
The Transmission Loss (TL) distribution.

However, as it can be seen in the above figure, there are some anomalies observed in the caustic regions. This is related to some kind of singularities in the classical Ray Method. To overcome this problem, a Gaussian Beam Ray Tracing Method as an alternative is proposed that it gives more realistic results. Accordingly, the cured Transmission Loss (TL) distribution is shown below.



The Transmission Loss (TL) distribution calculated by an alternative method.

In the new Transmission Loss (TL) distribution, no anomalies as before are observed. The probability of the detection can also be calculated by the Ray Method as shown below.

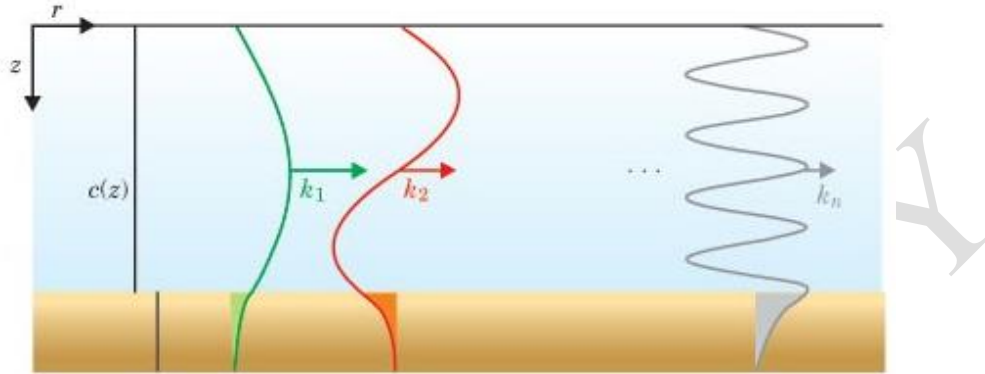


The probability of the detection.

Evaluating the above figure, **avoidance zones** can be easily defined.

5.2.2. The Wave Method (Mode Theory)

The acoustic propagation is described in terms of independent solutions called **Normal Modes**. The solution is composed of the summing of the modes. They satisfy the necessary boundary and continuity conditions. To explain this, first of all, consider the underwater medium in two-dimensional coordinates (r, z) where r and z show that range and depth in shown below.



The underwater medium in two-dimensional coordinates (r, z) .

Now, let us remember Helmholtz equation in 2D cylindrical coordinate ($\partial(\cdot)/\partial\theta = 0$)

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2} + k^2(z) \right] u(r, z) = f(r, z_0).$$

where $k(z) = \omega/c(z)$. The acoustic field (the solution) over can be written by a separation of variables technique ($u(r, z) = R(r)Z(z)$) for **an unforced equation**

$$\frac{d^2 Z(z)}{dz^2} + k_z^2 Z(z) = 0$$

$$\frac{d^2 R(r)}{dr^2} + \frac{1}{r} \frac{dR(r)}{dr} + k_r^2 R(r) = 0$$

where these differential equations are known as Sturm-Liouville type and Bessel differential equations, respectively. k_z and k_r are the wavelength along z and r directions, respectively ($k^2 = k_r^2 + k_z^2$). Using projection over the series expansion of the source function and some features of the Sturm-Liouville type differential equations such as orthogonality, the solution is obtained as

$$u(r, z) = \sum_{n=0}^{\infty} u_n(r, z) = \sum_{n=0}^{\infty} A_n \sin(k_{zn} z_0) \sin(k_{zn} z) \frac{e^{ik_{rn} r}}{\sqrt{k_{rn} r}}$$

In this solution, some features are

- Every $u_n(r, z)$ term is known as a **mode** or a **normal mode**.
- Each $u_n(r, z)$ term is solution and satisfies the boundary and continuity conditions.
- The solution is a combination of all $u_n(r, z)$.

Some highlighted features of the Mode Method can be listed as

- It is valid at all frequencies.
- It provides a complete and precise solution.
- The acoustic fields describing the acoustic propagation are represented by modes.
- It is difficult to understand mathematically.
- It is difficult to apply real boundary conditions.
- The propagation losses can be calculated everywhere in the problem space.
- It is generally desired for passive transmissions of the low frequencies.

Common MODE codes are

- KRAKEN

In the Normal Mode method, the key point is to calculate the wavenumber along the range direction k_{rn} as

$$k_{rn} = \sqrt{k^2 - k_{zn}^2}$$

where two main cases are evaluated

- k_{rn} is a real number

$$k_{rn} = \sqrt{k^2 - k_{zn}^2} > 0 \Rightarrow e^{ik_{rn}r} ; \text{ Propagation along } r$$

- k_{rn} is an imaginary number

$$k_{rn} = \sqrt{k^2 - k_{zn}^2} > 0 \Rightarrow e^{-k_{rn}r} ; \text{ Damping along } r.$$

Now, we have to find a critical condition for these two cases that is known as a **cut-off condition**

$$k_{rn} = \sqrt{k^2 - k_{zn}^2} = 0 \Rightarrow k^2 = k_{zn}^2 \Rightarrow \frac{\omega^2}{c^2} = k_{zn}^2 \Rightarrow \boxed{f = f_{cut-off} = \frac{c}{2\pi} k_{zn}}$$

where f is an source frequency which can principally have arbitrary values. However, $f_{cut-off}$ is a function of c and k_{zn} . It means that $f_{cut-off}$ has discrete values since k_{zn} can be principally formulated for a simple geometry as

$$k_{zn} = \frac{n\pi}{d}$$

where $n = 0, 1, 2, 3, \dots$ is an integer number and d is the depth of the water column. In fact, k_{zn} is found by solving the following equation along depth

$$\left[\frac{\partial^2}{\partial z^2} + k_{zn}^2 \right] Z(z) = F(z_0)$$

with application of the suitable boundary conditions. This leads to a discrete cut-off frequency formula

$$f_{cut-off} = f_n = \frac{c}{2\pi} \left(\frac{n\pi}{d} \right)$$

Thus, linking the source frequency f , one can conclude that

- $f > f_n$ corresponds to the propagation along r ,
- $f < f_n$ corresponds to the damping along r .

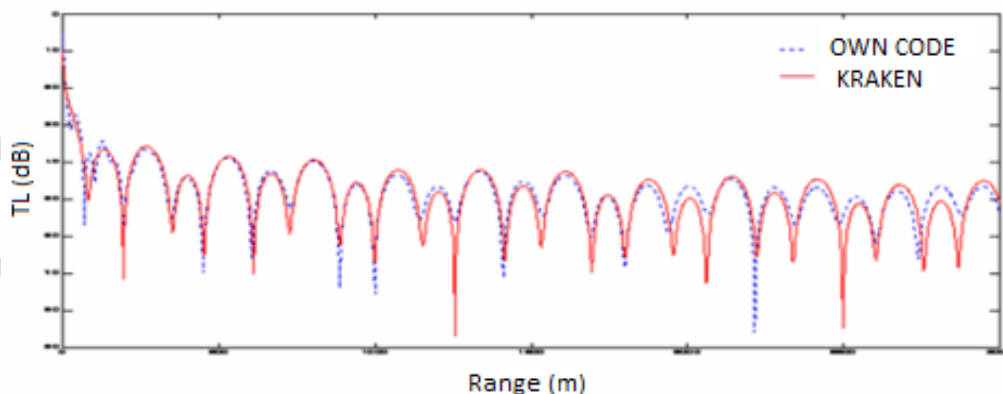
Accordingly, a finite number propagation of modes is possible due to the finite values of the source frequencies. The damping modes are known as **evanescent modes** while the others are known as **radiated modes**.

As an example, let us reconsider the Pekeris waveguide as shown below.



A schematic view of the Pekeris waveguide.

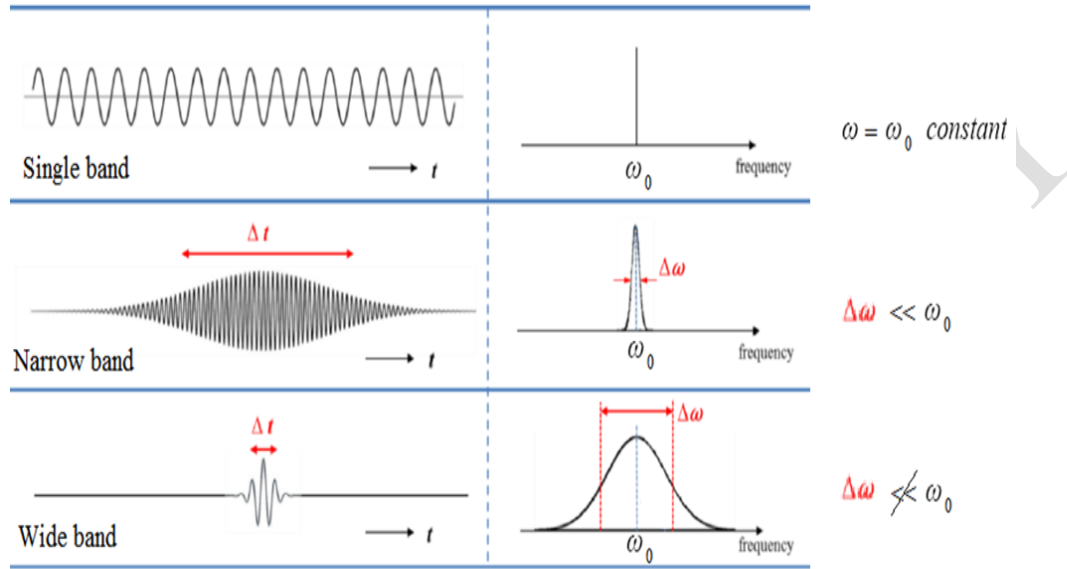
The problem space has two-layers of water and sediment parts. A target is present as a noise source and hydrophones are assumed to be receiver. Accordingly, the Normal Mode calculated transmission loss variation over the range for a one position of the source and hydrophone is shown below. The results are calculated and compared by KRAKEN program and OUR CODE (KITMİR) [Sertlek et al, 2008].



The transmission loss variation over the range.

5.3. TIME DOMAIN CALCULATIONS

In fact, the solution of Helmholtz equation is efficient for a narrow-band source. However, in practice, the time domain calculations are necessary for wide-band signals. They have many frequencies. The wide-band signals can have different forms such as impulsive type, Gaussian type, modulated type and so on. In this sense, let us remember again the concept of the single, narrow-band and wide-band signals, inspecting the following figure.



Single band, narrow band and wide band signals.

There are two main ways for the time domain calculations that are

- applications of the direct time domain methods,
- using inverse Fourier (or Laplace) transform.

5.3.1. Direct Time Domain Methods

One of the most widely used numerical method is Finite Difference Time Domain (FDTD) method in this area. By using the FDTD method, the acoustic wave equation can be directly solvable in time domain. To explore, the mentality of the FDTD method, let us consider one dimensional acoustic wave equation for simplicity

$$\frac{\partial^2 u(x, t)}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 u(x, t)}{\partial t^2} = f(x_0, t).$$

Now, using numerical central differences for analytical derivatives, an iterative discrete form of the equation is obtained as

$$u_i^{n+1} = 2u_i^n - u_i^{n-1} + (c\Delta t)^2 \left[\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2} \right]$$

where the acoustic pressure has a discrete form of

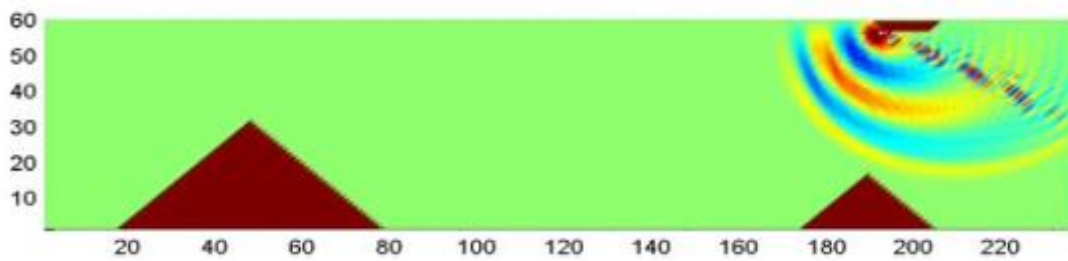
$$u(x, t) = u(i\Delta x, n\Delta t) = u(i, n) = u_i^{n+1}.$$

and, for example, the monochromatic source $f(x_0, t)$ can be applied as a boundary condition as

$$f(x_0, t) = \delta(x - x_0)\sin(\omega_0 t)$$

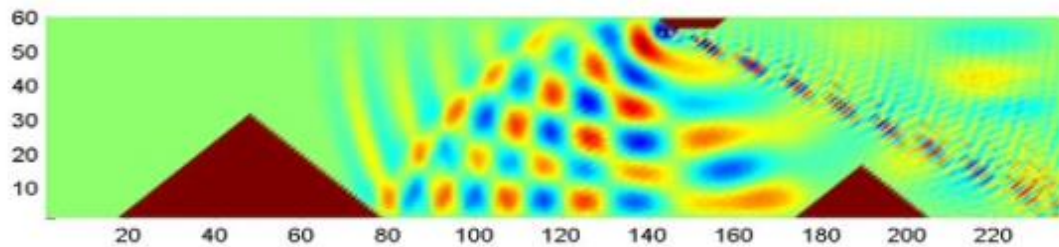
where $\delta(\cdot)$ is Dirac-Delta function and ω_0 is the angular frequency.

Using the FDTD method, a 2D underwater problem is solved as an example. In this problem, a surface ship moving with a constant speed and two underwater hills (small and big) are considered. The upper and lower boundaries of the problem space are considered as a pressure-release boundary condition. A single frequency source is excited at the lower part of the surface ship. At the beginning of the surface ship position, the acoustic field distribution is shown below. In this case, the radiation of the acoustic waves from the surface ship is clear in the form of cylindrical spreading.

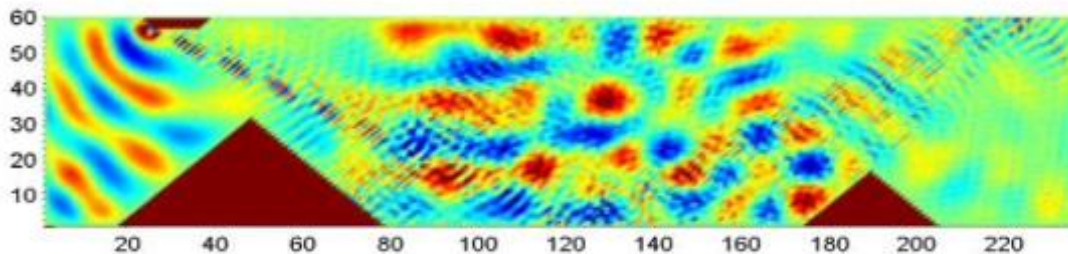


The acoustic field distribution at the beginning position of the surface ship.

Two more results corresponding to the middle and end of the surface ship positions are also shown below.



The acoustic field distribution at the middle position of the surface ship.

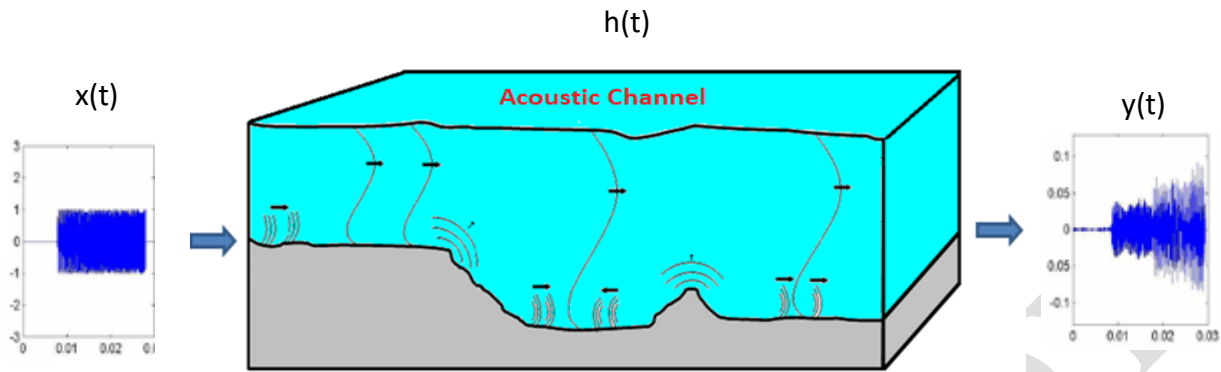


The acoustic field distribution at the end position of the surface ship.

At the middle position case, the construction of the modes is clearly visible. At the end position of the scenario, due to diffractions, more complex field distribution is observed.

5.3.2. Inverse Fourier Transform

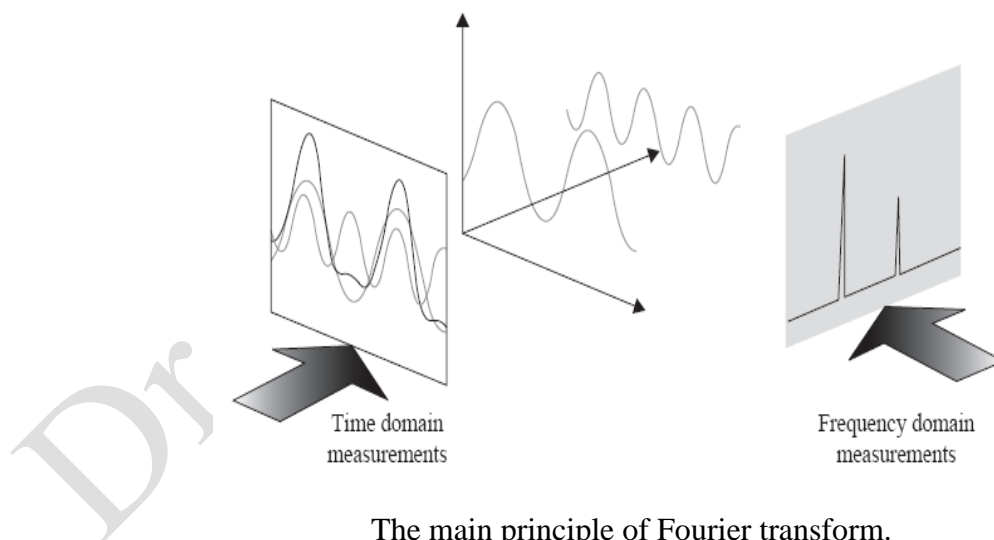
In reality, the underwater acoustic environment has very complex geometry and structure shown below.



The realistic underwater acoustic environment.

In this environment, an output signal in time domain can be calculated by using inverse Fourier transform using the frequency domain (Helmholtz equation) solution for an input signal. To perform this, the solution of the Helmholtz equation solution must be obtained for every frequency component of the source signal. It means that if the source signal has wide band, the frequency domain solution must be repeated many times. Inasmuch as the spectrum must be computed for at least several hundred frequencies, it is necessary to evaluate the integrand tens of thousands of times. This will lead to huge computational times.

The main principle of Fourier transform is shown below.



The main principle of Fourier transform.

For continuous functions, an integral Fourier transform is used to transform frequency domain $f(\omega)$ to time domain $f(t)$ as (or vice versa)

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\omega) e^{-i\omega t} dt.$$

However, for discrete data obtained from experiments and/or numerical solutions, a discrete Fourier transform based on Fourier series is used as

$$f(t) = \sum_{n=-\infty}^{\infty} a[n]e^{-in\omega_0 t}$$

where the coefficient $a[n]$

$$a[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)e^{in\omega_0 t} dt.$$

Using some special operations, a Fast Fourier Transform (FFT) is widely used today. In the inverse FFT operations, a maximum frequency of the signal and a frequency resolution are

$$f_{max} = \frac{1}{2\Delta t}$$

$$\Delta f = \frac{1}{T} = \frac{1}{N\Delta t}$$

where Δt is the time steps and N is the number of the time samples.

6. NOISE SOURCES

The main noise sources in the underwater acoustic medium are

- thermal noise,
- sea ambient noise,
- ship (vessel) noise.

6.1. THERMAL NOISE

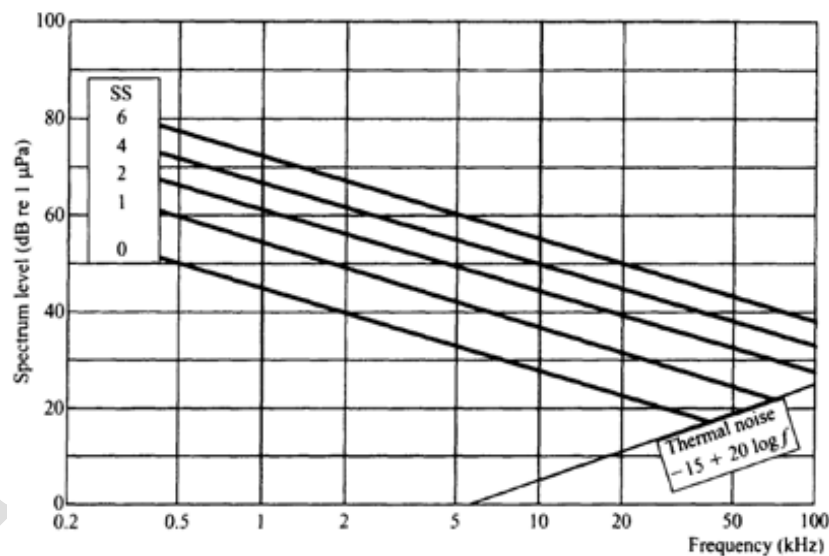
The thermal agitation of the molecules of water producing pressure fluctuations at the face of the hydrophone. It can be formulated experimentally as

$$N_{thermal} [dB] = -15 + 20 \log(f_{[kHz]})$$

where also considering the thermal agitation of the electrons in the sonar system, this is generally negligible compared with the sea ambient noise.

6.2. SEA AMBIENT NOISE

The spectrum level (dB re 1 μ Pa) of the sea ambient noise for the various sea states (SS) is shown in the following figure. As it is shown, the thermal noise forms a lower bound to the sea ambient noise.



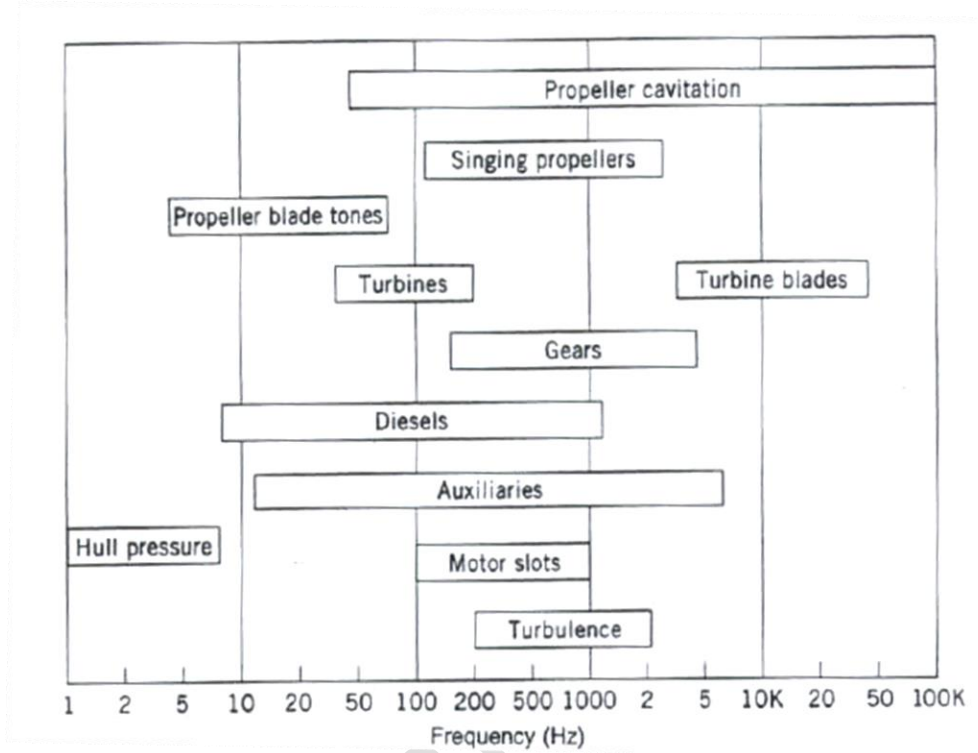
The spectrum level (dB re 1 μ Pa) of the sea ambient noise.

The sea states are related to the wave height and the wind speed. The rain and the biological noises from the sea animals also increases the sea ambient noise.

6.3. SHIP (VESSEL) NOISE

The ship (vessel) noise is an only controllable parameter. Using special noise reduction (controlling) techniques, it can be minimized. In fact, it is related to the Source Level (*SL*) in the sonar equation.

The frequency spectra of different ship noise sources are shown below.



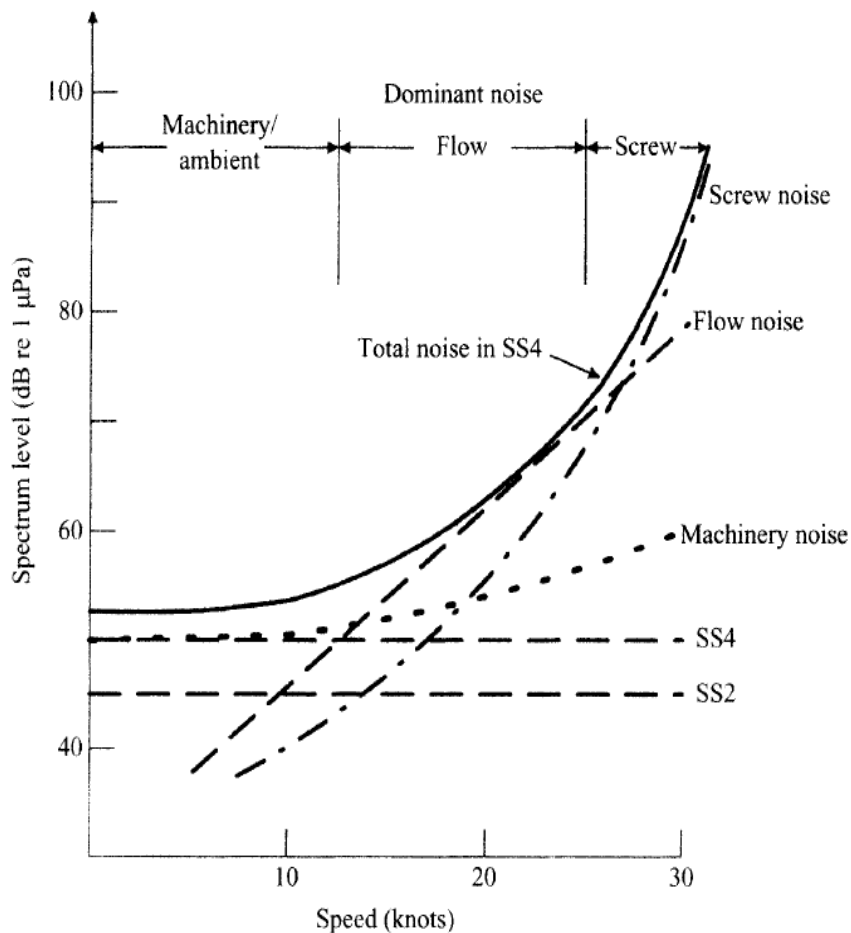
The frequency spectra of ship noise sources.

The three main noise sources in the ship are

- machinery noise (propulsion and auxiliary),
- propeller noise,
- flow noise.

According to the above figure, the propeller **blade tones** can be related to mechanical vibration and resonances at low speeds. Therefore, they can also be effective in their harmonics. The propeller cavitation is a wide-band noise. Therefore, in the next chapter, the **cavitation** phenomenon is evaluated in detail.

The ship speed is also an important factor for evaluation of the ship noise. The variation of ship noise with its speed is shown below [Waite, 2002].



The variation of ship noise with its speed.

The machinery noise is dominant unless the vessel is quiet or in high sea states. The flow noise is dominant at medium speeds. The propeller (screw) noise begins to dominate at the high speeds.

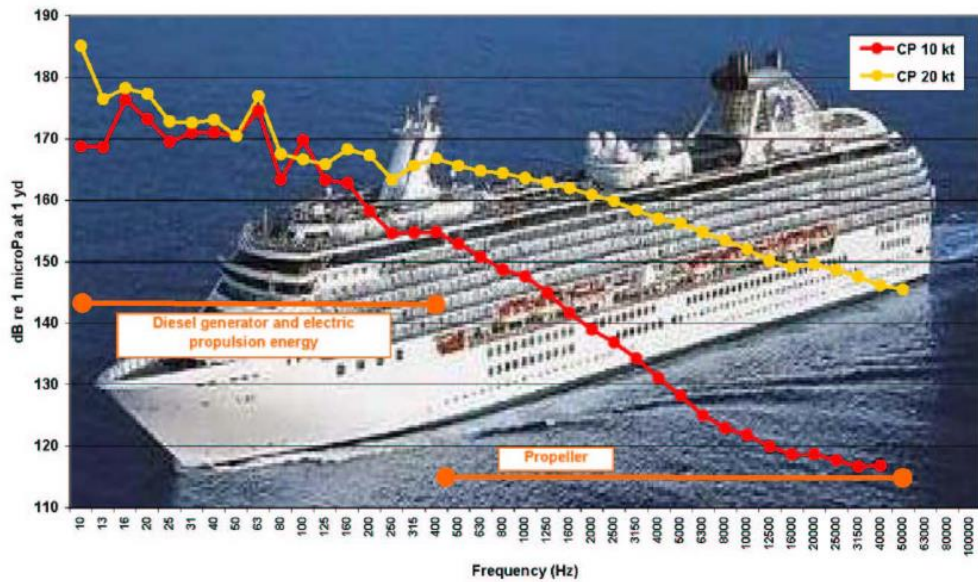
6.3.1. Ship Acoustic Signatures

The acoustic signatures of the ship can be calculated or measured in the sense of

- frequency spectrum (wide band),
- field distributions (radiation pattern),

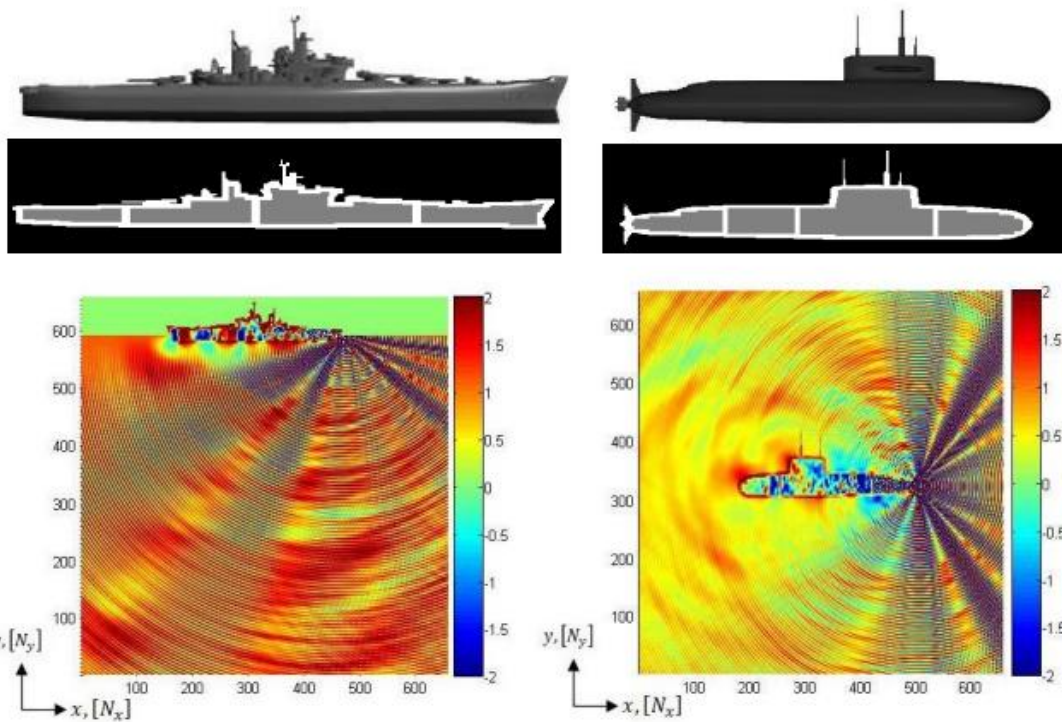
In fact, due to the very complex geometry and very complex multi-physical phenomena, a full calculation of the ship signature is a very challenging task.

As an example, the measured frequency spectrum of a cruise ship Coral Princess is shown below for its two different speeds [Kipple, 2004].

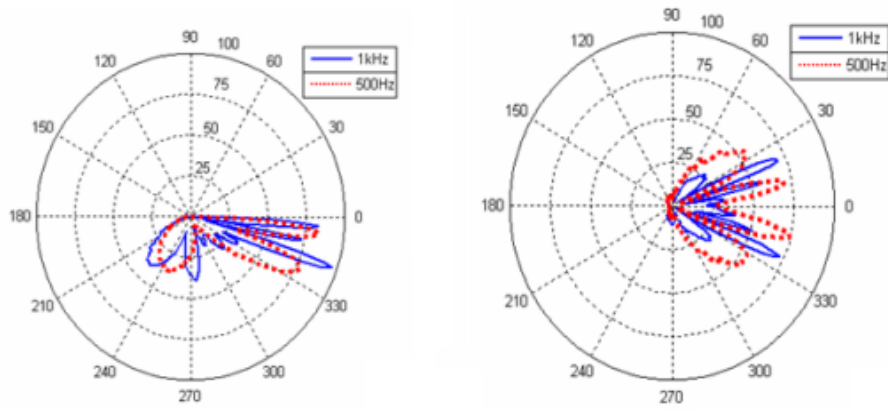


The measured underwater radiated noise of the cruise ship Coral Princess.

As another example, the field distributions at 1 kHz (the radiation patterns at 500 Hz and 1 kHz) of a hypothetical surface ship and a hypothetical submarine are calculated by the FDTD method and shown below for its two different speeds [Başaran and Aksoy, 2008].



Snapshots of the acoustic pressure field distributions produced by the noise sources at 1 kHz for a battleship and a submarine.



The radiation patterns of the acoustic signatures for a battleship (left) and a submarine (right) for only the propeller noises at 500 Hz and 1 kHz.

Dr. Serkan AKSOY

7. CAVITATION

Solid objects moving in the liquid leave spaces behind, causing low pressure regions. If the speed of the moving object is high enough, medium molecules dissolve due to the pressure drop, causing gas bubbles. However, most of these bubbles explode (burst) before they reach the sea surface. As a result of these explosions, an acoustic noise called **cavitation** occurs¹. In fact, it is a hydrodynamic phenomenon and a resultant loss in power by absorption and scattering within the bubbles. Two main types of the cavitations are

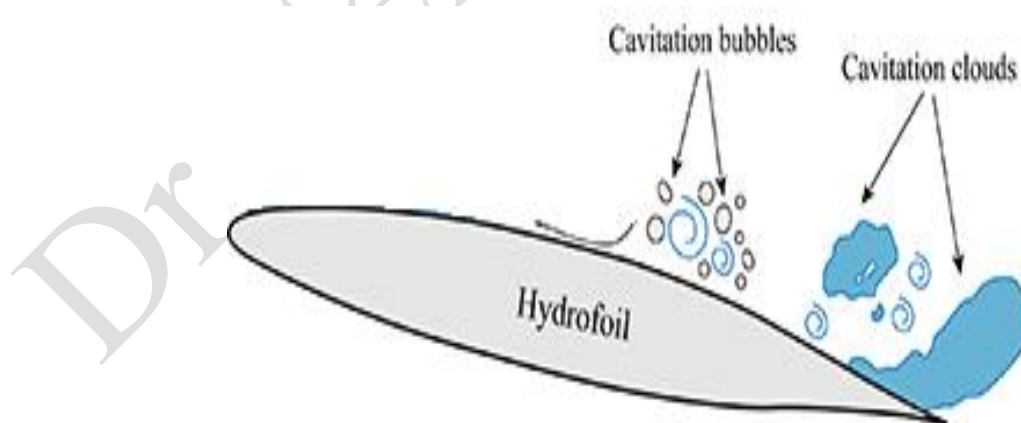
- propeller cavitation,
- flow cavitation.

The propeller cavitation is strongly produced by a propeller rotation. The propeller cavitation is shown below.



The hydrodynamic propeller cavitation.

The origin of the flow induced noise due to flow cavitation is shown below.



The schematic view of the flow cavitation.

¹ It can be related to the Source Level (*SL*) in the sonar equation.

7.1. CALCULATION FOR THE CAVITATION

The loss calculations due to the bubbles loss (cavitation) can be integrated with Helmholtz equation. To perform this, the wavelength (k) is considered as a complex-valued number

$$k = k' + ik''.$$

If there is no bubble losses ($k'' = 0$), k will be a real-valued number. Otherwise, the bubble losses are modeled by k'' that

$$k'' \simeq \frac{\omega}{c} \left(1 + \frac{A_1}{2} - i \frac{A_2}{2} \right)$$
$$A_1 = N \left(\frac{4\pi a}{\omega^2} \right) c_0^2 \frac{(\omega_r/\omega)^2 - 1}{((\omega_r/\omega)^2 - 1)^2 + \delta^2}$$
$$A_2 = N \left(\frac{4\pi a}{\omega^2} \right) c_0^2 \frac{\delta}{((\omega_r/\omega)^2 - 1)^2 + \delta^2}.$$

where a is the bubble radius, N is the number of bubbles in the unit volume, ω_r is the resonance frequency, δ is the damping constant.

These equations show that the bubbly medium is a kind of **a dispersive (frequency dependent) medium**. It means that the bubbles have strong frequency dependencies in which the wide band signals will be distorted due this material dispersion.

8. TARGET STRENGTH

Target Strength (TS) is acoustic echo returned by underwater targets: submarines, ships, torpedoes, mines, fish. It is formulated as

$$TS = 10 \log \left(\frac{I_{received}}{I_{source}} \right).$$

The Target Strength (TS) of some underwater acoustic targets can be given by considering related canonical objects as an approximation.

An underwater mine can be simulated by a sphere in which its TS is given by

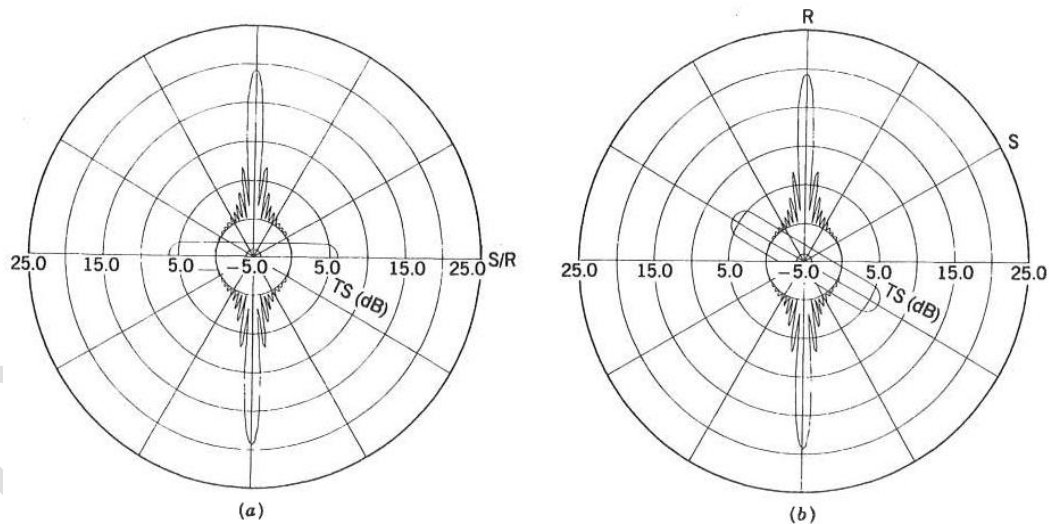
$$TS_{mine} = TS_{sphere} = 20 \log \left(\frac{a}{\lambda} \right)$$

where a is the mine radius. A torpedo can also be simulated by a cylinder in which its TS is

$$TS_{torpedo} = TS_{cylinder} = 10 \log \left(\frac{aL^2}{2\lambda} \left(\frac{\sin x}{X} \right)^2 \cos^2 \theta \right)$$

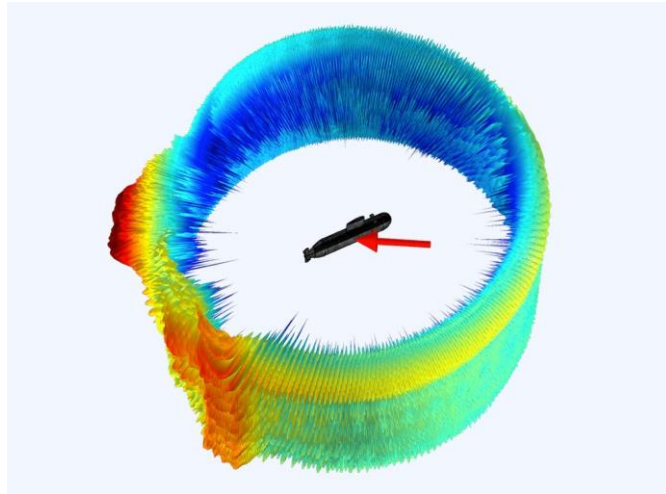
where a and L show the torpedo radius and torpedo length. $\lambda = c/f$ is the wavelength $X = (2\pi L/\lambda) \sin \theta$. It should be noted that the Target Strength (TS) of the torpedo is dependent on frequency (f) and angle (θ).

Using same mentality of cylinder equivalency for a submarine, the angular dependency of the submarine TS can be drawn below for a monostatic case and a bistatic case [Crocker, 1998].



The monostatic (left) and the bistatic (right) TS of a submarine (S: Source, R: Receiver).

In fact, a submarine TS must be evaluated for both elevation angle and azimuth angle. For this aim, a schematic TS representation of a more realistic submarine is shown below [Web].



A schematic TS representation of a more realistic submarine.

8.1. REDUCTION OF TARGET STRENGTH

The reduction the ship target strength can be performed by a **passive way** or an **active way**. In the former way, coating with/without shaping are applied while active noise cancellation is applied in the latter way.

8.1.1. Coating

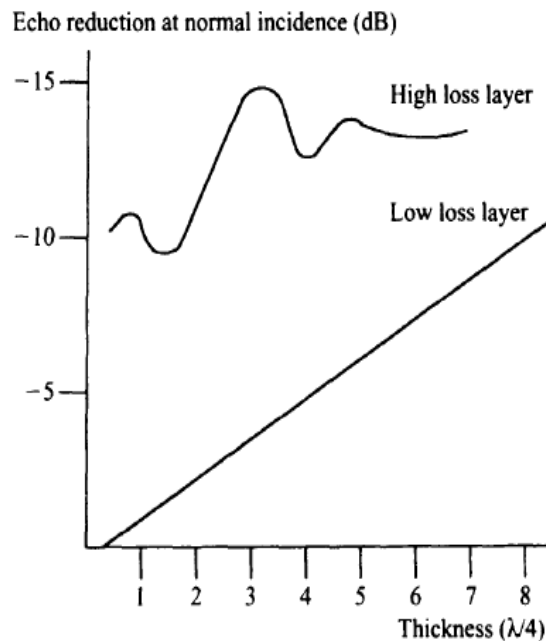
The application of the coating on a submarine surface and its effect on the target strength reduction is show below.



Aspect	TS (dB)		
	Small	Large: clad	Large
Beam	5	10	25
Intermediate	3	8	15
Bow/stern	0	5	10

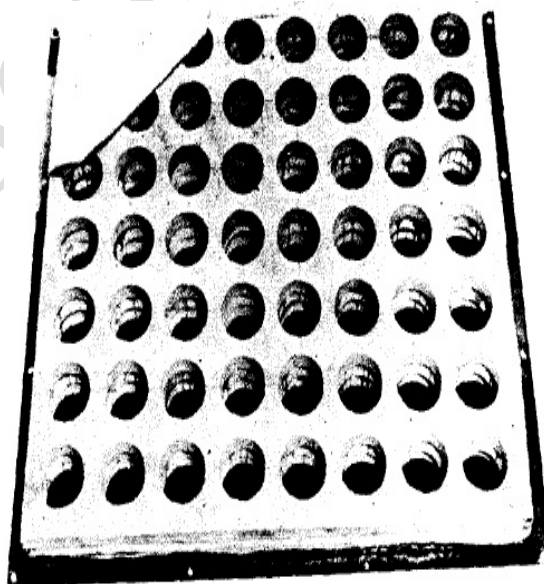
The application of the coating on a submarine surface and the reduction levels.

The coating can be applied as a **thick layer** absorber material or a **thin layer** absorber material. For the thick layers, the effect of absorber thickness on the echo reduction at normal incidence is shown below for a high loss layer and a low loss layer. The variation is rather linear for the low loss layer [Waite, 2002].



The effect of absorber thickness on the echo reduction.

The thin layer absorbers have special geometries and structures and can also be used for the reduction of the target strength. They are based on the excitation of resonances. One of the thin layer absorber examples is shown below.

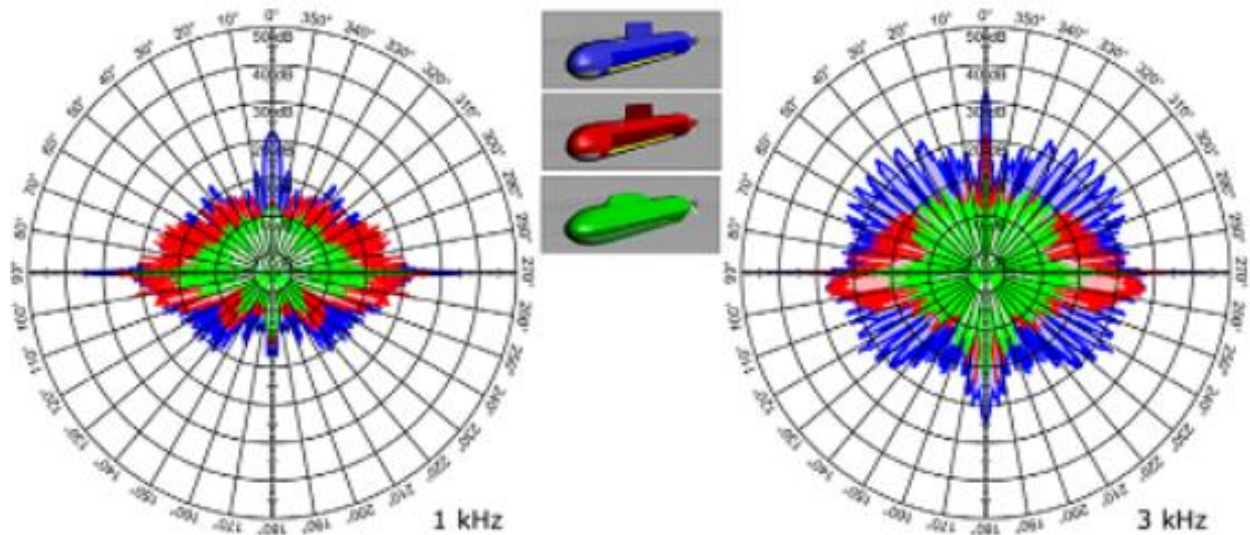


One of the thin layer absorber.

Depending on depth and temperature, some resonance shift occurs in the thin layer absorbers.

8.1.2. Coating/Shaping

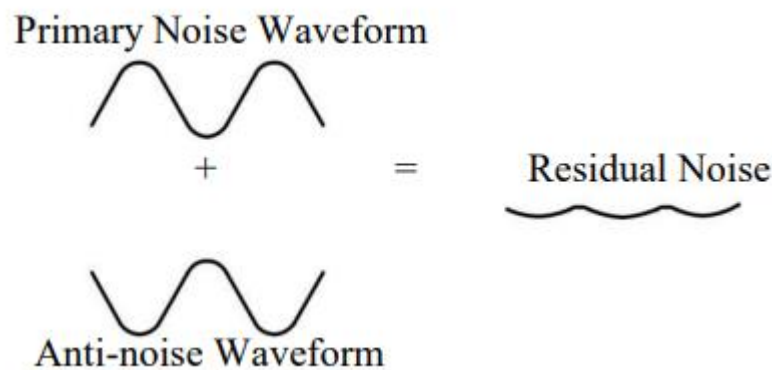
The shaping is also very effective method to reduce the target strength of the ships. If one can apply the shaping with coating, it gives more satisfactory reduced results. As an example, monostatic target strengths for uncoated, coated and stealth submarine are compared below in polar coordinates [Avsic, 2019]. As we can see, the shaping is applied for the stealth submarine (green).



The monostatic target strengths for uncoated (blue), coated (red), stealth submarine (green).

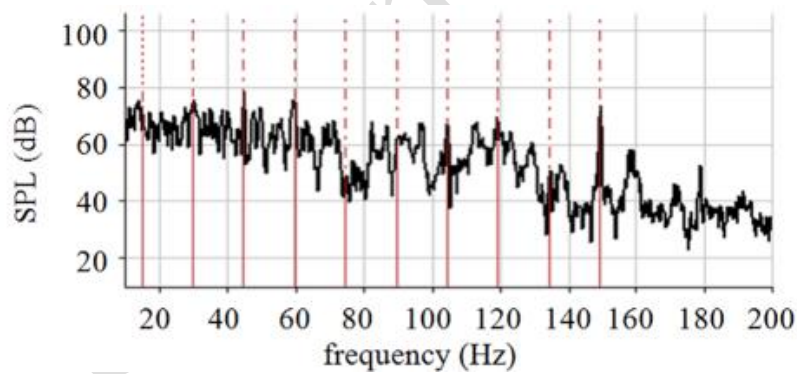
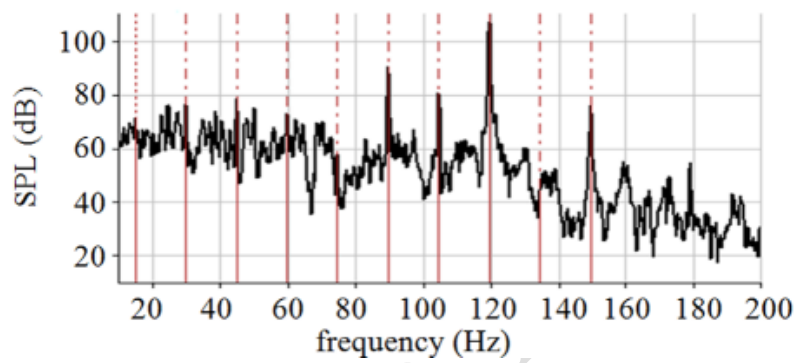
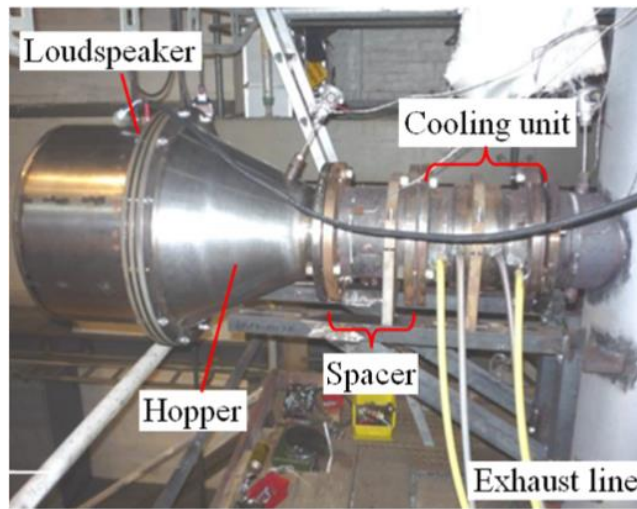
8.2. Active Noise Cancellation

Using anti-noise waveform, a primary (source) noise waveform can be minimized. This is known as **Active Noise Cancellation (ANC)**. The main principle of the ANC is shown below. Accordingly, after the addition of the two signals, a residual noise is only observed.



The main principle of the ANC.

In practice, the application of the ANC must be carefully planned. This is due to the finding proper and efficient positioning number and intensity of the anti-noise sources. An example of the ANC application and the results are shown below for a diesel engine in submarine [Sachau et al, 2016].



The ANC application and the results (without the ANC (upper), with the ANC (below)).

9. REFERENCES

- Avsic T., (2019), An underwater vehicle shape with reduced acoustic backscatter, Proceedings of the 23rd International Congress on Acoustics, 1935-1942, 9-13 September, Aachen, Germany.
- Başaran E., Aksoy S., (2008), Ship signature calculations by finite difference time domain method, Undersea Defense Conference (UDT 2008), No: 11A.1, 10-12 June 2008, Glasgow, UK.
- Crocker M. J., (1998), Handbook of Acoustics, John Wiley & Sons Inc.
- Kipple B. M., (2004), Coral Princess - Underwater Acoustic Levels, Technical Report, Naval Surface Warfare Center, Bremerton, WA.
- Pekeris C.L., (1948), Theory of propagation of explosive sound in shallow water, Geol Soc. Amer. Mem., 27 (Part II),1-117.
- Sertlek H. Ö., Aksoy S., Bölükbaş D., Akgün S., (2008), Application of the mode matching technique to fluid-solid layered acoustic problems, The Journal of the Acoustical Society of America, 123-5, 3597, 2008.
- Sachau D., Jukkert S., Hövelmann N., (2016), Development and experimental verification of a robust active noise control system for a diesel engine in submarines, Journal of Sound and Vibration, 375, 1- 18.
- Urick_R. J., (1982), Sound Propagation in the Sea, Peninsula Publishing.
- Waite A. D., (2002), Sonar for Practising Engineering, 3 ed., John & Wiley Sons Ltd.
- Web, (2022), Submarine Target Strength, COMSOL Software.