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Computational efficient methods for the uncertainty quantification of eddy current testing problems and their practical applications

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ABSTRACT

Eddy current testing is a widely used approach for the non-destructive evaluation of electrically conductive materials. The accuracy of material parameter estimation depends on the sensitivity of the measured signals to the properties of interest, as well as on the impact of noise and other disturbances. Challenges arise in independent and unique parameter estimation when multiple parameters affect the signals in similar ways, reducing identifiability. This paper introduces an efficient computational framework to assess the estimability and identifiability of parameters in eddy current testing, with a focus on metallic coated steel sheets. The approach utilizes stochastic and deterministic analytical models, as well as finite element simulations, to quantify this uncertainty.

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I. INTRODUCTION

Typically, in Eddy Current Testing (ECT), the impedance of the excitation coil or the induced voltage in a pick-up coil is analyzed. For selected applications, advanced and highly accurate analytical models have been developed, but their mathematical inversion is impossible. Therefore, estimators are used to determine the unknown material parameters. This paper proposes methods to quantify the uncertainty associated with model-based parameter estimates.

The accuracy of parameter estimation depends on factors such as measurement noise, the chosen signal processing method, and the estimator applied. In this work, a sinusoidal multi-frequency approach[1,2] combined with Fourier transform-based signal processing is investigated. Therefore, a differential coil,[1] as illustrated in Fig. 1, is used. The voltage induced in the pick-up coils is influenced by the coil geometry, the electrical and magnetic properties of the materials involved, as well as the liftoff. Note, the presented approach is also applicable to other coil geometries and applications.

The unknown material parameters can be determined by formulating a model-based estimation approach. This paper aims to quantify the impact of different estimator setups on the variance of the estimated parameters. To achieve this, a method based on the directional derivatives of the induced voltage with respect to individual parameters is employed. This approach is particularly suitable for analyzing simple ECT systems with a limited number of parameters to be estimated and for assessing the structural identifiability of the parameters.[3] For more complex problems, as well as for determining practical or statistical identifiability,[3] the inspection of directional derivatives becomes intractable. In such cases, improved analytical approaches must be used instead. In the course

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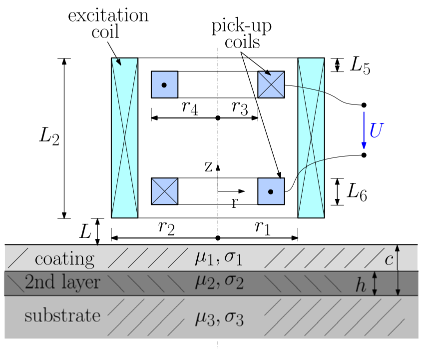

FIG. 1. Investigated differential coil setup next to a three-layer sheet.

of this study, selected practical ECT problems involving the parameter estimation of laminated steel sheets are analyzed. Both linear ferromagnetic material properties and nonlinear material behavior are analyzed and compared.

II. ANALYTICAL EDDY CURRENT MODEL

When a linear, homogeneous, and isotropic sheet material is assumed, analytical models are well-documented in the literature.[1] For a layered sheet, as shown in Fig. 1, the complex valued induced differential voltage U in the pick-up coils of the coil system can be determined using

$$U(f;\boldsymbol{\theta}) = \frac{\mathrm{j}\omega\mu_0\pi I N_1 N_2}{(r_2-r_1)(r_4-r_3)L_2L_6}\int_0^\infty \frac{1}{\alpha^6}J(r_2,r_1)J(r_4,r_3) \times \mathrm{e}^{-2\alpha L}(\mathrm{e}^{-\alpha(L_2-2L_5-L_6)}-1)(\mathrm{e}^{-\alpha L_5}-\mathrm{e}^{-\alpha(L_6+L_5)}) \times (\mathrm{e}^{-\alpha L_2}-1)R\,\mathrm{d}\alpha = \zeta I. \tag{1}$$

The variables, assumptions and relationships are introduced in Refs. 1 and 2. As demonstrated in Ref. 4, it is relatively straightforward to determine the sheet factor R for multi-layer sheets.

III. NOISE MODEL

The measurement noise significantly impacts the accuracy of the practical parameter estimability. Increased measurement noise results in higher parameter-estimation uncertainty. In this work, the measured quantity is the induced differential voltage in the pick-up coils when a sinusoidal current is impressed in the excitation coil. The FFT algorithm is used to determine the amplitude and phase of the fundamental wave for each excitation frequency. Furthermore, the variance of the current noise σ_I^2 is assumed to be approximately constant across all frequencies, implying a white Gaussian noise characteristic (AWGN). The EC model considers the deterministic complex induced voltage. Thus, the impact of current noise $\sigma_\mathrm{I,tf}^2$, excluding thermal and quantization noise which contribute to the overall noise measured in the current data acquisition, must be transformed into separate real and imaginary components of the voltage noise by

$$\hat{\sigma}_\mathrm{U,re} = \sqrt{\mathrm{Re}(\zeta)^2\sigma_\mathrm{I,tf,re}^2 + \mathrm{Im}(\zeta)^2\sigma_\mathrm{I,tf,im}^2} \tag{2}$$

and

$$\hat{\sigma}_\mathrm{U,im} = \sqrt{\mathrm{Re}(\zeta)^2\sigma_\mathrm{I,tf,im}^2 + \mathrm{Im}(\zeta)^2\sigma_\mathrm{I,tf,re}^2} \tag{3}$$

as derived in Ref. 5. This describes how the current noise, after passing through the EC system and the signal-processing chain, affects the measured complex induced voltage. More details about the used approximations and how the FFT-based signal processing is taken into account can be found in Ref. 5. This provides the basis for developing a parameter estimation accuracy model based on the noisy measurement signal.

IV. UNCERTAINTY QUANTIFICATION

The Fisher information matrix (FIM) $\mathbf{I}(\boldsymbol{\theta})$ measures how much information an observable random variable provides about the unknown parameters of a statistical model. The main diagonal of the inverse of the FIM provides a lower bound for the variance of any unbiased parameter estimator, the so-called Cramér–Rao Lower Bound (CRLB).[6,7] This standard theory in statistical signal processing is discussed in detail in Ref. 7. For taking into account the noise, the covariance matrix $\boldsymbol{\Sigma}$ is given by

$$\boldsymbol{\Sigma} = \kappa\begin{bmatrix}\sigma_\mathrm{U,re}^2(f;\boldsymbol{\theta}) & 0\\ 0 & \sigma_\mathrm{U,im}^2(f;\boldsymbol{\theta})\end{bmatrix}, \tag{4}$$

assuming that the noise in the real and imaginary parts is uncorrelated. Unmodeled influences and the use of a simple unweighted least squares estimator instead of a maximum likelihood estimator are reflected by a factor κ. As shown in Ref. 5, $\kappa = 20$ is a typical value. This factor depends on the number and selection of excitation frequencies and must be determined individually for each analysis. The FIM is then given by

$$\mathbf{I}(\boldsymbol{\theta}) = \sum_{f=f_\mathrm{min}}^{f_\mathrm{max}}\left(\frac{\partial \boldsymbol{U}(f;\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right)^\mathrm{T}\boldsymbol{\Sigma}^{-1}\left(\frac{\partial \boldsymbol{U}(f;\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right) \tag{5}$$

with

$$\frac{\partial \boldsymbol{U}(f;\boldsymbol{\theta})}{\partial\theta_j} = \begin{bmatrix}\dfrac{\partial\mathrm{Re}(U(f;\boldsymbol{\theta}))}{\partial\theta_j}\\[2mm] \dfrac{\partial\mathrm{Im}(U(f;\boldsymbol{\theta}))}{\partial\theta_j}\end{bmatrix}. \tag{6}$$

θ_j represents the various n parameters to be estimated, such as coating thickness or substrate conductivity. Since the thin coating thickness is in the micrometer-range and the conductivity, for example, is in the megasiemens-per-meter range, this results in a very ill-conditioned FIM, which means that the matrix should not be inverted without further treatment. A normalization of the matrix provides a workaround. If we introduce a diagonal normalization matrix $\mathbf{K}$, we obtain a better conditioned matrix $\mathbf{I}^* = \mathbf{KIK}^\mathbf{T}$ with $\mathbf{K} = \mathrm{diag}\left(\frac{1}{\sqrt{\mathrm{diag}\,(\mathbf{I})}}\right)$. This matrix can therefore easier be inverted numerically. To obtain the original desired inverse $\mathbf{I}^{-1}$, $\mathbf{I}^{*-1}$ must be denormalized again as follows $\mathbf{I}^{-1} = \mathbf{KI}^{*-1}\mathbf{K}^\mathbf{T}$. The variances of the parameters $\boldsymbol{\theta}$ are given by the diagonal elements of the inverse Fisher information matrix $\mathbf{I}^{-1}$, $\mathrm{Var}\,(\theta_j) = [\mathbf{I}^{-1}]_{jj}$ for $j = 1,\ldots,n$. In Sec. IV A, different methods for evaluating the estimability of parameters in ECT problems are examined. For the investigations

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the following coil setup is used: $r_1 = 24$ mm, $r_2 = 24.8$ mm, $r_3 = 23.7$ mm, $r_4 = 24$ mm, $L_2 = 10$ mm, $L_5 = 0$ mm and $L_6 = 5$ mm. The excitation current amplitude is $I = 0.2$ A, the current noise is $\sigma_I^2 = 150^2$ μA, and the thermal noise is $\sigma_{I,\mathrm{th}}^2 = 30^2$ μA, and the number of acquired samples $N = 10\,000$.[5] The used frequencies are defined as $\vec{f} = [10\,000, 50\,750, 100\,500, 150\,250, 200\,000]$ Hz.

A. Directional derivatives for parameter identifiability

A simple method based on the directional derivatives in (6) is suitable for analyzing ECT problems with a small number of influencing parameters to determine the structural identifiability[3] by visual investigation.

1. Analysis of linear substrate material

For a two-layer model, the directional derivatives are calculated at different discrete excitation frequencies and are graphically illustrated in Ref. 8. This approach eliminates the need for a noise model and determines which parameters can be independently inferred from the measured signals. If directional derivative vectors consistently point in the same direction, it suggests that the corresponding parameters are not independently identifiable. As shown in Ref. 8, it is evident that, for a two-layer sheet, the substrate permeability and substrate conductivity cannot be independently determined. This raises the question of whether this dependency is inherent to magnetically linear materials or if it also extends to nonlinear substrate materials.

2. Analysis of nonlinear substrate material

The proposed method is also well-suited for investigating the estimability of magnetically nonlinear substrate materials, as it does not require an analytical model and can instead be based on results obtained through finite element (FE) simulations. For this analysis, a simple uncoated ferromagnetic sheet with a nonlinear BH curve is assumed. The magnetization behavior $\mu = f(H)$ for the FE simulation is modeled using the anhysteretic curve of the Jiles–Atherton model.[9] A critical question is whether, for nonlinear materials, the conductivity and all parameters defining the anhysteretic curve (M_s, a, α) can be independently identified by leveraging different magnetic operating points, achieved through variations in excitation currents and frequencies. In this study, performed by using FEMM,[10] the analysis is limited to the fundamental wave, neglecting higher harmonics. In Fig. 2, the normalized directional derivatives with respect to the influencing parameters are shown. It is evident that the parameter M_s and the conductivity σ always have exactly opposite effects on the complex induced voltage, independent of the selected current, frequency or BH curve. The structural non-identifiability of these parameters arises from their dependence on the magnetic flux distribution, which is fundamentally defined by the material properties. Consequently, the two parameters cannot be distinguished based on EC losses.

B. Analytical method for practical parameter identifiability of multi-layer systems

For multi-layer structures, a more advanced method is required, as the large number of parameters makes it difficult to clearly determine the dependencies and the strength of these dependencies, complicating the classification of parameters as estimable or non-estimable. Depending on the choice of parameters

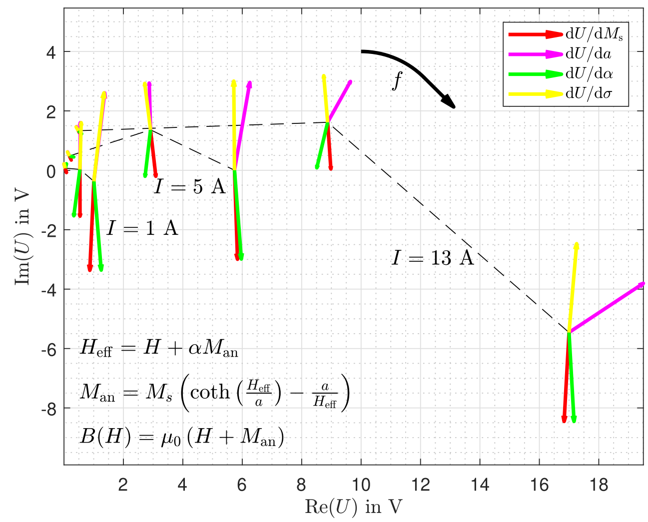

FIG. 2. Directional derivatives to the Jiles–Atherton anhysteretic curve parameters and the conductivity of a one-layer material.

to be estimated, there is a gradual transition between estimable and non-estimable, which must be analyzed using Monte Carlo experiments. This approach allows for a targeted analysis of which parameter combinations, the number of parameters to be estimated, or the choice of parameters to be estimated result in certain parameters being well or poorly identifiable. However, this poses a computational challenge. For a three-layer EC problem with eight influencing parameters $p = 8$, and assuming $\mu_1 \approx 1$ for the coating, there are $2^p - 1 = 255$ possible parameter estimation combinations. These combinations include single-parameter estimations, such as $\boldsymbol{\theta} = [\theta_1]$, as well as multi-parameter estimations, such as $\boldsymbol{\theta} = [\theta_1, \theta_2]$, $\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3]$, $\boldsymbol{\theta} = [\theta_1, \theta_4, \theta_5]$, and so on. In each case, the remaining parameters are assumed to be known. Calculating, inverting, and extracting the FIM for one estimator configuration takes approximately five seconds on average when considering five excitation frequencies. Evaluated on an i9-10900F processor, a Monte Carlo experiment with $M_c = 150\,000$ experiments would result in a computation time of about six years, making it impractical. It is evident that the air gap is always estimable, reducing the number of parameters to be estimated to $p = 7$. A more efficient approach is to compute the FIM matrices only for the case of all $p = 7$ parameters to be estimated. This results in M_c 7×7 FIM matrices, which can then be selectively partitioned into the required submatrices for specific subset parameter estimators as indicated for two selected one-parameter (green) and two-parameters (orange) estimators in the following FIM matrix computed by utilizing (5):

$$\mathbf{I} = \begin{bmatrix} \boxed{a_{11}} & \boxed{a_{12}} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\ \boxed{a_{21}} & \boxed{a_{22}} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\ a_{31} & a_{32} & \boxed{a_{33}} & a_{34} & a_{35} & a_{36} & a_{37} \\ a_{41} & a_{42} & a_{43} & \boxed{a_{44}} & a_{45} & \boxed{a_{46}} & a_{47} \\ a_{51} & a_{52} & a_{53} & a_{54} & \boxed{a_{55}} & a_{56} & a_{57} \\ a_{61} & a_{62} & a_{63} & \boxed{a_{64}} & a_{65} & \boxed{a_{66}} & a_{67} \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} \end{bmatrix}.$$

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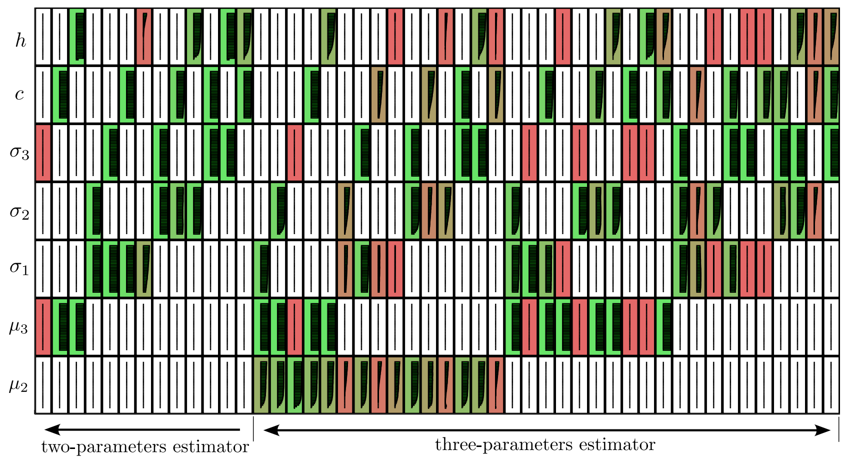

FIG. 3. Relative estimability representation with histograms of selected two and three parameters estimators of a three-layer sheet metal. Due to space limitations, the presentation of additional estimators has been excluded.

This approach is valid, because the full matrix already contains all the information about all parameters, although the matrix is singular. The submatrices can then be inverted to evaluate the estimability. This reduces the computation time to approximately three days. In this paper, the relative estimability is defined as $\mathrm{Var}(\theta_j) < 0.01^2\,\theta_j^2$. For a Monte Carlo experiment with $\mu_2 \in [1, 500]$, $\mu_3 \in [1, 500]$, $\sigma_1 \in [5, 17]$ MS/m, $\sigma_2 \in [2, 17]$ MS/m, $\sigma_3 \in [2, 7]$ MS/m, $c \in [10, 25]$ μm, and $h \in [0, 10]$ μm, the results for selected two- and three-parameters estimators are shown in Fig. 3. In Fig. 4, a selected detail for a specific three-parameters estimator is presented. The white fields indicate that these parameters are not estimated and assumed to be known. The individual parameter estimability is visualized in the form of a histogram. The histogram reveals the parameter ranges for which the respective parameters are estimable. The background color represents how frequently the individual parameter could be estimated, using a color gradient from green to red. If a parameter combination is entirely non-estimable, indicated by the histogram being zero across the entire parameter range, this suggests structural non-identifiability. This behavior can be observed in the directional derivatives, where arrows point either in the same or opposite directions. For structural non-identifiability analysis, no noise model is required, as the FIM becomes singular. Even when parameters are structurally non-identifiable, it can still be beneficial to use an estimator that includes them. This eliminates the need to predefine these parameters and provides the estimator with additional degrees of freedom, enabling potentially a more accurate estimation of other identifiable parameters. For example, for a two-layer structure, a coating thickness estimator with additional degrees of freedom for μ_2 and/or σ_2 allows unbiased thickness estimation without prior knowledge of the substrate material parameters, as fictitious substrate parameters are estimated while the coating thickness estimate converges to the true value. Without this approach, if the substrate parameters are not known exactly and only the coating thickness is estimated, the resulting coating thickness estimate will be biased.

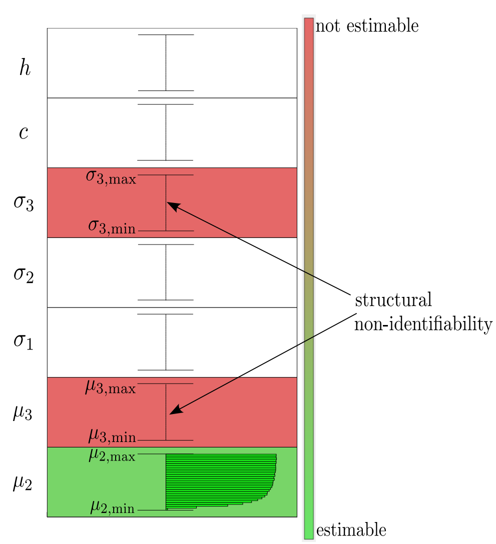

FIG. 4. Detail of Fig. 3 of a μ_2, μ_3, and σ_3 estimator with relative individual estimability histograms.

V. CONCLUSION

This paper presents computationally efficient methods to quantify the estimability of parameters using eddy current testing, applicable to both linear and nonlinear EC problems. These methods enable the identification of structural and practical non-identifiability of parameters, considering factors such as the selected parameter estimator, the number of parameters to be estimated, noise, and the chosen parameter settings. In this work, the methods were evaluated for metallically coated sheets. Note that the presented approach can be adapted to various eddy current problems. If no analytical eddy current model is available, finite element simulations can be utilized as an alternative.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

M. Koll: Conceptualization (equal); Data curation (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Software (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **D. Wöckinger**: Conceptualization (equal); Investigation (equal); Methodology (equal); Project administration (equal); Supervision (equal); Writing – review & editing (equal). **G. Bramerdorfer**: Conceptualization (equal); Funding acquisition (equal); Project administration (equal); Resources (equal); Supervision (equal); Writing – review & editing (equal). **S. Schuster**: Conceptualization (equal); Investigation (equal); Methodology (equal); Software (equal); Supervision (equal); Validation (equal); Writing – review & editing (equal). **S. Scheiblhofer**: Conceptualization (equal); Investigation (equal); Methodology (equal); Software (equal); Writing – review & editing (equal). **N. Gstöttenbauer**: Conceptualization (equal); Methodology (equal); Project administration (equal); Writing – review & editing (equal). **J. Reisinger**: Conceptualization (equal); Funding acquisition (equal); Project administration (equal); Resources (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

[1] C. V. Dodd, J. W. Luquire, W. E. Deeds, and W. G. Spoeri, "Some eddy-current problems and their integral solutions," Technical Report No. ORNL-4482, Oak Ridge National Laboratory, 1969.
[2] M. Koll, D. Wöckinger, C. Dobler, G. Goldbeck, G. Bramerdorfer, S. Schuster, S. Scheiblhofer, N. Gstottenbauer, and J. Reisinger, COMPEL, 2024.
[3] R. C. Smith, *Uncertainty Quantification: Theory, Implementation, and Applications* (SIAM, 2024).
[4] J. W. Luquire, W. E. Deeds, and C. V. Dodd, J. Appl. Phys. **41**, 3983–3991 (1970).
[5] M. Koll, D. Wöckinger, G. Bramerdorfer, S. Schuster, S. Scheiblhofer, N. Gstottenbauer, and J. Reisinger, IEEE Sens. Lett. **9**, 1–4 (2025).
[6] C. R. Rao, "Information and accuracy attainable in estimation of statistical parameters," Bull. Calcutta Math. Soc. (1945).
[7] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory* (Prentice Hall, 1993).
[8] M. Koll, D. Wöckinger, C. Dobler, G. Goldbeck, G. Bramerdorfer, S. Schuster, S. Scheiblhofer, N. Gstottenbauer, and J. Reisinger, J. Magn. Magn. Mater. **592**, 171754 (2024).
[9] D. C. Jiles and D. L. Atherton, J. Appl. Phys. **55**, 2115–2120 (1984).
[10] D. Meeker, FEMM **4**, 162 (2010).

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