

Article

VK-RRT*: A Multi-Constraint Coupling Approach to Energy-Efficient AUV Path Planning in Three-Dimensional Ocean Current Environments

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Abstract

To address unclear multi-constraint coupling mechanisms, energy estimation errors, and poor dynamic feasibility in autonomous underwater vehicle path planning, this paper proposes an energy-efficient velocity-kinodynamic rapidly exploring random tree star algorithm. The method integrates a velocity–energy model, a velocity–curvature model, and a velocity–safe-distance model into a planning framework. Four targeted improvements are incorporated: a potential-field-guided sampling, an adaptive step-length expansion, a kinodynamic feasibility constraint, and a multi-objective cost function. Simulation experiments across four scenarios and five start–goal tasks show that the proposed method eliminates all curvature violations, reduces propulsion energy by up to 49.4% and improves mean minimum obstacle clearance by 34.2% and 47.4% over the two baselines. Ablation studies confirm that each module contributes to overall performance.

Keywords: autonomous underwater vehicle; path planning; multi-constraint coupling; ocean current; RRT*; energy efficiency

1. Introduction

Autonomous underwater vehicles (AUVs) play an increasingly important role in ocean resource exploration, oceanographic surveys, and maritime defence [1,2]. Given the limited onboard battery capacity in long-duration deployments, path planning directly determines mission energy consumption, spatial coverage efficiency, and operational safety [3,4]. In these tasks, AUVs often operate in environments characterised by unstructured obstacles, complex seabed topography, spatially varying ocean currents, and strong hydrodynamic disturbances, all of which impose stringent demands on path planning [1,4–7]. Generating paths that avoid obstacles, exploit ambient currents, and satisfy kinodynamic constraints remains a key bottleneck for improving AUV operational effectiveness.

The operating speed of an AUV is the physical quantity that ties its energy consumption, manoeuvrability, and safety requirements together. In non-uniform flow fields, including mesoscale eddies and shear layers, the relative speed between the vehicle and the ambient current directly determines propulsion energy, so favourable currents can substantially reduce energy consumption while adverse currents sharply amplify it [8]. Conventional planners often optimise only geometric path length or approximate currents as constant disturbances, which fails to capture speed-dependent energy variation [5,9–12].



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Beyond its effect on energy consumption, operating speed also constrains manoeuvrability, because the minimum turning radius of torpedo-shaped AUVs without direct lateral thrust varies nonlinearly with forward speed. Many existing methods assume a speed-independent turning radius or model the AUV as a point mass, producing sharp-cornered paths that may exceed the vehicle's physical manoeuvring capability [8,13–16]. Forward speed also affects safety clearance, since braking distance increases nonlinearly with speed. Conventional obstacle inflation applies a fixed clearance regardless of velocity, simultaneously wasting manoeuvring space at low speed and leaving insufficient stopping margins at high speed [10,13,17,18]. Because current-induced energy variation, velocity-dependent curvature limits, and speed-varying safety requirements are all governed by AUV operating speed, these three constraints are physically coupled and cannot be treated as mutually independent.

Recent AUV path-planning studies have improved current utilisation and search efficiency. Current-aware planners introduce flow information into travel time, path length, or energy-related objectives, enabling the planned route to exploit favourable currents and reduce exposure to adverse flow regions [5,9–11,16,19,20]. However, their feasibility evaluation relies on fixed safety margins and simplified manoeuvrability assumptions, without linking operating speed to turning radius or braking distance during node expansion. Recent rapidly exploring random tree (RRT) variants further improve convergence and geometric regularity through informed sampling, bidirectional growth, adaptive step length, angle constraints, pruning, or curve smoothing [14–16]. These mechanisms shorten or smooth the generated path, but the imposed angle and smoothness constraints are generally geometric rather than velocity dependent, and obstacle avoidance is often evaluated using a fixed clearance instead of a braking distance that varies with AUV speed. Learning-based methods provide adaptive decision-making in unknown or time-varying environments [2,8,21–24], but require extensive training, impose large computational costs ill-suited to onboard AUV resources, and produce decisions governed by learned parameters rather than explicit hydrodynamic and kinodynamic constraints. Their generalisation also degrades when the flow field or obstacle layout deviates from the training distribution. To address these gaps, this paper proposes an energy-efficient velocity-kinodynamic rapidly exploring random tree star algorithm (VK-RRT*). The principal contributions are as follows.

1. Three coupling models, namely the velocity–energy (V–E), velocity–curvature (V–C), and velocity–safe-distance (V–S) models, are formulated to quantify the velocity-dependent relationships among propulsion energy, turning feasibility, and braking clearance.
2. A kinodynamic constraint-enforcement mechanism and a multi-objective cost function are proposed to integrate the three coupling models into the RRT* framework. The V–C and V–S models impose a heading-change constraint and a dynamic-inflation collision check at the node expansion stage. The V–E model drives a composite cost that steers parent selection toward energy-efficient edges.
3. A composite potential-field-guided sampling strategy and an adaptive step-length expansion mechanism are developed to improve exploration efficiency. The potential field biases sampling toward low-energy, high-safety regions. The adaptive step couples obstacle proximity with flow induction to shorten expansions near obstacles and lengthen them in open corridors.

The remainder of this paper is organised as follows. Section 2 establishes the three-dimensional ocean environment model. Section 3 presents the derivation of the three velocity-dependent coupling models. Section 4 presents the VK-RRT* algorithm and its

four integrated mechanisms. Section 5 validates the proposed method through simulation studies. Section 6 concludes the paper and discusses future work.

2. Ocean Environment Modelling

A realistic underwater environment model is a prerequisite for validating three-dimensional path-planning algorithms. The three-dimensional ocean environment is decomposed into three components: seabed topography, suspended underwater obstacles, and a three-dimensional ocean current field.

Seabed relief is primarily characterised by underwater ridges exhibiting a gentle central peak that tapers toward the surroundings [25]. A superposition of Gaussian basis functions is adopted to represent the undulating terrain:

$$Z(x, y) = z_0 + \sum_{i=1}^n h_i \exp \left[-\left(\frac{x - x_i}{\sigma_{xi}} \right)^2 - \left(\frac{y - y_i}{\sigma_{yi}} \right)^2 \right] \quad (1)$$

where z_0 is the reference seabed depth; n is the number of ridge features; (x_i, y_i) is the plan-view centre of the i -th ridge; h_i is its peak height; σ_{xi} and σ_{yi} are width coefficients in the x - and y -directions, controlling the spatial extent and slope of each ridge.

Irregular underwater objects, such as sunken structures and marine organisms, often have complex geometries. An ellipsoidal model provides a compact approximation that captures elongated, flattened, and spherical shapes through three independent semi-axes. The surface of a single ellipsoidal obstacle is defined as

$$S(x, y, z) = \left(\frac{x - x_0}{a} \right)^2 + \left(\frac{y - y_0}{b} \right)^2 + \left(\frac{z - z_0}{c} \right)^2 = 1 \quad (2)$$

where (x_0, y_0, z_0) is the ellipsoid centre; a , b , and c are the semi-axis lengths along the three principal directions. The model reduces to a sphere when $a = b = c$ and represents flattened or elongated shapes when the semi-axes differ.

Real ocean currents contain complex eddies and shear layers. A superposition of Lamb–Oseen vortex models is employed to construct a non-uniform three-dimensional current disturbance field [26]. The Lamb–Oseen model accounts for fluid viscosity, producing a velocity field that is continuous and singularity-free at the vortex core and decays smoothly in the far field, in closer agreement with physical observations. The velocity components of a single three-dimensional Lamb–Oseen vortex at spatial position $\mathbf{r} = (x, y, z)^T$ are

$$\begin{aligned} V_x(\mathbf{r}) &= -\frac{\Gamma(y - y_0)}{2\pi\|\mathbf{r} - \mathbf{r}_0\|^2} \left[1 - e^{-\|\mathbf{r} - \mathbf{r}_0\|^2/\delta^2} \right] \\ V_y(\mathbf{r}) &= \frac{\Gamma(x - x_0)}{2\pi\|\mathbf{r} - \mathbf{r}_0\|^2} \left[1 - e^{-\|\mathbf{r} - \mathbf{r}_0\|^2/\delta^2} \right] \\ V_z(\mathbf{r}) &= \frac{\Gamma}{\pi\delta^2} e^{-\|\mathbf{r} - \mathbf{r}_0\|^2/\delta^2} \end{aligned} \quad (3)$$

where V_x , V_y , V_z are the Cartesian current velocity components; $\mathbf{r}_0 = (x_0, y_0, z_0)^T$ is the vortex centre; Γ is the circulation strength, whose sign determines the rotation direction; δ is the vortex core radius, characterising the spatial extent of the vortex influence. The total current velocity at position p is obtained by superimposing the contributions from all N_v vortices:

$$\mathbf{v}_c(p) = \sum_{j=1}^{N_v} [V_{x,j}, V_{y,j}, V_{z,j}]^T \quad (4)$$

3. Velocity Constraint Coupling Models

During missions in three-dimensional ocean environments, AUV motion is governed by geometric, kinematic, kinodynamic, and hydrodynamic constraints. Traditional path-planning methods commonly treat maximum speed, turning radius, and safety clearance as mutually independent static thresholds, neglecting the coupling among these quantities. This paper derives three coupling models: the velocity-energy (V-E) model, the velocity-curvature (V-C) model, and the velocity-safe-distance (V-S) model. These models provide quantitative and physically interpretable bases for cost evaluation and constraint enforcement in the planning algorithm. All coupling model curves presented in this section are illustrative, computed using the representative AUV parameters listed in Section 5.1.

3.1. Velocity-Energy Coupling Model (V-E)

Because AUV endurance is constrained by onboard energy capacity, the effect of ocean currents on energy consumption must be characterised for reliable mission planning. Let $\mathbf{v}$ denote the AUV ground velocity vector and $\mathbf{v}_c$ the ambient current velocity vector. The relative speed V_r determines the effective hydrodynamic drag; the ground speed is V . Within the typical operating speed range, the hydrodynamic resistance of an AUV can be approximated by a quadratic function of the relative speed [27,28]:

$$D(V_r) = \frac{1}{2} \rho C_D A V_r^2 \quad (5)$$

where ρ is the seawater density, C_D is the equivalent drag coefficient, and A is the reference cross-sectional area. Under steady-state navigation, the thruster force balances drag, and the propulsion power is

$$P(V_r) \approx D(V_r) \cdot V_r = \frac{1}{2} \rho C_D A V_r^3 \quad (6)$$

Propulsion power therefore grows approximately as the cube of the relative speed. For a path segment traversed at constant ground speed V , the specific energy consumption (energy per unit distance) can be approximated as

$$e(V_r) = \frac{P(V_r)}{V} \approx \frac{\rho C_D A V_r^3}{2V} \quad (7)$$

This model establishes a quantitative link between relative flow speed and specific energy consumption (Figure 1). On with-current segments, V_r decreases and $e(V_r)$ is substantially reduced; on counter-current segments, V_r increases and energy consumption rises sharply.

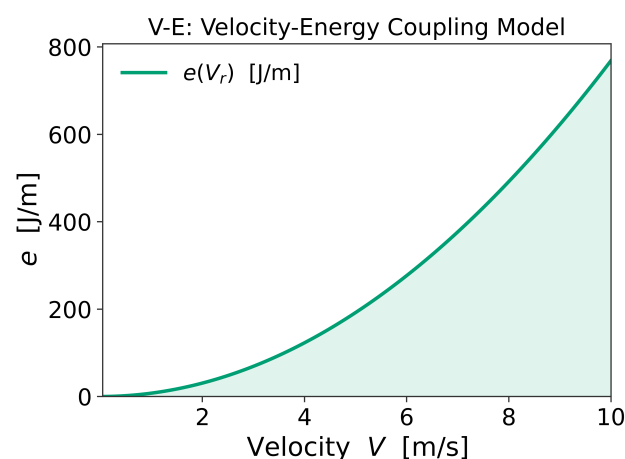


Figure 1. Velocity-energy (V-E) coupling model.

3.2. Velocity-Curvature Coupling Model (V-C)

The manoeuvrability of an AUV in three-dimensional space is bounded by both geometric and kinodynamic limits. The centripetal acceleration required to follow a path of curvature κ at speed V is

$$a_n = V^2 \kappa \quad (8)$$

where a_n is the normal (centripetal) acceleration. The upper bound $\bar{a}$ on centripetal acceleration is determined by the AUV's hydrodynamic characteristics and the lateral thrust authority of the propulsion system, which can be limited by thruster saturation and actuator constraints [29]. The speed-dependent curvature bound follows from the standard lateral acceleration relation used in speed planning, where the admissible speed along a path is constrained by the path curvature [30]. Accounting for both structural and kinodynamic limits, the V-C constraint envelope in the speed-curvature plane is formulated as

$$\kappa_{\max}(V) = \begin{cases} \kappa_g, & V \leq V_c \\ \bar{a}/V^2, & V > V_c \end{cases} \quad (9)$$

where κ_g is the geometric curvature limit imposed by maximum control-surface deflection or minimum turning radius; $\bar{a}$ is the maximum allowable centripetal acceleration; and $V_c = \sqrt{\bar{a}/\kappa_g}$ is the critical speed at which the geometric and kinodynamic constraints intersect. The maximum feasible curvature $\kappa_{\max}(V)$ exhibits a piecewise character, which partitions the operating regime into two distinct regions (Figure 2).

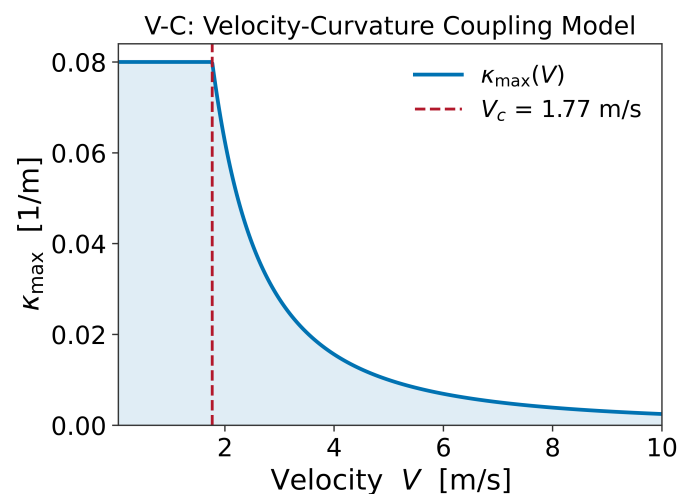


Figure 2. Velocity-curvature (V-C) coupling model.

- Geometry-saturated regime ($V \leq V_c$): The centrifugal force $V^2\kappa$ has not yet reached the kinodynamic limit. Manoeuvrability is constrained by vehicle geometry alone and the maximum curvature equals the constant κ_g , permitting tight turns at low speed.
- Kinodynamically limited regime ($V > V_c$): As speed exceeds the critical value, the limiting factor switches from geometry to kinodynamics. The maximum allowable curvature must decrease as $\bar{a}/V^2$, requiring progressively gentler turns at higher speeds.

The curvature at every position along the path is required to satisfy $\kappa(s) \leq \kappa_{\max}(V(s))$, ensuring that the control and propulsive forces remain within the physical capability of the vehicle.

3.3. Velocity-Safe-Distance Coupling Model (V-S)

A dynamic safety distance model that explicitly incorporates AUV speed is essential for kinodynamically feasible planning. Conventional obstacle inflation methods apply a fixed inflation radius, ignoring the speed-dependent braking dynamics of the vehicle.

During an emergency stop, the thruster delivers the maximum reverse braking thrust T_b . The longitudinal dynamics of the AUV are then governed by

$$m \frac{dV}{dt} = - \left(T_b + \frac{1}{2} \rho C_D A V^2 \right) \quad (10)$$

where m is the total system mass. The braking distance from initial speed V to rest is obtained by integrating

$$D_b(V) = \int_0^V \frac{mv}{T_b + \frac{1}{2} \rho C_D A v^2} dv \quad (11)$$

where v is the integration variable representing instantaneous speed. This expression shows that braking distance increases nonlinearly with initial speed, with reverse thrust dominating at low speed and hydrodynamic drag becoming more influential at high speed.

Combining the physical braking distance with the distance covered during system reaction time t_r and a fixed safety buffer δ_s , the dynamic safe distance is defined as

$$D_s(V) = D_b(V) + V t_r + \delta_s \quad (12)$$

The dynamic safety distance follows the safe-stopping structure used in formal obstacle-avoidance models, where the clearance must include braking distance and reaction distance terms [31]. The collision-avoidance constraint in the planning framework is therefore speed dependent:

$$\|p(s) - p_k\| \geq D_s(V(s)), \quad k = 1, \dots, N_o \quad (13)$$

where $p(s)$ is the AUV position at arc length s ; p_k is the nearest surface point of the k -th obstacle to $p(s)$; and N_o is the total number of obstacles. This formulation automatically enforces larger clearances at high speed and permits closer approach during low-speed fine manoeuvring, improving both safety and operational flexibility (Figure 3).

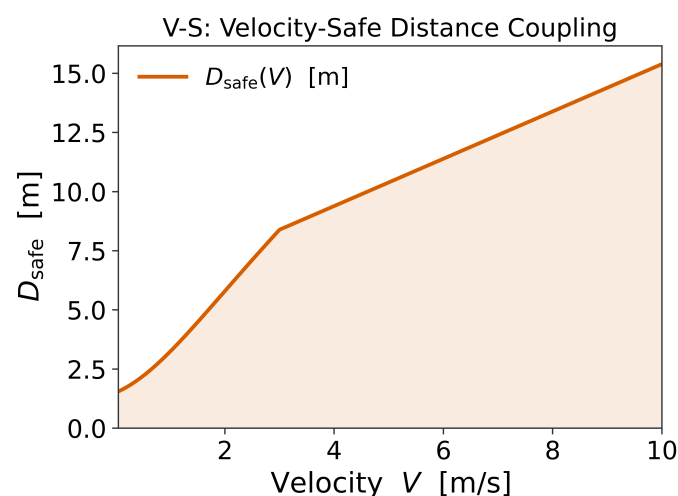


Figure 3. Velocity-safe-distance (V-S) coupling model.

4. Energy-Efficient Velocity-Kinodynamic RRT* (VK-RRT*)

Traditional RRT* exhibits three key limitations in underwater flow environments: blind random sampling with no awareness of environmental structure, no accounting for flow-field energy consumption, and no guarantee of kinodynamic feasibility in the generated paths. Building on the coupling models derived in Section 3, VK-RRT* introduces four coordinated mechanisms: potential-guided sampling, adaptive step-length expansion, kinodynamic constraint, and multi-objective cost evaluation. These mechanisms are designed to generate paths that are energy efficient, kinodynamically feasible, and safe with respect to speed-dependent obstacle clearance. The overall framework is illustrated in Figure 4.

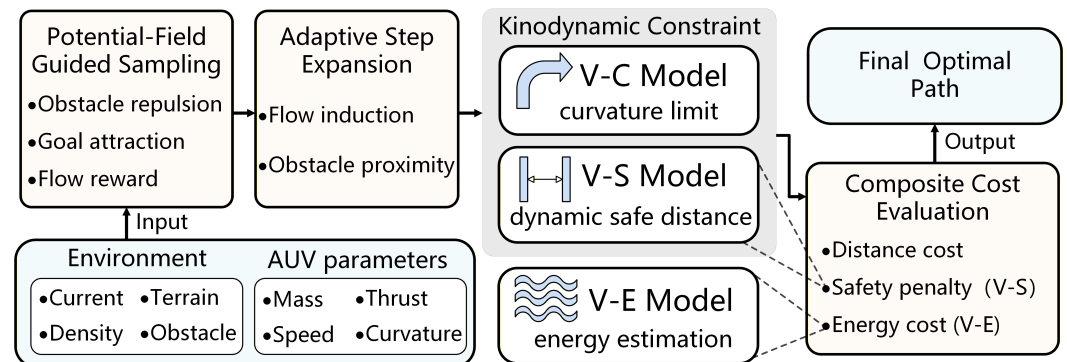


Figure 4. Overall framework of the proposed flow-field VK-RRT* algorithm. The V-E, V-C, and V-S coupling models provide the physical basis for energy estimation, curvature feasibility, and dynamic safety enforcement, respectively.

4.1. Hybrid Heuristic Sampling Guided by Composite Potential Field

Uniform random sampling in three-dimensional space can slow convergence and lead to unnecessary exploration. A hybrid heuristic sampling strategy is introduced, in which a composite potential field maps obstacle distribution, goal direction, and hydrodynamic characteristics into a scalar field over the planning space. A candidate competition mechanism then biases the random sampling process toward low-energy, high-safety regions. For any candidate point p , the potential is the sum of four terms:

$$U(p) = U_o(p) + U_t(p) + U_a(p) + U_f(p) \quad (14)$$

where U_o is the obstacle repulsion term, U_t is the terrain repulsion term, U_a is the goal attraction term, and U_f is the flow-field reward term. The obstacle and terrain repulsion terms are, respectively,

$$U_o(p) = \sum_{k=1}^{N_o} \begin{cases} \frac{w_r}{\max(d_k(p), \varepsilon)^2}, & d_k(p) < R_r \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

$$U_t(p) = \begin{cases} \frac{w_r}{\max(d_z(p), \varepsilon)^2}, & d_z(p) < R_r \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

where $d_k(p)$ is the signed distance from p to the boundary of the k -th ellipsoidal obstacle; $d_z(p) = z - z_b(x, y)$ is the vertical clearance above the seabed; R_r is the repulsion influence radius; w_r is the repulsion weight; and $\varepsilon > 0$ is a small constant that prevents singularity at the obstacle boundary. The goal attraction term is

$$U_a(p) = -\frac{w_a}{\|p - q_g\|} \quad (17)$$

where q_g is the goal position and $w_a > 0$ is the attraction weight.

The flow-field reward term is constructed from the projection of the local current velocity onto the goal direction:

$$U_f(p) = -w_f \max(\mathbf{v}_c(p)^T \hat{\mathbf{e}}_g, 0), \quad \hat{\mathbf{e}}_g = \frac{q_g - p}{\|q_g - p\|} \quad (18)$$

where $w_f > 0$ is the flow reward weight and $\hat{\mathbf{e}}_g$ is the unit vector pointing from p toward the goal. A positive projection ($\mathbf{v}_c^T \hat{\mathbf{e}}_g > 0$) lowers the potential at p via the negative term, increasing the likelihood of its selection as an expansion target. When the current opposes the goal direction, the term is zeroed to avoid extra penalisation.

4.2. Adaptive Step-Length Expansion Strategy

Conventional RRT* uses a fixed expansion step length, which limits its adaptability in complex three-dimensional ocean environments. A small fixed step in open water produces unnecessarily dense nodes, raising interpolation cost and slowing convergence. A large fixed step near obstacles risks crossing obstacle boundaries. AUV dynamics in favourable and opposing currents also differ fundamentally. An adaptive step-length strategy is therefore proposed that couples spatial obstacle density with local flow dynamics.

An obstacle-proximity factor β is defined as a function of the distance d_o from the sampled point to the nearest obstacle:

$$\beta(d_o) = \begin{cases} 0, & d_o \leq R_s \\ 1 - e^{-\lambda(d_o - R_s)}, & R_s < d_o \leq D_o \\ 1, & d_o > D_o \end{cases} \quad (19)$$

where R_s is the base safety radius below which expansion is suppressed; D_o is the open-water distance threshold above which the environment is considered unobstructed; $\lambda > 0$ is a sensitivity parameter. As the sampled point approaches an obstacle, β decreases rapidly, compressing the step length for fine-grained local exploration; in open water, β is set as 1.

A flow-induction factor α is defined based on the projection of the local current velocity onto the desired expansion direction $\hat{\mathbf{u}}$:

$$v_p = \mathbf{v}_c \cdot \hat{\mathbf{u}} \quad (20)$$

$$\alpha(v_p) = 1 + \gamma \frac{v_p}{V_{\max}} \quad (21)$$

where $\hat{\mathbf{u}}$ is the unit vector from the nearest tree node toward the random sample; $V_{\max}$ is the maximum AUV design speed; γ is the flow influence weight. A positive v_p (with-current) yields $\alpha > 1$, amplifying the step, and a negative v_p (counter-current) yields $\alpha < 1$, compressing it. Combining both factors, the adaptive step length is

$$L_a = \text{clip}(L_0 \cdot \alpha(v_p) \cdot \beta(d_o), L_{\min}, L_{\max}) \quad (22)$$

where L_0 is the nominal base step length, and $L_{\min}$ and $L_{\max}$ are lower and upper bounds that prevent abrupt step changes. Through this strategy, the algorithm naturally aligns with prevailing currents in open regions and maintains safe clearance in obstacle-cluttered areas.

4.3. Kinodynamic Constraint Checking

Although potential-field guidance promotes convergence toward high-quality paths, the raw RRT* path consists of piecewise linear segments that may violate kinodynamic

limits at junction nodes. A two-level kinodynamic feasibility check is therefore introduced to screen candidate nodes during tree expansion and ensure that the planned path respects the AUV's manoeuvrability limits.

4.3.1. V-C-Based Heading-Change Constraint

For a candidate path segment traversed at reference cruise speed v_0 , the maximum admissible heading change is approximated as

$$\psi_{\max} \approx \min(\|\mathbf{d}_p\|, \|\mathbf{d}_n\|) \cdot \kappa_{\max}(v_0) \quad (23)$$

where $\mathbf{d}_p$ and $\mathbf{d}_n$ are the displacement vectors of the parent segment and the candidate new segment, respectively, and $\kappa_{\max}(v_0)$ is evaluated from the V-C model. Using the shorter of the two segments as the reference length provides a conservative curvature bound. Let θ_p denote the heading angle of the parent segment and θ_n that of the candidate segment. A candidate node q_n is rejected if the heading change exceeds the admissible limit:

$$|\theta_n - \theta_p| > \psi_{\max}(v_0) \Rightarrow \text{Reject } q_n \quad (24)$$

This prevents sharp, kinematically infeasible turns from appearing in the generated path.

4.3.2. V-S-Based Dynamic Collision Detection

To prevent paths from passing too close to obstacles, where flow disturbances or braking manoeuvres could cause collision, the V-S model is used as a collision-detection threshold. The dynamic safe distance at cruise speed v_0 is

$$D_s(v_0) = D_b(v_0) + v_0 t_r + \delta_s \quad (25)$$

The collision check radius is set to $R_c = r_0 + D_s(v_0)$, where r_0 is the AUV body radius. The collision detection is

$$\text{dist}(q_n, O_b) < R_c \Rightarrow \text{Reject } q_n \quad (26)$$

where O_b denotes the set of obstacle boundaries. For terrain clearance, the condition is

$$z > z_b(x, y) + \max(0.5 D_s(v_0), \varepsilon_g) \quad (27)$$

where ε_g is a minimum vertical clearance constant. This dynamic inflation creates a speed-dependent exclusion zone around each obstacle, guiding the planner toward open corridors and reducing collision risk caused by insufficient braking distance. When the environment is too narrow to provide any path point satisfying the required safety margin, the planner selects the candidate with the maximum available clearance to the obstacle boundary.

4.4. Multi-Objective Composite Cost Function

In the parent selection and rewiring steps of RRT*, the quality of a candidate edge is evaluated by $\text{Cost}(q_p, q_n)$. To overcome the limitation of optimising geometric distance alone, three sub-costs are defined and combined into a normalised multi-objective composite cost.

The geometric distance cost J_d reflects path length:

$$J_d = L = \|q_n - q_p\| \quad (28)$$

The velocity–energy cost J_e drives the planner to exploit ambient currents. The AUV is assumed to traverse the segment at constant ground speed v_0 . Let $\hat{\mathbf{n}} = (q_n - q_p)/L$ be

the unit direction vector of the segment and $q_m = (q_p + q_n)/2$ be the segment midpoint at which the current is sampled. The relative speed is

$$V_r = \|v_0 \hat{\mathbf{n}} - \mathbf{v}_c(q_m)\| \quad (29)$$

and the energy cost is

$$J_e = e(V_r) \cdot L = \frac{\rho C_D A V_r^3}{2v_0} \cdot L \quad (30)$$

Because J_e scales as the cube of V_r , with-current segments are strongly incentivised and counter-current segments are heavily penalised. The energy cost J_e is computed using the constant speed. Small speed controller errors may shift the absolute energy value, especially in regions with sharp current gradients. Although speed-tracking errors may affect the absolute value of the estimated energy, the same calculation rule is applied to all path segments. As a result, J_e can still be used to compare the relative energy demand of different path segments during planning.

The proximity penalty cost J_s complements binary collision checking by guiding the planner away from obstacles. Let d_k be the signed distance from q_m to the k -th obstacle. When $d_k < 2D_s(v_0)$, a linear penalty is accumulated for each obstacle,

$$\phi(d_k) = \frac{\max(0, D_s(v_0) - d_k)}{D_s(v_0)} \quad (31)$$

and the safety cost for the segment is

$$J_s = \left(\sum_{k=1}^{N_o} \phi(d_k) \right) \cdot L \quad (32)$$

when q_m lies outside the influence range of all obstacles $J_s = 0$. The penalty increases linearly as clearance falls below $D_s(v_0)$, continuously steering the planner toward paths with ample clearance.

Because the three sub-costs differ substantially in physical units and numerical scale, a static reference normalisation based on the maximum expansion step length L_{max} is applied. Normalisation bases are defined as

$$\tilde{L} = L_{max}, \quad \tilde{E} = e(V_{max}) \cdot L_{max}, \quad \tilde{S} = N_o \cdot L_{max} \quad (33)$$

These bases are chosen to place the three sub-costs in comparable numerical ranges under typical operating conditions. The composite cost is

$$\text{Cost}(q_p, q_n) = w_1 \frac{J_d}{\tilde{L}} + w_2 \frac{J_e}{\tilde{E}} + w_3 \frac{J_s}{\tilde{S}} \quad (34)$$

where w_1 , w_2 , and w_3 are weights. Through this design, the parent selection and rewiring steps are actively biased away from high-energy, low-clearance, and obstacle-proximate expansions.

5. Simulation Experiments and Analysis

5.1. Simulation Environment Setup

A high-fidelity three-dimensional simulation platform is implemented, incorporating realistic seabed topography, ellipsoidal suspended obstacles, and a composite Lamb–Oseen current field (Figure 5).

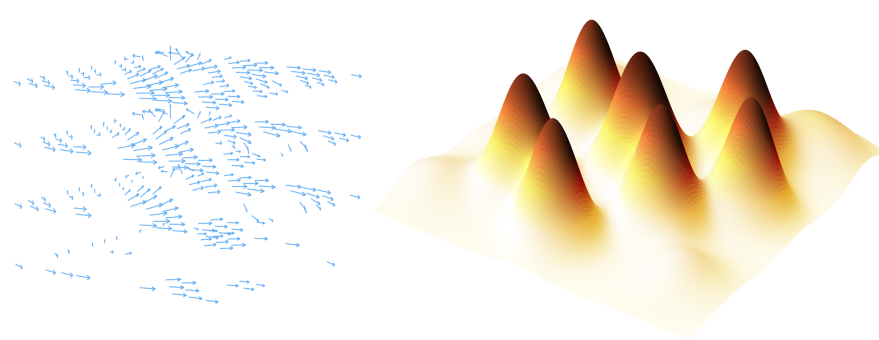


Figure 5. Three-dimensional Lamb–Oseen current field and seafloor obstacle terrain. Blue arrows indicate ocean current vectors.

The planning volume spans $100\text{ m} \times 100\text{ m} \times 55\text{ m}$. The seabed is generated by superimposing 14 Gaussian ridges on a reference depth of $z_0 = 2\text{ m}$, with peak heights ranging from 13 m to 65 m and width coefficients ranging from 9 m to 13 m, producing a rugged terrain. The current field is composed of three Lamb–Oseen vortices, whose parameters are listed in Table 1.

Table 1. Lamb–Oseen vortex parameters.

Vortex	Centre (x_0, y_0, z_0) (m)	Γ	δ (m)
1	(38, 36, 35)	+60.0	22.0
2	(68, 64, 30)	−45.0	17.0
3	(22, 74, 42)	+30.0	13.0

The hyperparameters are summarised in Table 2. The AUV physical parameters adopted in all experiments are shown in Table 3. For the composite cost, w_1 , w_2 , and w_3 regulate the relative preference for path length, propulsion energy, and obstacle clearance after normalisation. Since energy-efficient navigation is the main objective in current-disturbed environments, a larger weight is assigned to w_2 , while moderate values of w_1 and w_3 limit excessive detours and maintain clearance preference. In the potential-field sampler, w_a , w_r , and w_f regulate goal attraction, obstacle repulsion, and current-assisted sampling. For the adaptive step length, $L_{\min}$ and $L_{\max}$ define the allowable search resolution, while λ and γ control the sensitivity to obstacle proximity and current projection. These parameters may influence the trade-off among convergence speed, energy consumption, path compactness, and clearance conservativeness. The empirical weights mainly determine the planning preference, such as whether the algorithm favours lower energy consumption, shorter path length, or larger obstacle clearance. By contrast, the basic feasibility of the path is directly checked by the V–C heading-change constraint and the V–S dynamic collision constraint during node expansion. Therefore, moderate variations in these weights are more likely to change the quantitative performance, such as energy, path length, clearance, and convergence speed, rather than directly causing collision or curvature violation.

Two baseline algorithms are selected for comparison: RRT*, which performs standard uniform random sampling, and Informed-RRT*, which restricts sampling to the prolate hyperellipsoid defined by the current best path cost [32,33]. Both baselines use Euclidean distance as their sole cost function and apply a fixed collision margin of 1.0 m. To ensure a fair comparison, all three algorithms share the same maximum iteration count, goal-region radius, and rewiring radius.

Table 2. Algorithm hyperparameters.

Parameter	Symbol	Value
Distance-cost weight	w_1	0.5
Energy-cost weight	w_2	2.0
Safety-cost weight	w_3	0.5
Repulsion weight	w_r	1.0
Attraction weight	w_a	2.5
Flow reward weight	w_f	0.6
Repulsion influence radius	R_r	10.0
Base step length	L_0	3.5
Minimum step length	$L_{\min}$	1.75
Maximum step length	$L_{\max}$	7.0
Proximity sensitivity	λ	0.5
Flow induction weight	γ	0.3
Maximum iterations	$I_{\max}$	7000
Goal-region radius	r_g	5.0 m
Rewiring radius	r_w	12.0 m

Table 3. AUV physical parameters.

Parameter	Symbol	Value
Cruise speed	v_0	1.5 m/s
Maximum speed	$V_{\max}$	3.0 m/s
Geometric curvature limit	κ_g	0.08 m^{-1}
Max. centripetal acceleration	$\bar{a}$	0.25 m/s^2
Critical speed	V_c	1.768 m/s
Seawater density	ρ	1025 kg/m^3
Drag coefficient	C_D	0.30
Reference cross-sectional area	A	0.05 m^2
System mass (incl. added mass)	m	50 kg
Maximum reverse thrust	T_b	30 N
Reaction time	t_r	1.0 s
Safety buffer	δ_s	1.5 m

To evaluate algorithmic robustness under varying environmental conditions, four test scenarios are constructed with different combinations of vortex-strength triplets ($\Gamma_1, \Gamma_2, \Gamma_3$) and obstacle densities N_o , as summarised in Table 4. Scenario S1 adopts a moderate current field (60, −45, 30) with 10 obstacles. Scenario S2 adopts a weak current field (18, −14, 10) with 14 obstacles. Scenarios S3 and S4 share a strong current configuration (100, −80, 55) with 6 and 14 obstacles, respectively. This design covers a representative range of current intensities and obstacle densities, enabling systematic evaluation of algorithmic performance under diverse operating conditions.

5.2. Path-Planning Results and Multi-Scenario Comparison

All three algorithms successfully find collision-free paths from the start position (40, 85, 10) m to the goal position (5, 5, 35) m within the maximum iteration budget. Representative planning results under scenario S1 are shown in Figure 6. The RRT* path contains noticeable detours and local sharp bends because uniform sampling does not account for environmental structure. Since RRT* does not account for current cost and the additionally placed obstacles do not intersect its solution path, κ^* and $d_{\min}$ remain unchanged across scenarios. Informed-RRT* produces a geometrically more compact path owing to focused sampling within the informed ellipsoid, yet non-smooth junction nodes remain visible at several waypoints. VK-RRT* selects wider detours around obstacle clusters, aligns

with the prevailing current direction in open corridors, and yields a noticeably smoother overall trajectory.

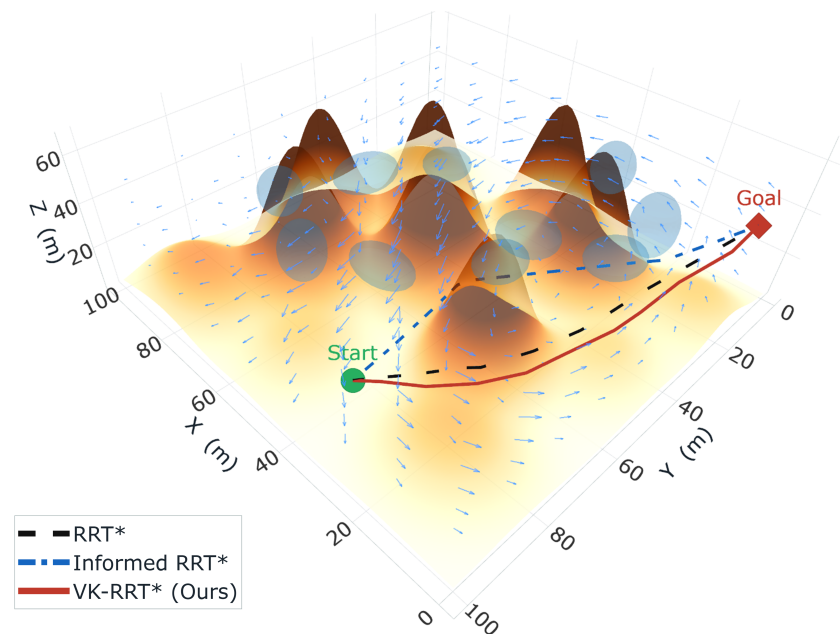


Figure 6. Three-dimensional path-planning results under scenario S1. Seabed topography is rendered as a brown surface, suspended obstacles as grey ellipsoids, and blue arrows as ocean current vectors.

Quantitative results across four physical metrics under the four test scenarios are reported in Table 4, where bold values indicate the best performance for each metric within the corresponding scenario. The four evaluation metrics are total path length L (m), total propulsion energy E (J), computed via the V-E model, maximum path curvature κ^* (m^{-1}), and minimum obstacle clearance $d_{\min}$ (m).

Table 4. Performance comparison of three path planners under four test scenarios.

Scenario	Current	Obstacles	Algorithm	L [m]	E [J]	κ^* [m^{-1}]	$d_{\min}$ [m]
S1	(60, −45, 30)	$N_o = 10$	RRT*	98.0	1377.9	0.0887	3.21
			Informed RRT*	95.8	1854.1	0.1044	2.59
			VK-RRT* (Ours)	99.5	1319.7	0.0475	4.13
S2	(18, −14, 10)	$N_o = 14$	RRT*	99.4	1571.3	0.0887	3.21
			Informed RRT*	96.1	1661.6	0.0620	2.52
			VK-RRT* (Ours)	100.3	1558.6	0.0411	4.65
S3	(100, −80, 55)	$N_o = 6$	RRT*	98.0	1295.2	0.0887	3.21
			Informed RRT*	95.6	2208.8	0.0797	2.65
			VK-RRT* (Ours)	99.9	1148.1	0.0390	3.88
S4	(100, −80, 55)	$N_o = 14$	RRT*	99.4	1301.0	0.0887	3.21
			Informed RRT*	96.1	2356.2	0.0620	2.52
			VK-RRT* (Ours)	98.9	1192.2	0.0541	4.03

Current: vortex-strength triplet $(\Gamma_1, \Gamma_2, \Gamma_3)$. Obstacles: total number of ellipsoidal obstacles.

In terms of energy efficiency, VK-RRT* achieves the lowest propulsion energy in all four scenarios. The advantage grows with current intensity: under the moderate current field (S1–S2), the energy savings over the two baselines are modest, whereas under the strong current field (S3–S4), reductions reach up to 11.4% relative to RRT* and 49.4% relative to Informed RRT*. This trend is attributable to the lack of flow-field awareness: as

current intensity increases, counter-current traversal becomes more probable, and the cubic dependence of propulsion power on relative speed amplifies energy waste in adverse-flow regions. The flow-potential-guided sampling and V-E cost evaluation of VK-RRT* steer expansion toward current-assisted corridors, suppressing this amplification effect.

The curvature and clearance results reflect the kinodynamic constraints enforced by VK-RRT*. The maximum curvature κ^* of VK-RRT* remains strictly below $\kappa_{\max} = 0.08 \text{ m}^{-1}$ in all scenarios, whereas RRT* consistently exceeds this bound and Informed RRT* does so in S1. The minimum obstacle clearance $d_{\min}$ of VK-RRT* is the largest in all scenarios, with average improvements of 30.0% over RRT* and 62.6% over Informed RRT* across the four scenarios. This consistent clearance advantage is a direct consequence of the V-S dynamic inflation mechanism, which enforces a speed-dependent exclusion zone rather than a fixed static margin.

Regarding path length, VK-RRT* generally produces slightly longer paths than the baselines. This is an expected consequence of the multi-objective cost function: a small geometric detour is exchanged for reduced energy consumption, satisfied curvature constraints, and larger obstacle clearance.

Because RRT-based planners rely on random sampling, the representative single-run results may be affected by the selected random seed. To further evaluate robustness, each algorithm is repeatedly executed over 10 independent random seeds under the same four scenarios. The resulting mean and standard deviation are reported in Table 5, and the corresponding distributions are shown in Figure 7.

Table 5. Multi-seed performance comparison of three path planners under four test scenarios. Values are reported as mean $\pm$ standard deviation over random seeds.

Scenario	Algorithm	L [m]	E [J]	κ^* [m^{-1}]	$d_{\min}$ [m]
S1	RRT*	97.32 $\pm$ 1.24	1748.81 $\pm$ 264.50	0.076 $\pm$ 0.016	2.69 $\pm$ 0.56
	Informed RRT*	95.90 $\pm$ 0.48	1890.51 $\pm$ 38.67	0.086 $\pm$ 0.029	2.42 $\pm$ 0.55
	VK-RRT* (Ours)	99.25 $\pm$ 1.05	1330.25 $\pm$ 36.77	0.054 $\pm$ 0.009	3.47 $\pm$ 0.85
S2	RRT*	97.40 $\pm$ 1.28	1642.52 $\pm$ 64.21	0.078 $\pm$ 0.018	2.48 $\pm$ 0.44
	Informed RRT*	95.98 $\pm$ 0.47	1672.30 $\pm$ 31.02	0.073 $\pm$ 0.029	2.14 $\pm$ 0.25
	VK-RRT* (Ours)	98.84 $\pm$ 0.57	1554.26 $\pm$ 29.41	0.060 $\pm$ 0.013	3.44 $\pm$ 0.96
S3	RRT*	96.73 $\pm$ 1.08	2107.48 $\pm$ 437.19	0.069 $\pm$ 0.022	2.53 $\pm$ 0.33
	Informed RRT*	95.71 $\pm$ 0.41	2300.21 $\pm$ 55.32	0.076 $\pm$ 0.023	2.61 $\pm$ 0.40
	VK-RRT* (Ours)	99.84 $\pm$ 1.19	1200.62 $\pm$ 33.27	0.054 $\pm$ 0.013	3.60 $\pm$ 0.82
S4	RRT*	97.40 $\pm$ 1.28	2008.19 $\pm$ 499.45	0.078 $\pm$ 0.018	2.48 $\pm$ 0.44
	Informed RRT*	95.98 $\pm$ 0.47	2320.89 $\pm$ 59.65	0.073 $\pm$ 0.029	2.14 $\pm$ 0.25
	VK-RRT* (Ours)	100.05 $\pm$ 1.20	1173.59 $\pm$ 40.42	0.045 $\pm$ 0.007	3.15 $\pm$ 0.52

The multi-seed results confirm that VK-RRT* achieves a 100% success rate in all four scenarios and consistently gives the lowest mean propulsion energy. Relative to RRT*, the mean energy reductions are 23.9%, 5.4%, 43.0%, and 41.6% in S1–S4, respectively; relative to Informed RRT*, the corresponding reductions are 29.6%, 7.1%, 47.8%, and 49.4%. The advantage grows in the strong-current scenarios because purely geometric planners are more likely to cross adverse-flow regions, where the dependence of propulsion power on relative speed amplifies energy consumption.

The distributions in Figure 7 also show that the energy spread of VK-RRT* is much smaller than that of RRT* in the strong-current cases. This indicates that the V-E cost not only reduces the mean propulsion energy but also makes the energy performance less sensitive to random-seed variability in high-current environments. The curvature and clearance results reflect the kinodynamic constraints enforced by VK-RRT*. The maximum curvature of VK-RRT* remains below $\kappa_{\max}$ in all scenarios, while the baselines show larger

curvature dispersion and multiple violations. VK-RRT* also gives the largest mean obstacle clearance in all four scenarios, with average improvements of 34.2% over RRT* and 47.4% over Informed RRT*. This consistent clearance advantage is a direct consequence of the V-S dynamic inflation mechanism, which enforces a speed-dependent exclusion zone rather than a fixed static margin. Regarding path length, VK-RRT* produces slightly longer paths than the two baselines. This is an expected consequence of the multi-objective cost function: a small geometric detour is exchanged for reduced energy consumption, satisfied curvature constraints, and larger obstacle clearance.

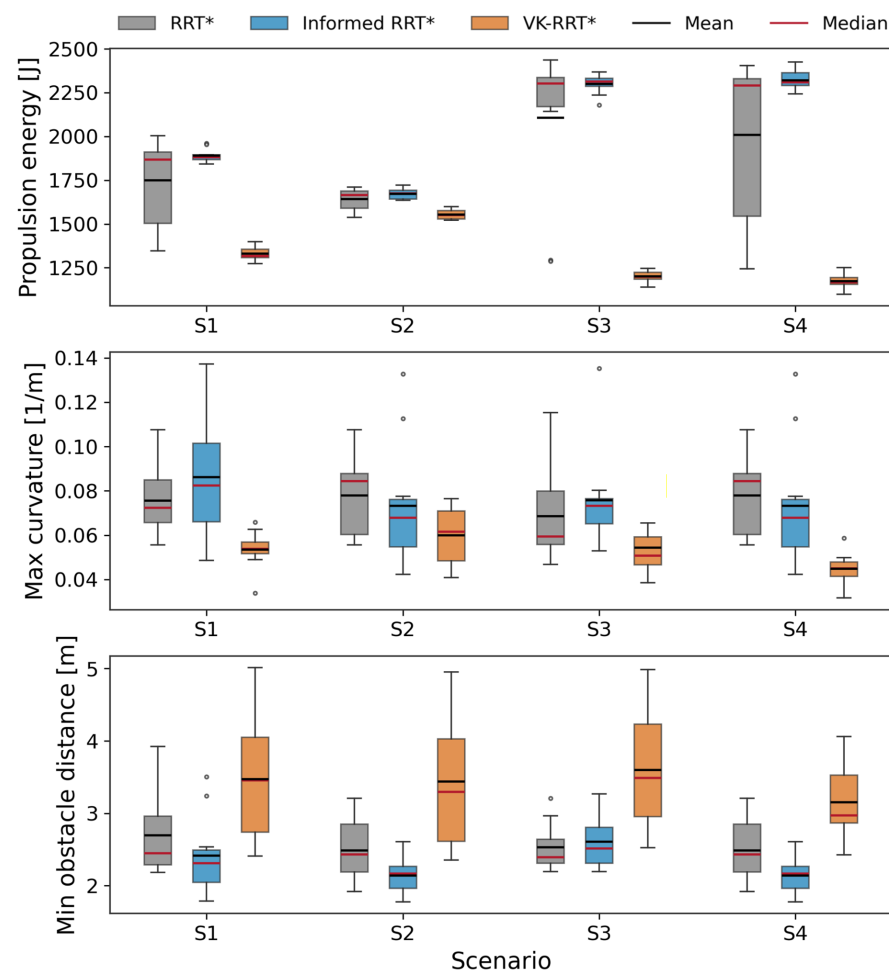


Figure 7. Multi-seed distributions of propulsion energy, maximum curvature, and minimum obstacle distance under scenarios S1–S4. Each boxplot summarises 10 independent random seeds; black and red horizontal lines denote the mean and median, respectively.

5.3. Path Metric Analysis for Scenario S1

To provide deeper insight into the physical behaviour along the planned paths, four metrics are extracted from the scenario S1 results and plotted as functions of length in Figure 8.

The curves in Figure 8a show that the curvatures of RRT* and Informed RRT* exceed the constraint bound $\kappa_{\max}$, indicating that these paths require turns beyond the AUV's manoeuvrability limits. By contrast, the curvature of VK-RRT* remains strictly below the bound throughout the trajectory, and its overall variation is smaller, confirming that the V-C hard constraint ensures geometric smoothness at every path node.

The specific energy values in Figure 8b show that VK-RRT* consumes less energy than the two baselines, particularly on with-current segments where the relative speed V_r is

reduced. Because RRT* and Informed RRT* optimise geometric distance only, they do not penalise counter-current traversal and therefore incur higher energy costs in adverse-flow regions. By combining the flow-potential term during sampling with the V-E energy cost during cost evaluation, VK-RRT* preferentially expands through low-drag corridors.

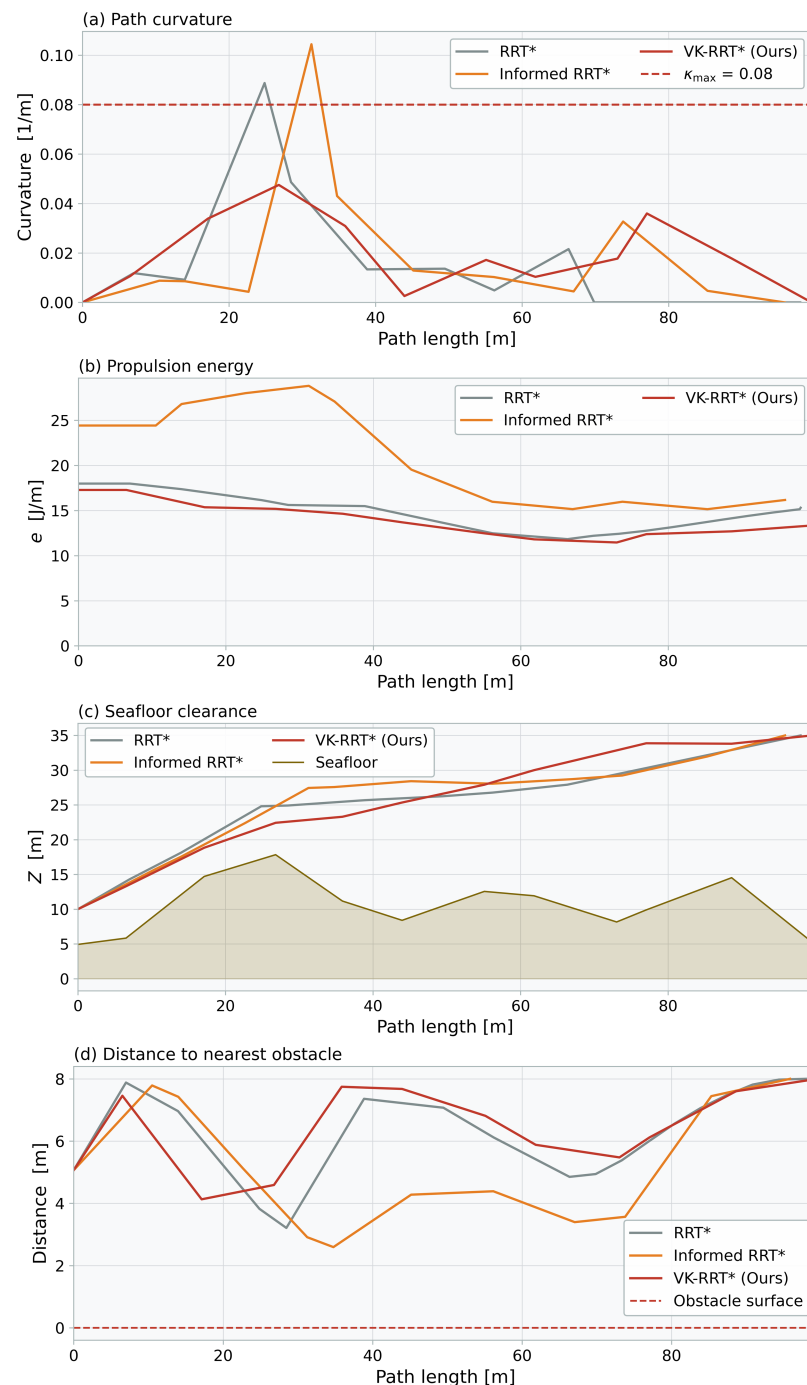


Figure 8. Metric curves along length for the three algorithms under scenario S1. (a) Path curvature with the kinematic constraint bound. (b) Specific propulsion energy e along each path. (c) Altitude curves relative to seafloor topography. (d) Distance from path to nearest obstacle.

The altitude curves in Figure 8c show that VK-RRT* maintains a safe margin above the seabed topography throughout the trajectory. The distance curves in Figure 8d further confirm this advantage. VK-RRT* achieves the highest obstacle clearance in most cases, whereas both baselines contain segments where the clearance drops to levels insufficient

for safe braking. These results validate the effectiveness of the V-S-based dynamic inflation mechanism in enforcing speed-dependent safety margins.

5.4. Adaptive Step-Length and Heading-Change Analysis

The adaptive step-length strategy directly affects the maximum admissible heading change through the relation Equation (23). A shorter adaptive step compresses the allowable heading-change bound, imposing a tighter turning constraint at that node. Figure 9 plots the actual heading-change angle of VK-RRT* and the corresponding V-C upper bound $\psi_{\max}$ along the path under scenario S1.

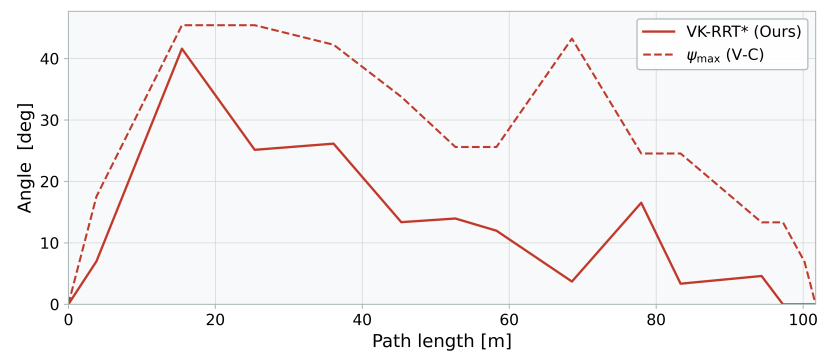


Figure 9. Heading-change angle of VK-RRT* and the V-C admissible bound $\psi_{\max}$ along the path under scenario S1. The actual heading change remains below the bound at every node.

The actual heading change remains strictly below $\psi_{\max}$ at every node along the trajectory, confirming that the V-C-based heading-change constraint is satisfied throughout the path. In regions where the adaptive step is compressed by the obstacle proximity factor β , the bound $\psi_{\max}$ decreases accordingly, tightening the turning constraint near obstacles. In open corridors where the flow-induction factor α amplifies the step, $\psi_{\max}$ increases, permitting larger heading adjustments that allow the tree to align with the prevailing current direction. This coupling between adaptive step length and kinodynamic feasibility ensures that the planned path satisfies the V-C constraint under spatially varying expansion granularity.

5.5. Task Generalisation Evaluation

The preceding analysis focused on a single start-goal configuration. To assess whether the performance advantages of VK-RRT* generalise across different traversal geometries, five distinct start-goal tasks (T0–T4) are defined and evaluated under both the moderate-current scenario S1 and the strong-current, high-obstacle-density scenario S4. The tasks span diagonal, lateral, and ascending trajectories, exposing the planner to different obstacle clusters and current regimes. Three-dimensional paths are visualised in Figure 10, and quantitative results are reported in Table 6, where all three algorithms share the same random seed.

Across the 10 task scenario combinations, VK-RRT* achieves the lowest propulsion energy in 8 cases, with the energy advantage increasing with current intensity. The two exceptions occur in task T3 under both scenarios, where the start-goal vector aligns closely with a favourable current corridor so that a geometrically short path coincidentally benefits from the currents. In this task, VK-RRT* selects a wider route to satisfy the safety clearance constraint, trading energy for improved obstacle margin.

The curvature results confirm that VK-RRT* satisfies $\kappa^* < \kappa_{\max} = 0.08 \text{ m}^{-1}$ in all cases. Both baselines produce violations in multiple tasks, with Informed RRT* reaching $\kappa^* = 0.1769 \text{ m}^{-1}$ in T3-S4. VK-RRT* generally yields the largest minimum clearance

$d_{\min}$ across the tested cases. In the two marginal exceptions, the compared algorithm attains a slightly larger clearance by taking a detoured route, but this improvement is accompanied by higher energy consumption. The consistent clearance advantage is a direct result of the V-S dynamic inflation mechanism. Informed RRT* produces the shortest paths in most tasks because of its ellipsoid-focused sampling strategy. The length penalty of VK-RRT* represents the geometric cost of satisfying energy, curvature, and safety objectives simultaneously.

Table 6. Performance comparison of three planners over five start–goal tasks under the S1 and S4 environments.

Task	Start	Goal	Algorithm	L [m]	E [J]	κ^* [m ^{−1}]	$d_{\min}$ [m]
S1: Current (60, −45, 30), Obstacle number $N_o = 10$							
T0	(40, 85, 10)	(5, 5, 35)	RRT*	98.0	1377.9	0.0887	3.21
			Informed RRT*	95.9	1841.9	0.1373	2.54
			VK-RRT* (Ours)	100.3	1307.7	0.0520	3.54
T1	(12, 88, 18)	(88, 15, 18)	RRT*	112.0	3256.0	0.0488	2.16
			Informed RRT*	111.6	3274.0	0.0280	2.31
			VK-RRT* (Ours)	117.4	2257.1	0.0375	2.55
T2	(8, 55, 20)	(92, 30, 20)	RRT*	92.4	1818.4	0.0556	2.24
			Informed RRT*	91.9	1944.8	0.1158	3.61
			VK-RRT* (Ours)	93.2	1785.6	0.0445	3.13
T3	(12, 40, 20)	(90, 75, 22)	RRT*	98.4	1468.8	0.0981	2.09
			Informed RRT*	99.0	1516.6	0.1006	1.39
			VK-RRT* (Ours)	110.9	1892.3	0.0629	4.67
T4	(25, 90, 18)	(90, 45, 40)	RRT*	83.8	2474.4	0.0319	2.70
			Informed RRT*	83.2	2422.0	0.0710	1.58
			VK-RRT* (Ours)	103.1	1891.4	0.0735	5.41
S4: Current (100, −80, 55), Obstacle number $N_o = 14$							
T0	(40, 85, 10)	(5, 5, 35)	RRT*	98.5	1247.7	0.0679	2.11
			Informed RRT*	95.7	2356.8	0.0422	2.24
			VK-RRT* (Ours)	99.0	1181.4	0.0519	3.79
T1	(12, 88, 18)	(88, 15, 18)	RRT*	112.7	4469.3	0.0715	2.59
			Informed RRT*	111.9	4418.2	0.0646	2.19
			VK-RRT* (Ours)	115.6	2645.4	0.0441	2.93
T2	(8, 55, 20)	(92, 30, 20)	RRT*	92.7	2063.4	0.0715	4.11
			Informed RRT*	91.7	2286.5	0.0595	3.89
			VK-RRT* (Ours)	94.9	1922.5	0.0571	3.98
T3	(12, 40, 20)	(90, 75, 22)	RRT*	97.5	1374.2	0.0878	2.16
			Informed RRT*	96.4	1423.2	0.1769	1.64
			VK-RRT* (Ours)	110.2	2107.6	0.0773	2.42
T4	(25, 90, 18)	(90, 45, 40)	RRT*	84.2	3514.7	0.0753	2.69
			Informed RRT*	83.2	3506.3	0.0394	2.31
			VK-RRT* (Ours)	101.6	1958.6	0.0666	2.71

L —total path length. E —total propulsion energy. κ^* —maximum path curvature. $d_{\min}$ —the obstacle distance for each path.

These results confirm that the multi-objective design of VK-RRT* delivers consistent performance across a representative range of start–goal configurations, current intensities, and obstacle densities. The additional computational burden mainly comes from local current evaluation, velocity-dependent cost calculation, curvature feasibility checking, and dynamic safety-distance enforcement during node expansion and rewiring. Since these operations are performed locally for candidate nodes and neighbouring edges, they do not excessively increase the overall search burden. Moreover, this overhead is necessary for

enforcing the physical coupling among energy consumption, manoeuvrability, and safety clearance. Therefore, VK-RRT* can be extended to different obstacle layouts, current configurations, and AUV platforms by updating the corresponding environmental conditions and vehicle parameters.

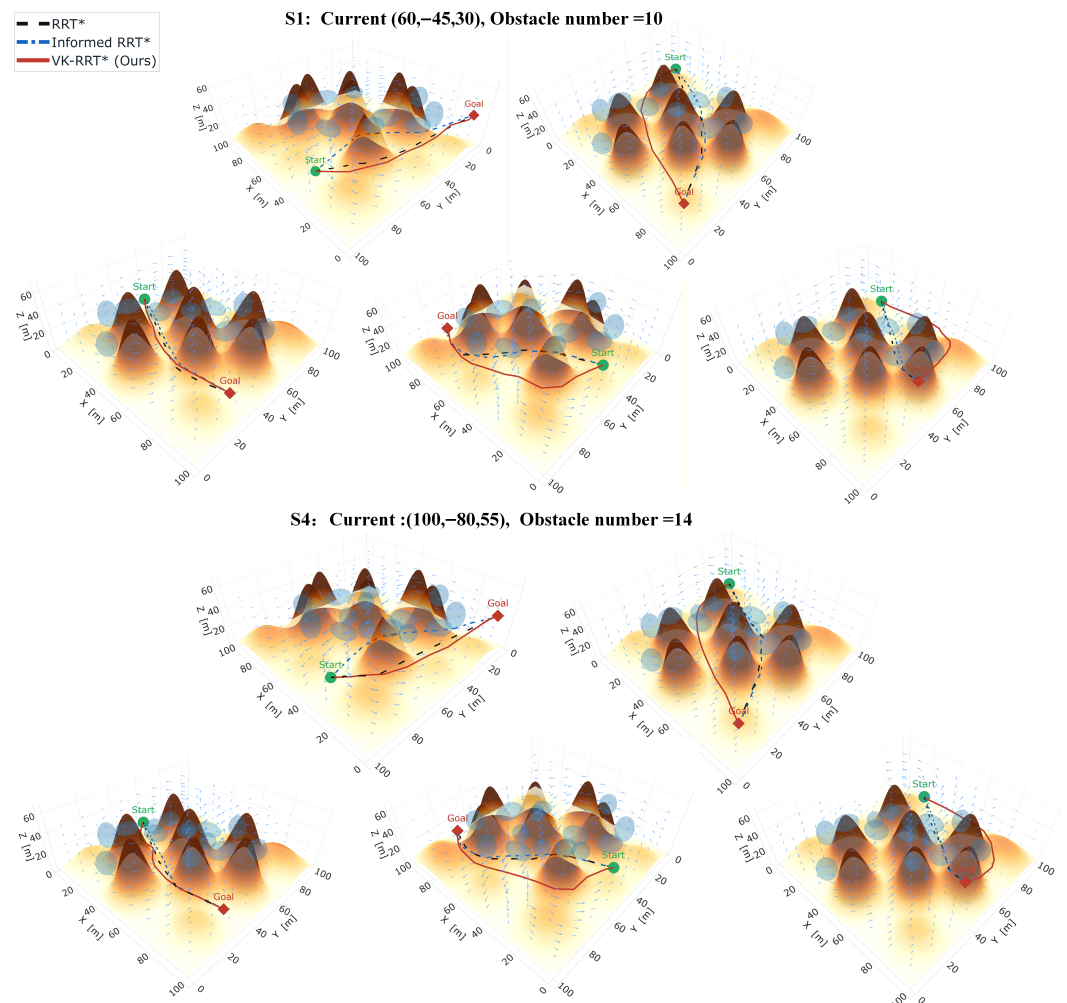


Figure 10. Three-dimensional path-planning results for five start-goal tasks (T0–T4) under scenarios S1 and S4.

5.6. Ablation Study

To isolate the contribution of each algorithmic component, an ablation study is conducted by systematically disabling one module at a time while keeping all other modules active. Five algorithm variants are evaluated: the full VK-RRT*, and four ablated versions obtained by removing one module at a time, referred to as without (w/o) Adaptive Step, w/o V-C, w/o V-S, and w/o V-E. Specifically, w/o Adaptive Step fixes the step length at L_0 , w/o V-C disables the heading-change constraint, w/o V-S replaces the dynamic inflation with no safety margin, and w/o V-E removes the energy cost J_e from the composite cost. Each variant is executed for 10 independent trials with different random seeds under scenario S1 to evaluate robustness, and results are reported in Figure 11.

As shown in the bottom-right panel of Figure 11, removing the V-S module causes the minimum obstacle clearance to decrease sharply relative to the full VK-RRT*. The median clearance falls below 1.0 m, and the lower whisker approaches zero, indicating that several trials generate paths with insufficient braking margin. Although the w/o V-S variant yields shorter paths and lower energy consumption, these gains are achieved by sacrificing safety clearance, making the resulting paths unsuitable for practical AUV deployment.

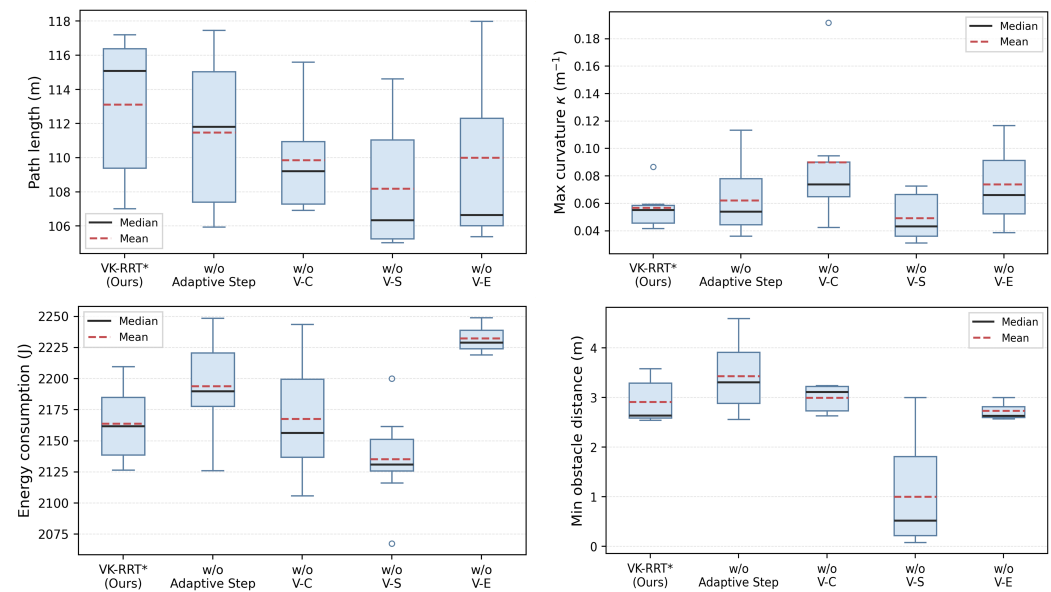


Figure 11. Ablation study results over 10 independent trials.

Disabling the V-C heading-change constraint (w/o V-C) produces the most significant curvature degradation. As shown in the top-right panel, the median maximum curvature rises from approximately 0.054 m^{-1} to 0.075 m^{-1} , with an outlier exceeding 0.18 m^{-1} . The box of this variant lies almost entirely above the kinodynamic bound $\kappa_{\max} = 0.08 \text{ m}^{-1}$, confirming that the V-C constraint is essential for guaranteeing curvature feasibility.

Removing the energy cost term (w/o V-E) produces the highest energy consumption among all variants. The energy consumption shows that the entire box of this variant lies above 2220 J. The interquartile range is notably compact, indicating that the energy penalty is consistent across trials. In the max curvature, the distribution of this variant shifts upward compared with the full algorithm.

The w/o adaptive step variant shows only moderate changes across the four metrics. Its minimum obstacle distance is slightly higher than that of the full VK-RRT*, but this does not indicate stronger safety planning. Rather, the fixed step length limits fine-grained local expansion near obstacles, making the planner more likely to choose conservative detours instead of passing through feasible narrow corridors. These detours increase obstacle clearance statistically, but they can also lengthen the path and lead to higher energy consumption. In open or current-assisted regions, the fixed step length further limits the ability to exploit favourable flow directions. Therefore, the adaptive step-length module mainly improves local scale adjustability, flow-field adaptability, energy efficiency, and curvature smoothness, while obstacle safety is primarily enforced by the V-S dynamic safety distance constraint.

The ablation results confirm that all four modules contribute distinct and complementary roles to the overall performance of VK-RRT*. The V-S module is critical for maintaining safe obstacle clearance, the V-C module guarantees curvature feasibility, the V-E module reduces propulsion energy consumption, and the adaptive step-length mechanism improves energy efficiency and curvature smoothness. Removing any single module leads to measurable degradation in at least one performance metric, validating the necessity of the complete multi-constraint coupling design.

6. Conclusions and Future Work

Unlike existing methods that treat energy consumption, turning feasibility, and obstacle clearance as independent static thresholds, this paper presented VK-RRT*, a planning

framework that physically couples all three quantities through AUV operating speed. Three velocity-dependent coupling models were derived from first principles: the V–E model capturing the cubic relationship between propulsion power and relative flow speed, the V–C model defining a piecewise manoeuvrability envelope in the speed–curvature plane, and the V–S model linking braking distance to instantaneous speed. These models were integrated into the RRT* framework through four coordinated mechanisms, comprising potential-field-guided sampling, adaptive step-length expansion, a kinodynamic feasibility constraint, and a multi-objective cost function, yielding a solution that is simultaneously energy efficient, kinodynamically feasible, and safety guaranteed.

Experiments across four scenarios and five start–goal tasks confirm that VK-RRT* consistently generates energy-efficient, curvature-feasible, and safety-aware paths under different current intensities, obstacle densities, and traversal geometries. In the multi-seed evaluation, the method maintained a 100% success rate across all tested scenarios, reduced mean propulsion energy by up to 43.0% and 49.4% relative to the baselines, and improved mean minimum obstacle clearance by 34.2% and 47.4%. The ablation study further confirmed that each module contributes a distinct and complementary role. Future work will extend the experimental validation to real ocean environments, where the planner can be tested under time-varying currents, complex seabed terrain, and sensing uncertainty. Beyond single-AUV navigation, cooperative multi-AUV planning will also be explored to address coordination, distributed decision-making, and inter-vehicle safety.

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