

## Article

# ACWMA: An Adaptive Cooperative WMA for 3D Path Planning of UUVs in Complex Marine Environment

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## Abstract

Three-dimensional (3D) path planning for Unmanned Underwater Vehicles (UUVs) in typical marine operating conditions presents high-dimensional, non-convex optimization challenges due to undulating seabed topography, underwater threat sources, and coupled multi-physical constraints. Existing studies lack multi-strategy collaborative optimization mechanisms specifically designed for UUV 3D marine navigation constraints, thereby hindering the simultaneous achievement of real-time performance, safety, and energy efficiency in path planning. This paper first develops a comprehensive multi-dimensional cost function based on the dynamic characteristics of UUV underwater 3D navigation, operational rules for typical marine operating conditions, and safe navigation requirements through mathematical modeling, thereby formally transforming the UUV 3D path planning problem in typical marine operating conditions into a multi-constrained non-linear global optimization problem. To address this challenge, an Adaptive Cooperative WMA (ACWMA) is proposed. The key improvements include: (i) an adaptive parameter switching and Lévy flight disturbance mechanism to balance exploration and exploitation capabilities; (ii) an optimal value leadership strategy to accelerate convergence; and (iii) a team collaborative learning mechanism to enhance population optimization efficiency. Algorithm benchmark performance is validated using the CEC 2017 standard test suite, while comparative and ablation experiments are conducted in multi-gradient complex marine 3D scenarios. The statistical significance of the algorithm performance improvement is verified using the Wilcoxon rank-sum test. The proposed ACWMA achieves a significant performance improvement of 8.71% over the suboptimal WMA in terms of core performance metrics and generates low-energy-consumption 3D paths that satisfy multiple constraints. These findings provide valuable engineering insights for 3D path planning in UUV autonomous operations within typical marine operating conditions.

**Keywords:** UUV; ACWMA; meta-heuristic algorithms; 3D path planning; typical marine operating conditions



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## 1. Introduction

Trajectory planning for unmanned platforms in typical marine operating conditions, including Unmanned Underwater Vehicles (UUVs), Autonomous Underwater Vehicles (AUVs), Unmanned Surface Vehicles (USVs), and ocean mission-oriented Unmanned Aerial Vehicles (UAVs), is inherently a high-dimensional, non-convex, strongly coupled multi-constraint and multi-objective optimization problem. The core challenges arise from the coupled constraints of the platforms themselves. The planned trajectory must strictly

conform to the kinematic and dynamic characteristics of the unmanned systems, including depth boundaries, maximum turning rate, climb/dive angle limits, energy consumption limits, and mission duration constraints, so as to guarantee the executability of the trajectory. Due to the coupling effects of the above constraints, traditional deterministic planning methods commonly suffer from inherent drawbacks in solving such problems, such as the tendency to fall into local optima, poor adaptability to non-convex constraints, and insufficient replanning efficiency in dynamic environments.

Metaheuristic algorithms have been widely recognized as the mainstream technical framework for solving the trajectory planning problem of marine unmanned vehicles, owing to their core advantages of gradient-free dependence, strong global search capability, and excellent adaptability to complex non-convex problems, as validated in numerous studies [1–5]. Among them, classical metaheuristic algorithms, including Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), and Whale Optimization Algorithm (WOA), have been extensively applied to trajectory planning tasks for marine unmanned platforms such as UUVs and USVs. These algorithms have laid the algorithmic foundation for this field due to their simple structure, fast convergence, and ease of engineering implementation [6–8]. Nevertheless, with marine mission scenarios evolving toward complexity, dynamics, and multi-vehicle cooperation, the inherent drawbacks of classical metaheuristic algorithms have become increasingly prominent. Specifically, they suffer from three critical bottlenecks: premature convergence under high-dimensional constraints, the inherent trade-off between convergence speed and optimization accuracy, and insufficient robustness in dynamic environments, which severely restrict their practical performance in engineering applications [9–11].

To address the aforementioned limitations of conventional metaheuristic algorithms, researchers have conducted extensive investigations into algorithm performance enhancement and scenario adaptation and have established three mainstream research paradigms: single-algorithm mechanism improvement, multi-algorithm hybrid optimization, and scenario-oriented customized design, with a series of phased achievements obtained. However, many open scientific issues and technical bottlenecks remain in the existing literature. The related advances and limitations are elaborated below.

In the direction of mechanism optimization for single algorithms, existing studies mainly aim to make targeted upgrades to compensate for the inherent defects of classical algorithms through methods such as iterative strategy improvement, population evolution mechanism reconstruction, and adaptive parameter adjustment. For instance, the ECFDBO algorithm proposed [12] effectively improves the convergence speed and optimization accuracy in the scenario of 3D UUV path planning and alleviates the local optimum problem by optimizing the iterative update strategy of the algorithm. To address the issues of multi-agent cooperative planning and high-dimensional optimization, relevant studies have proposed a multi-population evolution-based WOA variant (MEWOA) [13]. Through the differentiated division of labor and cooperative evolution mechanism of sub-populations, this variant enhances the global optimization capability of the algorithm in high-dimensional multimodal problems, thereby achieving performance improvement in complex single UUV path planning scenarios. For the 3D UAV path planning scenario, the Improved Whale Migration Algorithm (IWMA) realizes the balance between the exploration and exploitation capabilities of the algorithm through chaotic initialization, a multi-layer cooperative search framework, and adaptive mechanism design [14]. Despite the fact that the aforementioned improvements have achieved algorithm performance enhancement in specific scenarios, significant limitations still exist: most improvement strategies are only designed for 2D or simplified 3D scenarios, resulting in insufficient expandability to high-dimensional and typical marine operating conditions; the optimiza-



tion objectives of the algorithms are single, and practical engineering conditions such as dynamic marine disturbances and equipment dynamics constraints are not fully considered, leading to an obvious gap between theoretical simulation and engineering application; the scenario universality of the improvement mechanisms is weak, and in complex tasks such as multi-constraint coupling and multi-equipment collaboration, problems such as degradation of optimization accuracy and premature convergence are prone to occur.

In the field of multi-algorithm fusion optimization, the academic community has attempted to break through the capability boundaries of single algorithms by integrating the performance advantages of different algorithms, which has become a research hotspot in the field of path planning in recent years. For instance, Hu Zhiyuan et al. proposed the PACO fusion algorithm, which combines the global optimization capability of Particle Swarm Optimization (PSO) with the positive feedback mechanism of Ant Colony Optimization (ACO), realizing efficient solutions for global 3D path planning for UUVs with path length as the core objective [7]. Wang et al. addressed the underwater AUV search path planning problem for randomly moving targets by integrating genetic algorithms (GAs) with target position controlled mutation strategy, and verified the effectiveness of the proposed method for path planning of moving targets under random conditions [15]. To solve the path planning problem in 3D obstacle environments, a study combined the mutation strategy of Differential Evolution (DE) with the Symbiotic Organisms Search (SOS) algorithm to propose the HDSOS algorithm, which expands the adaptability of the algorithm in complex obstacle scenarios [16]. However, existing research on multi-algorithm fusion generally faces unavoidable core contradictions: although the multi-algorithm fusion mechanism improves optimization performance to a certain extent, it inevitably leads to a significant increase in computational complexity and overhead, which directly restricts the real-time re-planning capability of the algorithm in dynamic marine environments; most fusion strategies are only designed for global planning of single equipment in static environments, lacking an adaptive response mechanism to dynamic obstacles and time-varying hydrological disturbances, resulting in insufficient algorithm robustness; the design of the fusion framework lacks universal theoretical guidance and is mostly a targeted combination for specific scenarios, leading to weak scenario migration capability.

In the direction of scenario-specific customized design, existing studies have developed a series of scenario-adaptive solutions for the subdivided task scenarios of marine trajectory planning, integrating environmental modeling, control theory, and planning algorithms. For instance, the APFG method proposed by Sun et al. alleviates the problems of local oscillation and local minima in three-dimensional trajectory planning through obstacle rasterization modeling and potential field function improvement [17]; Feng et al. presented the optimal search path planning for the UUV battlefield ambush scenario considering the ocean current environment, which combines the Hybrid Quantum-Behaved Particle Swarm Optimization (HQPSO) and Simulated Annealing (SA) algorithms [18]; and for the AUV search and rescue task in dynamic and unknown environments, the EDOBP algorithm achieves adaptive trajectory planning in unknown scenarios [19]. In addition, for subdivided scenarios such as multi-UUV cooperative operation and path search in marine flow field environments, the academic community has also proposed corresponding solutions including model-driven methods and fusion optimization algorithms [20–22]. However, the aforementioned scenario-specific solutions still have common limitations: the scenario-exclusive design leads to insufficient generality of the algorithms, making it difficult to adapt to various typical marine operating conditions and diverse task requirements; the environmental modeling methods of some approaches (e.g., rasterization modeling) further increase the computational complexity, resulting in insufficient planning stability in extremely complex environments; most studies fail to fully consider practical engineering

constraints such as underwater perception delay, limited communication, and formation collision avoidance, which makes it difficult to guarantee the engineering practicality of the algorithms.

Although a diversified technical system has been formed for the research on path planning of unmanned equipment in typical marine operating conditions, there are still two core scientific bottlenecks that urgently need to be broken through in this field: first, the coordinated optimization problem of algorithm optimization accuracy, computational efficiency, and environmental robustness under multi-constraint coupling scenarios has not been solved, and the inherent contradictions in algorithm performance remain unbroken; second, the balance mechanism between the scenario universality and task-specific adaptability of algorithms still needs to be constructed, and a general planning framework suitable for complex dynamic marine environments has not yet been formed. To address the above core scientific issues, this paper proposes an ACWMA. Through comparative experiments with mainstream meta-heuristic algorithms, it is verified that the proposed algorithm can generate high-quality and feasible solutions for the three-dimensional path planning of UUVs in typical marine operating conditions.

The core innovative contributions of this study are reflected in the following three aspects:

- Closely focusing on the dynamic characteristics of UUVs during three-dimensional (3D) underwater navigation, the operational specifications of typical marine operating conditions, and the constraints of safe navigation, this study standardizes and transforms the 3D path planning problem of UUVs in typical marine operating conditions into a multi-constrained nonlinear global optimization problem through mathematical modeling, laying a rigorous problem modeling foundation for algorithm solving.
- A coordinated control strategy integrating adaptive parameter dynamic switching, Lévy flight random disturbance, and population team collaborative learning is designed. This strategy breaks through the technical bottleneck that traditional algorithms are prone to falling into local optimality, greatly improves the convergence rate and optimization efficiency of the algorithm, and significantly enhances its global optimization capability in high-dimensional and complex constrained scenarios.
- Simulation experiments are carried out based on measured topographic data of real marine environments. The results fully verify the effectiveness of the ACWMA in solving the UUV path planning problem in complex marine scenarios and further demonstrate its feasibility and promotion value in practical engineering applications.

The organizational structure of the subsequent content of this paper is as follows: Section 2 elaborates on the construction process of the cost function for UUV path planning in detail; Section 3 systematically introduces the design principle and implementation details of the proposed ACWMA; Section 4 presents the settings of multiple groups of comparative experiments and the results of quantitative analysis; the Section 5 summarizes the research work of the entire paper and draws relevant conclusions.

## 2. Mathematical Modeling of 3D UUV Path Planning

3D UUV path planning under complex marine environmental constraints is a core technology for ensuring the efficient operation of underwater vehicles. Its primary objective is to generate a feasible, optimal, and robust navigation trajectory that adapts to typical marine operating conditions and satisfies multiple operational constraints, thereby achieving safe and precise underwater mission execution. This study focuses on the 3D UUV path planning problem, which involves both the various constraints that must be strictly adhered to during UUV navigation and the construction of cost functions for guiding algorithmic optimization.

## 2.1. Construction of Cost Functions for UUV Path Planning

### 2.1.1. Path Length

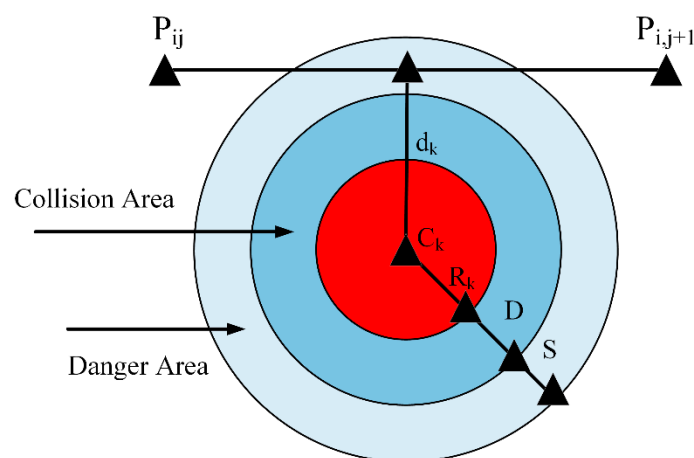
In UUV path planning, distance cost serves as one of the core metrics for evaluating path quality, fundamentally determined by the total navigation distance from the starting point to the destination. To construct a solvable optimization model, this study discretizes the continuous UUV trajectory into a sequence of ordered three-dimensional path nodes, where each node  $P_{ij}$  is precisely characterized by its Cartesian coordinates  $P_{ij} = (x_{ij}, y_{ij}, z_{ij})$ . Within the discretized path sequence, the Euclidean distance between two adjacent nodes  $P_{ij}$  and  $P_{i,j+1}$  is defined as  $\|P_{ij}P_{i,j+1}\|$ , representing the actual navigation cost between consecutive waypoints. Accordingly, the total distance cost for the UUV is formulated as the cumulative sum of distances between all adjacent node pairs, expressed mathematically as follows:

$$F_1(x_i) = \sum_{j=1}^{n-1} \|P_{ij}P_{i,j+1}\| \quad (1)$$

where  $n$  denotes the total number of discrete nodes generated from the continuous navigation trajectory. The first node in the path sequence corresponds to the UUV departure point, while the  $n$ -th node represents its destination. The set  $X_i$  is defined as the ordered collection comprising all  $n$  path nodes that the UUV must traverse.

### 2.1.2. Threat Cost

The mission reliability of UUVs is critically dependent on navigation safety. However, the heterogeneous morphologies of obstacles in complex underwater environments pose significant modeling challenges, with conventional methods often inducing computational redundancy that constrains obstacle avoidance performance enhancement. To address this limitation, this study proposes a unified cylindrical abstraction modeling scheme, wherein the set  $K$  characterizes all underwater obstacles. For any  $k \in K$ , the path planning plane projection is parameterized by its center coordinates  $C_k(x_k, y_k)$  and radius  $R_k$ . This approach effectively reduces computational complexity while accurately preserving the spatial occupancy characteristics of obstacles. The obstacle projection distribution along the trajectory from node  $P_{ij}$  to  $P_{i,j+1}$  illustrated in Figure 1 further substantiates the applicability and superiority of this modeling methodology for UUV collision-free navigation control [23].



**Figure 1.** Projection of the UUV trajectory through obstacles.

To precisely quantify the threat cost associated with collision risk during UUV navigation, the core characterization parameters required for model construction are first explicitly defined:  $D$  denotes the UUV hull diameter,  $S$  specifies the mandatory safety margin distance

between the UUV and the collision domain, and  $d_k$  represents the perpendicular distance from the UUV navigation trajectory to the center of obstacle  $k$ . Based on these parameter definitions and constraint conditions, the threat cost function mathematical model for the UUV within the specific navigation segment from node  $P_{ij}$  to node  $P_{i,j+1}$  is systematically formulated for a given obstacle set  $K$ , with the specific expression presented as follows:

$$F_2(X_i) = \sum_{j=1}^{n-1} \sum_{k=1}^k T_k(P_{ij}P_{i,j+1}) \quad (2)$$

$$T_k(P_{ij}P_{i,j+1}) = \begin{cases} 0, & \text{if } d_k > S + D + R_k \\ (S + D + R_k) - d_k, & \text{if } D + R_k < d_k \leq S + D + R_k \\ \infty, & \text{if } d_k \leq D + R_k \end{cases} \quad (3)$$

### 2.1.3. Height Cost

The navigation altitude of UUVs constitutes a core parameter that directly determines mission execution accuracy and navigation safety. In search and detection missions, a significant nonlinear correlation exists between UUV navigation altitude and the quality (including clarity and spatial resolution) of imagery acquired by imaging payloads. This study formulates the UUV path planning problem as a cost minimization optimization problem subject to multiple constraints. To ensure navigation altitude remains within reasonable bounds, strict interval constraints are imposed, and a navigation altitude cost function is introduced to construct a rigid penalty mechanism. When the navigation altitude exceeds preset thresholds, the cost function value is substantially increased to guide the path planning algorithm in adaptively adjusting the navigation altitude, thereby constraining it within the predefined feasible domain and ensuring reliable UUV mission execution.

$$F_3(X_i) = \sum_{j=1}^n H_{ij} \quad (4)$$

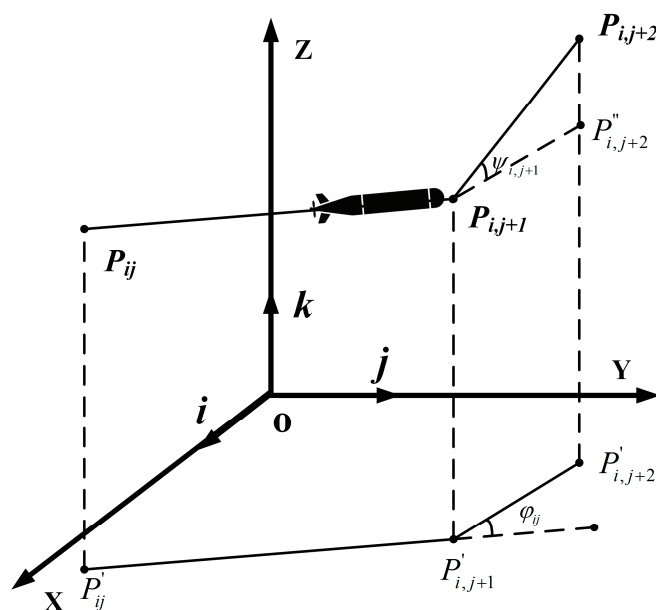
$$H_{ij} = \begin{cases} \left| h_{ij} - \frac{h_{\max} + h_{\min}}{2} \right|, & \text{if } h_{\min} \leq h_{ij} \leq h_{\max} \\ \infty, & \text{otherwise} \end{cases} \quad (5)$$

where  $h_{ij}$  denotes the altitude of the UUV above the seabed during navigation, and  $h_{\max}$  and  $h_{\min}$  represent the maximum and minimum bounds of the UUV allowable altitude range above the seabed, respectively.

### 2.1.4. Smoothness Cost

Smoothness cost serves as a critical metric for evaluating feasible trajectories generated by UUV path planning algorithms, with its essence lying in quantifying path turning rates and climbing rates to provide a quantitative foundation for assessing kinematic feasibility and energy efficiency. Figure 2 illustrates the 3D trajectory segment of the UUV from node  $P_{ij}$  to  $P_{i,j+2}$ , where  $P'_{ij}$ ,  $P'_{i,j+1}$ , and  $P'_{i,j+2}$  are the orthogonal projection points of the waypoints  $P_{ij}$ ,  $P_{i,j+1}$  and  $P_{i,j+2}$  onto the XOY plane, respectively. To characterize the turning behavior within the horizontal plane, the turning angle  $\varphi_{ij}$  is defined as the included angle between the extension of projected line segment  $P'_{ij}P'_{i,j+2}$  and line segment  $P'_{i,j+1}P'_{i,j+2}$ , where  $i$ ,  $j$ , and  $k$  represent the unit vectors along the X, Y, and Z axes, respectively. The calculation formula for the turning angle  $\varphi_{ij}$  is given as follows:

$$\varphi_{ij} = \arctan \left( \frac{\|P'_{ij}P'_{i,j+1} \times P'_{i,j+1}P'_{i,j+2}\|}{P'_{ij}P'_{i,j+1} \cdot P'_{i,j+1}P'_{i,j+2}} \right) \quad (6)$$



**Figure 2.** Path planning diagram of UUVs in a 3D coordinate system.

This turning angle  $\varphi_{ij}$  directly reflects the turning magnitude of the UUV within the horizontal plane; larger values indicate more severe turning maneuvers and inferior path smoothness, rendering it the core geometric parameter for constructing the smoothness cost function.

The climbing angle  $\psi_{ij}$  is defined as the included angle between the ascending trajectory of the UUV and the horizontal line. Here, the waypoint  $P''_{i,j+2}$  is the projection of waypoint  $P_{i,j+2}$  onto the plane containing the line segment  $P_{ij}P_{i,j+1}$ . On this basis, the angle between the line segment  $P_{i,j+1}P_{i,j+2}$  and the line segment  $P'_{i,j+1}P'_{i,j+2}$  is denoted as

$$\psi_{ij} = \arctan \left( \frac{Z_{i,j+1} - Z_{ij}}{\|P'_{ij}P'_{i,j+1}\|} \right) \quad (7)$$

This climbing angle  $\psi_{ij}$  directly reflects the climbing magnitude of the UUV within the vertical plane; a larger value indicates a steeper path ascent and serves as a critical parameter for constructing the smoothness cost function.

To enable quantitative assessment of UUV three-dimensional path smoothness, this paper proposes a smoothness cost function modeling framework based on local geometric characteristics. This framework extracts the horizontal turning angle  $\varphi_{ij}$  and climbing angle  $\psi_{ij}$  as core quantitative indicators through projection geometric analysis of the track point sequence. On this basis, the smoothness cost function can be expressed as:

$$F_4 = a_1 \sum_{j=1}^{n-2} \varphi_{ij} + a_2 \sum_{j=1}^{n-1} |\psi_{ij} - \psi_{i,j-1}| \quad (8)$$

In the formula,  $a_1$  and  $a_2$  are the penalty coefficients for the turning angle and climbing angle, respectively. Both the turning angle and the climbing angle are uniformly calculated and constrained in the radian system.

#### 2.1.5. Terrain Collision Cost

The terrain collision cost, denoted as  $F_5$ , serves as a critical constraint in UUV path planning to ensure that the UUV maintains a minimum safe clearance  $h_{\min} = 15$  from the seabed. To achieve a comprehensive assessment of track safety, this study adopts a



two-tier detection strategy: direct detection is performed for waypoints  $C_i$ , while each path segment is uniformly divided into  $K = 30$  sampling points  $C_{i,k}$  for refined detection. The mathematical formulation of the terrain collision cost is expressed as:

$$F_5 = \sum_{i=1}^N C_{clear}(c_i) + \sum_{i=1}^{N-1} \sum_{k=1}^K \frac{1}{K} C_{clear}(c_{ik}) \quad (9)$$

where the clearance altitude  $c = z^{abs} - H(x, y)$  is defined as the difference between the UUV absolute altitude and the terrain elevation. The collision cost function  $C_{clear}(c)$  adopts a piecewise design to implement differentiated penalties for distinct safety states:

$$C_{clear}(c) = \begin{cases} 0 & c \geq h_{\min} \\ 50 \cdot (h_{\min} - c) & 0 \leq c < h_{\min} \\ J_{\inf} \cdot |c| & c < 0 \end{cases} \quad (10)$$

The intra-segment sampling points  $P_{i,k}$  are generated through linear interpolation to ensure that the safety assessment within path segments possesses sufficient resolution:

$$P_{ik} = P_i + \frac{k}{K} (P_{i+1} - P_i), k = 1, 2, \dots, K \quad (11)$$

This model furnishes explicit safety constraints for UUV path planning algorithms, facilitating the generation of navigation trajectories that simultaneously achieve safety and efficiency.

#### 2.1.6. The Aggregate Cost Function

To address the UUV path planning problem and achieve a global optimality assessment of feasible paths  $X_i$ , this study constructs an aggregate cost function that integrates multidimensional constraints.

$$F(X_i) = \sum_{k=1}^5 b_k F_k(X_i) \quad (12)$$

where  $b_k$  denotes the weighting coefficient of the  $k$ -th cost function. The optimization objective function  $F(X_i)$  systematically integrates path feasibility, kinematic constraints, and mission performance metrics based on the core requirements of three-dimensional UUV path planning, thereby precisely establishing the optimization objective and constraint system for the UUV path planning problem in three-dimensional space.

This function not only comprehensively characterizes the multidimensional constraints during UUV navigation (e.g., terrain collision safety constraints, path smoothness constraints), but also achieves a high degree of alignment between optimization objectives and practical engineering requirements. It thereby furnishes a robust theoretical and mathematical foundation for the subsequent employment of metaheuristic algorithms to conduct three-dimensional UUV path planning optimization in this study, ensuring both the scientific rigor of the algorithmic optimization direction and the engineering practicality of the optimization results.

#### 2.2. A Spherical Vector-Based 3D UUV Path Planning Method

The spherical vector-based 3D UUV path planning method achieves a compact representation of three-dimensional spatial trajectories by encoding each feasible path  $X_i$  from the starting point to the destination as a set of vectors. To precisely characterize navigation attitude and displacement in three-dimensional space, the spherical vector designed by the proposed method consists of three components:  $\phi \in (-\pi, \pi)$ ,  $\theta \in (-\pi/2, \pi/2)$  and

$p \in (0, \text{path\_length})$ . Among them,  $\phi$  denotes the azimuth angle of the UUV,  $\theta$  represents the pitch attitude of the UUV, and  $p$  stands for the sailing range. Under this encoding framework, a feasible navigation path  $\Omega_i$  containing  $N$  nodes can be uniquely represented by a  $3N$ -dimensional spherical vector sequence.

$$\Omega_i = (p_{i1}, \theta_{i1}, \phi_{i1}, p_{i2}, \theta_{i2}, \phi_{i2}, \dots, p_{in}, \theta_{in}, \phi_{in}) \quad (13)$$

To address the efficiency requirements of 3D UUV path planning, the spherical vector method establishes a path representation model with explicit physical constraints by quantifying the correspondence among navigation magnitude, elevation angle, azimuth angle, and UUV motion parameters. This model enables obstacle avoidance and turning control through parametric synergistic regulation, while simultaneously guiding meta-heuristic algorithms to conduct directed search within the configuration space, effectively reducing redundant search space and enhancing both the global optimization capability of algorithms and the engineering adaptability of path planning.

### 3. Adaptive Collaborative Optimization Algorithm

#### 3.1. Fundamental Principles of the Whale Migrating Algorithm

The Whale Migrating Algorithm (WMA) is a novel bio-inspired metaheuristic optimization paradigm grounded in collaborative biological behaviors [24]. Its conceptual design originates from the collective coordination mechanisms observed in humpback whale long-distance migration, innovatively integrating leader-follower dynamic interactions with self-adaptive migration strategies to achieve an equilibrium between global exploration and local exploitation. This approach effectively addresses the fundamental limitations of conventional heuristic algorithms, particularly their susceptibility to local optima entrapment and restricted convergence capabilities, thereby furnishing a new technical paradigm for efficient resolution of complex optimization problems.

##### 3.1.1. Initialization of the WMA

Under the WMA algorithmic framework, population initialization constitutes a critical procedure for constructing the feasible solution space. For a  $D$ -dimensional optimization problem, we define the population size as  $N_{\text{pop}}$ , with each individual  $W_i \in \mathbb{R}^D$  representing a candidate solution. To ensure uniform distribution of the initial population within the feasible domain  $\Omega = \prod_{d=1}^D [L_d, U_d]$ , the individual generation adheres to the following strategy:

$$W_i = L + \text{rand}(1, D) \odot (U - L), \quad i = 1, \dots, N_{\text{pop}} \quad (14)$$

where  $\text{rand}(1, D)$  generates a  $D$ -dimensional uniform random vector in  $[0, 1]$ , and  $\odot$  denotes the Hadamard product. This initialization strategy ensures that all initial individuals are strictly confined within the feasible region, thereby providing the necessary foundation for the algorithm.

##### 3.1.2. Current Position of the Migrating Whale in the Ocean

The representation of swarm position is a critical component for achieving effective search guidance. To characterize the representative position of the swarm at any iteration, the algorithm first selects  $N_L$  individuals with the best objective function values from the current population to form a leader subset  $W_j$ . Based on this, the swarm mean position  $W_{\text{mean}}$  is defined as the arithmetic average of the positions within this subset:

$$W_{\text{Mean}} = \frac{1}{N_L} \sum_{j=1}^{N_L} W_j \quad (15)$$

This representation effectively smooths the stochasticity of a single leader by statistically aggregating information from high-quality individuals, providing a stable and directional reference point for swarm migration, thereby enhancing the global convergence performance of the algorithm.

### 3.1.3. The Movement of Inexperienced Juvenile Whales Toward Their Nearest Neighbors

Intra-population information exchange is realized through a hierarchical ranking and neighborhood guidance mechanism. First, the algorithm sorts the population in descending order of fitness values, partitioning individuals into a leader subset  $\{W_1, \dots, W_{N_L}\}$  and a juvenile whale group  $\{W_{N_L+1}, \dots, W_{N_{pop}}\}$ . For an inexperienced juvenile whale  $W_i (i = N_L + 1, \dots, N_{pop})$ , its movement behavior is primarily guided by the preceding individual  $W_{i-1}$  in the ranked sequence, emulating the natural behavior of juvenile whales imitating their companions. This mechanism achieves gradient-based learning within the population through local information transmission, effectively enhancing the search efficiency of the algorithm.

### 3.1.4. Juvenile Whales with Limited Experience Are Guided by Experienced Whales

The position update mechanism for juvenile whales is crucial for achieving efficient optimization. As the mean position of the leader group  $W_{mean}$  converges toward the global best  $W_{best}$ , the position update of a juvenile whale  $W_i (i = N_L + 1, \dots, N_{pop})$  needs to simultaneously integrate neighborhood guidance and global convergence information. Its update formula is given by:

$$\begin{aligned} W_i^{new} &= W_{Mean} + rand(1, D) \odot (W_{i-1} - W_i) + rand(1, D) \\ &\odot (W_{Best} - W_{Mean}), i \\ &= N_L + 1, \dots, N_{pop} \end{aligned} \quad (16)$$

The new position is accepted only if it yields a superior objective function value, i.e.,  $f(W_i^{new}) < f(W_i)$ . This mechanism effectively enhances the convergence speed and search accuracy of the algorithm through the synergy of local exploitation and global exploration.

### 3.1.5. New Regions Are Discovered and Explored by Experienced Whales

Within the global optimization framework of WMA, the regional exploration mechanism of the leader group constitutes the core for achieving global optimization. Inspired by the migratory behavior of humpback whales, this mechanism constructs the position update equation for leaders by integrating the position constraint vector  $r_1 \odot L$  and the directional guidance vector  $r_1 \odot r_2 \odot (U - L)$ :

$$W_i^{new} = W_i + r_1 \odot L + r_2 \odot (U - L), i = 1, \dots, N_L \quad (17)$$

where  $r_1, r_2 \in [0, 1]^D$  are random parameter vectors introduced to incorporate search stochasticity while ensuring the dominant role of position constraints. Upon completion of each iteration, the population is re-ranked according to fitness values, and the top  $N_L$  optimal individuals are selected as the leaders for the subsequent generation, thereby guaranteeing the timeliness and effectiveness of the guidance information.

## 3.2. Core Improvement Design of the ACWMA

This paper proposes an ACWMA with three core improvements: (1) an adaptive parameter switching mechanism coupled with Lévy flight perturbation to balance exploration and exploitation capabilities, thereby escaping local optima traps; (2) an optimal-value leadership strategy to strengthen the guiding role of elite individuals and accelerate conver-

gence speed; and (3) a collaborative team learning mechanism based on a three-level fitness gap model with adaptive learning rules to enhance population optimization efficiency.

### 3.2.1. Adaptive Parameter Switching Mechanism

To achieve a dynamic equilibrium between global exploration and local exploitation, this study proposes an adaptive parameter mechanism driven by the following time-varying probability function:

$$y(t) = e^{-2t} \cdot (\cos(2\pi u(t)))^2 \quad (18)$$

where  $t = \frac{it-1}{MaxIt-1} \in [0, 1]$  denotes the normalized iteration number, with it representing the current iteration and MaxIt the maximum number of iterations;  $u(t)$  is a random variable uniformly distributed within the interval  $[0, 1]$ , serving to introduce stochastic perturbations; and  $e^{-2t}$  represents an exponential decay factor that monotonically decreases throughout the iterative process, thereby regulating the trade-off between exploration and exploitation. The operational principle of this mechanism is elaborated as follows:

Early-stage iterations (it is relatively small,  $t \rightarrow 0$ ): The expected value of  $y(t)$  remains relatively high, biasing the algorithm toward optimal value-guided leadership strategies to enhance global exploration capabilities and mitigate premature convergence to local optima. Late-stage iterations (it approaches MaxIt,  $t \rightarrow 1$ ): The expected value of  $y(t)$  asymptotically approaches zero, prompting the algorithm to transition toward the original humpback whale migration strategy, thereby intensifying local exploitation and accelerating convergence toward the global optimum.

Based on the probability  $y(t)$ , the algorithm executes the following strategy selection at each iteration: If  $\text{rand} < y(t)$ , the original humpback whale migration strategy is employed to update the positions of selected individuals. If  $\text{rand} \geq y(t)$ , a Lévy flight perturbation strategy is introduced, wherein Lévy flight disturbances are performed around each whale [25].

$$W_i^{\text{new}} = W_i + \text{Levy}(\lambda) \odot (U - L), i = 1, \dots, N_L \quad (19)$$

where  $\text{Levy}(\lambda)$  denotes a random vector following the Levy distribution, and  $U$  and  $L$  represent the upper and lower bounds of the search space, respectively.

### 3.2.2. Optimal-Value Leadership Strategy

During the exploration phase of the algorithm, to enhance the directional convergence of the population toward the global optimum while mitigating premature convergence arising from excessive reliance on a single elite individual, this study integrates the optimal value leadership strategy with a collaborative team strategy for representative individuals within the population. Specifically, when the random number  $\text{rand} \sim U(0, 1)$  satisfies  $\text{rand} \leq 0.5$ , the individual updates its position via the optimal value leadership strategy, mathematically formulated as:

$$W_i^{\text{new}} = W_{i,j} + r \times (R_j - W_{i,j}) \quad (20)$$

where  $W_{i,j}$  denotes the position of the  $i$ -th individual in the  $j$ -th dimension of the decision space;  $r \sim U(0, 1)$  represents a uniformly distributed random number controlling the stochasticity of the step size to balance exploration and exploitation; and  $R_j$  signifies the position of the current optimal leader in the  $j$ -th dimension, serving as a guiding term that drives individuals toward more promising search regions. The threshold for strategy selection is set to 0.5 to ensure that, during the exploration phase, individuals switch equiprobably between “following the optimal leader” and “engaging in collaborative teamwork,” thereby achieving an effective equilibrium between convergence speed and population diversity.

### 3.2.3. Collaborative Team Learning Mechanism

During the exploration phase, when  $\text{rand} > 0.5$ , the algorithm adopts a collaborative team strategy. In the original WMA, experienced individuals rely solely on random migration within their own neighborhoods, a localized search pattern that tends to constrain the convergence rate. To address this limitation, this study introduces a collaborative team strategy within the elite subpopulation, establishing an update mechanism characterized by “elite guidance and swarm collaboration” to effectively accelerate the algorithm’s convergence.

#### 1. Modeling of the collaborative team strategy

The core design philosophy of this strategy involves establishing a hierarchical guidance mechanism, wherein superior individuals (elites) selected from the population guide inferior individuals (underperformers) to achieve efficient information transfer and collective population evolution. Specifically,  $W_{\text{better}}$  is randomly selected from the top five individuals with the highest fitness values to serve as the elite individual, while  $W_{\text{worse}}$  is randomly chosen from the remaining individuals as the underperformer. To quantify the evolutionary discrepancies among individuals, this study models three critical positional gaps: Gap1 between the leader and  $W_{\text{better}}$ , Gap2 between the leader and  $W_{\text{worse}}$ , and Gap3 between  $W_{\text{better}}$  and  $W_{\text{worse}}$ .

$$\begin{cases} \overrightarrow{\text{Gap}}_1 = W_{\text{best}} - W_{\text{better}} \\ \overrightarrow{\text{Gap}}_2 = W_{\text{best}} - W_{\text{worse}} \\ \overrightarrow{\text{Gap}}_3 = W_{\text{better}} - W_{\text{worse}} \end{cases} \quad (21)$$

By leveraging this gap information, the collaborative team strategy drives underperforming individuals toward superior regions, thereby significantly accelerating the convergence process while maintaining population diversity.

#### 2. Learning factor modeling

To precisely characterize the relative significance of positional discrepancies among individuals at different hierarchical levels and quantify their impact on the evolutionary process, this study introduces learning factors (LF) for the three critical spatial gaps (Gap1, Gap2, Gap3). These factors dynamically regulate the weights with which individuals acquire evolutionary information from different gaps, mathematically defined as follows:

$$LF_k = \frac{\|\overrightarrow{\text{Gap}}_k\|}{\sum_{k=1}^3 \|\overrightarrow{\text{Gap}}_k\|}, \quad (k = 1, 2, 3) \quad (22)$$

As evident from the above formulation, the learning factor  $LF_k$  essentially represents the proportional contribution of the  $k$ -th gap to the total discrepancy. A larger spatial gap between the corresponding groups yields a greater  $LF_k$ , implying that individuals will acquire more evolutionary information from this gap during the optimization process. Consequently, the search direction is adjusted more substantially, facilitating more efficient learning and evolutionary progress.

#### 3. Adaptive learning weight factor

To achieve precise modeling of differentiated learning behaviors among individuals and balance global information utilization with individual adaptive adjustment within the collaborative team strategy, this study introduces an adaptive learning weight factor. This factor dynamically modulates the intensity with which individuals acquire evolutionary

information from spatial gaps, contingent upon their relative fitness levels within the current population. Such modulation prevents high-quality individuals from deviating from optimal regions due to excessive learning while incentivizing underperforming individuals to leverage gap information for rapid improvement. The adaptive learning weight factor is mathematically defined as:

$$\alpha_i = \frac{fit_{i,t}}{\max(fit_t)} \quad (23)$$

where  $fit_{i,t}$  denotes the fitness value of the  $i$ -th individual at the  $t$ -th iteration, and  $\max(fit_t)$  represents the maximum fitness value of the entire population at the  $t$ -th iteration.

#### 4. Knowledge acquisition and position update mechanism

Under the collaborative team strategy framework, the evolutionary information acquired by individual  $i$  regarding the  $k$ -th spatial  $\overrightarrow{Gap_k}$  is quantitatively modeled through the knowledge acquisition vector  $\overrightarrow{KA_k}$ . This vector integrates the individual's adaptive learning weight  $\alpha_i$  with the learning factor  $LF_k$  corresponding to the respective gap, mathematically expressed as:

$$\overrightarrow{KA_k} = \alpha_i \cdot LF_k \cdot \overrightarrow{Gap_k}, (k = 1, 2, 3) \quad (24)$$

where  $\alpha_i$  regulates the individual learning intensity, and  $LF_k$  characterizes the relative significance of different gaps; their synergistic interaction determines the extent to which individuals assimilate information from specific gaps.

Based on the aforementioned knowledge acquisition vector, the position update equation for an individual can be expressed as:

$$W_i^{new} = W_i + \overrightarrow{KA_1} + \overrightarrow{KA_2} + \overrightarrow{KA_3} \quad (25)$$

During the exploration phase of elite individuals, the introduction of the collaborative team strategy imposes effective synergistic constraints on the entire population from the early iterations. Such constraints drive underperforming individuals  $W_{worse}$  to rapidly converge toward superior individuals  $W_{better}$  and the global optimal leader, significantly enhancing the convergence speed and evolutionary efficiency of the population.

The overall update strategy for the exploration phase of elite individuals can be summarized as follows:

$$\begin{cases} W_i^{new} = W_i + \overrightarrow{KA_1} + \overrightarrow{KA_2} + \overrightarrow{KA_3}, & \text{if } rand \leq 0.5 \\ W_i^{new} = W_{i,j} + r \times (R_j - W_{i,j}), & \text{if } rand > 0.5 \end{cases} \quad (26)$$

When  $rand \leq 0.5$ , individuals switch to the optimal-value leadership strategy; when  $rand > 0.5$ , individuals adopt the team collaboration strategy, so as to realize the dynamic balanced regulation of global exploration and local exploitation.

#### 3.2.4. Pseudocode of the ACWMA

By integrating three core innovations—an adaptive parameter switching mechanism, an optimal value leadership strategy, and a collaborative team learning mechanism—into the WMA framework, we propose a novel algorithm termed ACWMA. The detailed procedural steps are formally described in Algorithm 1.



**Algorithm 1:** ACWMA

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```

Step 1: Input Parameters
- N: Population size; - Max_iteration: Maximum number of iterations
- lb: Lower bound of variables; - ub: Upper bound of variables
- dim: Dimension of the problem (number of optimized variables)
Step 2: Initial Population Generation
for i = 1 : nPop
    % Generate initial positions uniformly at random
    % Calculate the fitness value of the current individual
    % Update the global optimal solution
end
Step 3: Main Iteration Loop
while it < MaxIt
    Step 3.1 Population Sorting: Classify leaders and followers
    Step 3.2 Calculate the mean position of the leader subpopulation
    Step 3.3 Calculate dynamic parameters t and y
        t: Linearly increasing parameter, controlling the decay rate of y
        y: Dynamic probability factor /* Equation (18) */
    Step 3.4 Traverse all individuals and update positions
    for i = 1 : nPop
        Step 3.4.1 Follower update (i > NL)
        if i > NL
            if rand() < y % Local exploitation based on leaders
                /* Equation (16) */;
            else % Strategy 2: Adaptive Lévy flight (global exploration)
                /* Equation (19) */;
            end
        Step 3.4.2 Leader update (i <= NL)
        else
            if rand() ≤ 0.5 % Strategy 1: Optimal-value leadership strategy is adopted
                /* Equation (20) */;
            else % Strategy 2: Team collaboration strategy is adopted
                % Calculate position gap /* Equation (21) */;
                % Fitness weight (larger weight for leaders with worse fitness)
                /* Equations (22) and (23) */;
                % Calculate guidance term (KA) /* Equation (24) */;
                % Position update: Current position + Guidance term /* Equation (25) */;
            end
        end
        Step 3.4.3 Calculate fitness of the new position and update the optimal solution
        /* Equation (26) */;
    end
end
Step 4: Output Results
% Optimal fitness value; % Optimal position

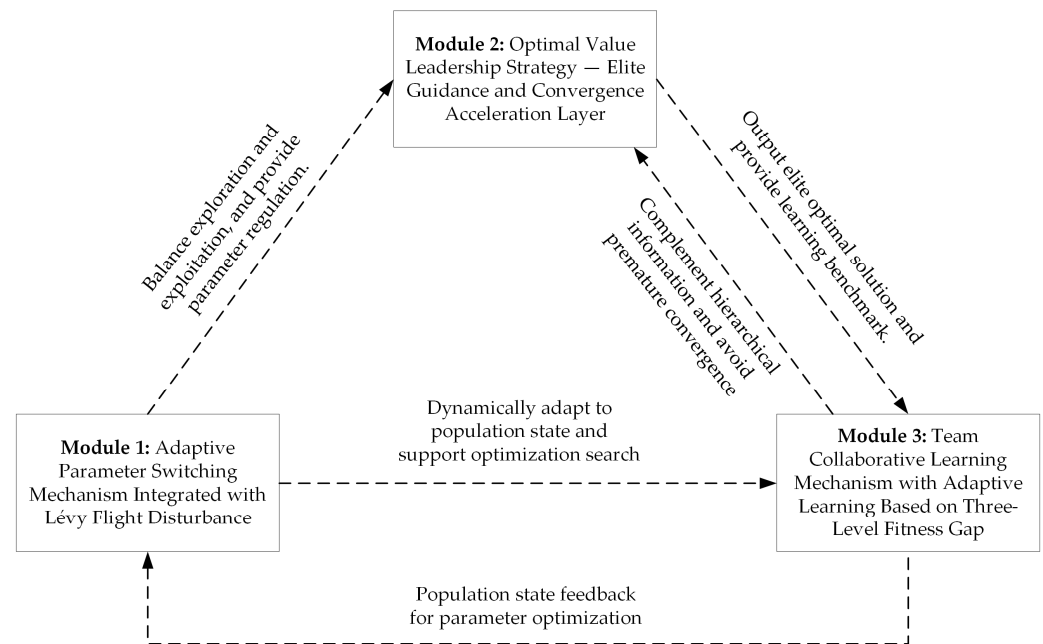
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**3.3. Synergistic Working Mechanism of ACWMA**

The proposed ACWMA in this paper consists of three mutually supportive and synergistic core improvement modules. Rather than functioning independently, these modules

cooperate organically to collectively enhance the optimization performance, convergence speed and global search capability of the algorithm, as shown in Figure 3.



**Figure 3.** Flowchart of the synergistic working mechanism of ACWMA.

First, the adaptive parameter switching mechanism serves as the fundamental regulatory core of the algorithm. Combined with the Lévy flight disturbance strategy, it dynamically balances the exploration and exploitation capabilities of the algorithm. When the algorithm falls into a local optimum, Lévy flight disturbance can break the local convergence trap and provide new search directions for population optimization. Meanwhile, the adaptive parameter switching mechanism adjusts the disturbance intensity and search strategy in real time according to the evolutionary state of the population, laying a stable optimization foundation for the efficient operation of the two subsequent modules.

Secondly, the optimal value leadership strategy gives full play to the guiding role of elite individuals. Taking the global optimal individual as the core, it guides the population to converge toward better solution regions, effectively shortening the optimization path and accelerating the convergence speed of the algorithm. Meanwhile, it provides clear learning objectives and evolutionary directions for the team collaborative learning mechanism, avoiding aimless searching of the population.

Finally, the team collaborative learning mechanism designs adaptive learning rules based on the three-level fitness gap model. The population is divided into different hierarchical levels. Through collaborative learning and information interaction among these levels, complementary advantages among individuals are realized, and the optimal information of elite individuals is transmitted to the entire population via the collaboration mechanism, thereby improving the overall optimization level of the population. Meanwhile, this mechanism remedies the “elite monopoly” problem that may exist in the optimal value leadership strategy and prevents premature convergence of the algorithm. In addition, the adaptive parameter switching mechanism provides dynamic regulation for collaborative learning. It adjusts relevant parameters according to the performance of population collaborative learning to optimize learning efficiency and further strengthen the global optimization capability of the algorithm.

The three modules function synergistically to form a complete optimization system of “Basic regulation—Guidance acceleration—Collaborative improvement”. The adaptive parameter switching mechanism guarantees the stability and exploration capability of the optimization process; the optimal value leadership strategy enhances the search directionality and convergence speed; and the team collaborative learning mechanism improves the overall optimization efficiency and robustness of the population. Cooperating and supporting one another, the three modules effectively address the inherent drawbacks of traditional algorithms, such as local optimum trapping, slow convergence and low optimization efficiency, thereby fully exerting the comprehensive superiority of the ACWMA.

## 4. Experimental Results and Analysis

### 4.1. Experimental Environment and Parameter Settings

To validate the effectiveness and feasibility of the proposed ACWMA, experiments were conducted using a real-world digital elevation model (DEM) obtained from field measurements using a 3D imaging sonar in a specific sea area of the South China Sea. The original topographic structure of this sea area was enhanced and reconstructed to generate a 3D terrain simulation environment comprising X, Y, and Z spatial coordinates, thereby closely approximating actual underwater operational conditions. Concurrently, underwater threat obstacles with a radius of  $r$  were deployed within this 3D terrain simulation scenario, represented geometrically as cylinders. The UUV diameter was set to 533 mm, with a navigation depth constraint of 400 m. All aforementioned parameters were standardized to provide a uniform testing benchmark for the equitable validation of subsequent algorithmic performance.

To verify the scientificity and rationality of the weight parameter configuration of each cost function, this study adopts the single-factor variable method to conduct weight sensitivity analysis so as to clarify the influence of each weight parameter on path planning performance. The safe distance between the UUV and obstacle targets is set to 5 m. Taking the reference weight combination ( $b_1 = 1$ ,  $b_2 = 5$ ,  $b_3 = 1$ ,  $b_4 = 5$ ,  $b_5 = 10$ ) as the benchmark, only a single weight parameter is changed each time while the remaining parameters are kept at their benchmark values. Path length, trajectory smoothness, obstacle avoidance safety, and algorithm convergence efficiency are selected as the core evaluation indicators to systematically analyze the effects of different weight values on UUV path planning performance.

The value gradients of each weight parameter are set as follows: the gradients of  $b_1$  and  $b_3$  are 0.5, 1.0 (benchmark), 2.0, 3.0, and 4.0; the gradients of  $b_2$  and  $b_4$  are 1, 3, 5 (benchmark), 7, and 9; and the gradients of  $b_5$  are 2, 5, 10 (benchmark), 15, and 20. A series of comparative simulation experiments demonstrate that the rationality of weight values directly determines overall path planning performance. If  $b_1$  and  $b_3$  are assigned excessively small values, the constraint on the corresponding optimization objectives will be insufficient, resulting in the planned path being too close to obstacles and degradation of trajectory stability. Excessively large values will lead to redundant paths and reduce planning efficiency. When  $b_2$  and  $b_4$  are set to relatively small values, the constraints on obstacle avoidance and trajectory smoothness are weakened; by contrast, excessively large values will cause an over-conservative planning strategy and unnecessarily long detoured paths. A relatively small value of  $b_5$  fails to impose adequate constraints on the comprehensive optimization objective, whereas an overlarge value increases the convergence burden of the algorithm and reduces the optimization efficiency.

Combined with all evaluation indicators, the reference weight combination adopted in this study ( $b_1 = 1$ ,  $b_2 = 5$ ,  $b_3 = 1$ ,  $b_4 = 5$ ,  $b_5 = 10$ ) can achieve an optimal balance among obstacle avoidance safety, path optimality and algorithm convergence efficiency. It not only effectively avoids the planning defects caused by inappropriate weight selection but also gives full play to the synergistic effect of each evaluation index, ensuring the scientificity and practicality of UUV path planning. The established weight configuration is verified to be reasonable and feasible.

To further validate the adaptability and superiority of the ACWMA under varying complex operational conditions, twelve test scenarios of different complexity levels were designed in this study to conduct comparative experiments. In each test scenario, all algorithms involved in the comparison strictly adhered to the UUV navigation dynamics characteristics when plotting path curves. Specifically,  $n = 10$  path nodes were arranged between the predefined starting point and target point to construct the complete UUV navigation trajectory. The starting coordinates of the UUV were fixed at (100, 80, 60), while the target coordinates were fixed at (950, 880, 70). The core objective of the 3D path planning was to guide the UUV from the starting point, effectively avoiding all underwater threat zones and safely reaching the target point.

Considering the inherent stochastic nature of metaheuristic algorithms, unified parameter configurations were adopted for all tested metaheuristic algorithms to ensure the reliability, reproducibility, and fairness of the experimental results: the population size was fixed at 100, and the maximum number of iterations was set to 1000. Each algorithm was independently executed 30 times, and statistical analyses were performed on the multiple experimental results to systematically verify the search efficiency and optimization performance advantages of the ACWMA in 3D UUV path planning problems compared with other benchmark algorithms.

All 3D UUV path planning simulation experiments in this study were conducted in the MATLAB 2025a environment. The experimental hardware platform was equipped with an Ultra9 275HX 24-core processor and 32 GB of memory, providing robust hardware support for efficient simulation computations.

#### 4.2. Benchmark Function Testing and Performance Validation

To verify the efficiency of the proposed ACWMA, comparative experiments were conducted against several state-of-the-art and classical metaheuristic algorithms, including the WMA, Greylag Goose Optimization (GGO) [26], Tornado Optimization Algorithm (TOC) [27], Ivy Algorithm (IVY) [28], Dung Beetle Optimizer (DBO) [29], WOA, and PSO. To comprehensively evaluate the performance of the ACWMA, the CEC 2017 benchmark test suite was uniformly employed for validation, with the dimensionality set to 10, 30, and 50 dimensions, respectively.

##### 4.2.1. Statistical Result Analysis of CEC 2017 Functions

To validate the global search efficacy and generalization capability of the proposed ACWMA, systematic experiments were conducted on the CEC 2017 standard benchmark suite comprising 29 test functions. The experimental results demonstrate that the optimization performance of the ACWMA significantly outperforms that of other selected mainstream metaheuristic algorithms, with detailed quantitative comparisons presented in Tables 1–3. Based on the aforementioned experimental data, this section provides an in-depth performance analysis across different function categories within the CEC 2017 test suite and systematically discusses the underlying mechanisms contributing to the algorithm's performance advantages as well as its potential engineering application value.

**Table 1.** Statistical analysis results on CEC 2017 benchmark function suite at 10 dimensions.

| Func | Measure | ACWMA                 | WMA                    | DBO                | GGO                | TOC                | IVY                | WOA                | PSO                   |
|------|---------|-----------------------|------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|-----------------------|
| F1   | Std     | $3.68 \times 10^3$    | $3.69 \times 10^3$     | $1.17 \times 10^9$ | $2.16 \times 10^8$ | $2.92 \times 10^8$ | $3.18 \times 10^8$ | $8.47 \times 10^6$ | $1.28 \times 10^7$    |
|      | Mean    | $1.48 \times 10^3$    | $3.84 \times 10^3$     | $9.96 \times 10^8$ | $1.28 \times 10^8$ | $2.80 \times 10^6$ | $3.91 \times 10^3$ | $4.03 \times 10^6$ | $7.67 \times 10^2$    |
|      | Rank    | 1                     | 2                      | 8                  | 5                  | 6                  | 7                  | 3                  | 4                     |
| F3   | Std     | $2.68 \times 10^{-5}$ | $8.44 \times 10^{-14}$ | $1.35 \times 10^3$ | $1.85 \times 10^3$ | $9.87 \times 10^3$ | $3.24 \times 10^2$ | $3.73 \times 10^3$ | $5.63 \times 10^{-9}$ |
|      | Mean    | $3.00 \times 10^2$    | $3.00 \times 10^2$     | $2.49 \times 10^3$ | $2.02 \times 10^3$ | $5.49 \times 10^2$ | $5.68 \times 10^2$ | $1.86 \times 10^3$ | $3.00 \times 10^2$    |
|      | Rank    | 3                     | 1                      | 5                  | 6                  | 8                  | 4                  | 7                  | 2                     |
| F4   | Std     | $7.33 \times 10^{-2}$ | $1.01 \times 10^1$     | $3.69 \times 10^1$ | $4.41 \times 10^1$ | $3.27 \times 10^1$ | $1.10 \times 10^1$ | $5.32 \times 10^1$ | $1.29 \times 10^1$    |
|      | Mean    | $4.00 \times 10^2$    | $4.00 \times 10^2$     | $4.79 \times 10^2$ | $4.20 \times 10^2$ | $4.08 \times 10^2$ | $4.06 \times 10^2$ | $4.40 \times 10^2$ | $4.08 \times 10^2$    |
|      | Rank    | 1                     | 2                      | 6                  | 7                  | 5                  | 3                  | 8                  | 4                     |
| F5   | Std     | $4.08 \times 10^0$    | $1.35 \times 10^1$     | $1.05 \times 10^1$ | $7.51 \times 10^0$ | $1.78 \times 10^1$ | $1.65 \times 10^1$ | $2.37 \times 10^1$ | $1.34 \times 10^1$    |
|      | Mean    | $5.08 \times 10^2$    | $5.28 \times 10^2$     | $5.51 \times 10^2$ | $5.23 \times 10^2$ | $5.30 \times 10^2$ | $5.30 \times 10^2$ | $5.48 \times 10^2$ | $5.31 \times 10^2$    |
|      | Rank    | 1                     | 5                      | 3                  | 2                  | 7                  | 6                  | 8                  | 4                     |
| F6   | Std     | $3.14 \times 10^{-2}$ | $1.01 \times 10^1$     | $6.35 \times 10^0$ | $4.85 \times 10^0$ | $9.85 \times 10^0$ | $6.60 \times 10^0$ | $1.48 \times 10^1$ | $3.54 \times 10^0$    |
|      | Mean    | $6.00 \times 10^2$    | $6.09 \times 10^2$     | $6.22 \times 10^2$ | $6.07 \times 10^2$ | $6.15 \times 10^2$ | $6.00 \times 10^2$ | $6.38 \times 10^2$ | $6.03 \times 10^2$    |
|      | Rank    | 1                     | 7                      | 4                  | 3                  | 6                  | 5                  | 8                  | 2                     |
| F7   | Std     | $5.72 \times 10^0$    | $1.57 \times 10^1$     | $1.07 \times 10^1$ | $9.94 \times 10^0$ | $1.69 \times 10^1$ | $2.35 \times 10^1$ | $2.63 \times 10^1$ | $9.25 \times 10^0$    |
|      | Mean    | $7.24 \times 10^2$    | $7.41 \times 10^2$     | $7.83 \times 10^2$ | $7.51 \times 10^2$ | $7.48 \times 10^2$ | $7.97 \times 10^2$ | $7.73 \times 10^2$ | $7.27 \times 10^2$    |
|      | Rank    | 1                     | 5                      | 4                  | 3                  | 6                  | 7                  | 8                  | 2                     |
| F8   | Std     | $3.80 \times 10^0$    | $1.28 \times 10^1$     | $5.42 \times 10^0$ | $5.37 \times 10^0$ | $1.13 \times 10^1$ | $9.76 \times 10^0$ | $2.04 \times 10^1$ | $7.98 \times 10^0$    |
|      | Mean    | $8.09 \times 10^2$    | $8.34 \times 10^2$     | $8.34 \times 10^2$ | $8.23 \times 10^2$ | $8.27 \times 10^2$ | $8.22 \times 10^2$ | $8.45 \times 10^2$ | $8.18 \times 10^2$    |
|      | Rank    | 1                     | 7                      | 3                  | 2                  | 6                  | 5                  | 8                  | 4                     |
| F9   | Std     | $1.22 \times 10^{-1}$ | $4.54 \times 10^1$     | $1.00 \times 10^2$ | $5.95 \times 10^1$ | $1.62 \times 10^2$ | $2.38 \times 10^2$ | $3.05 \times 10^2$ | $1.04 \times 10^2$    |
|      | Mean    | $9.00 \times 10^2$    | $9.12 \times 10^2$     | $1.06 \times 10^3$ | $9.66 \times 10^2$ | $9.48 \times 10^2$ | $1.53 \times 10^3$ | $1.38 \times 10^3$ | $9.09 \times 10^2$    |
|      | Rank    | 1                     | 2                      | 4                  | 3                  | 6                  | 7                  | 8                  | 5                     |
| F10  | Std     | $1.66 \times 10^2$    | $2.97 \times 10^2$     | $2.62 \times 10^2$ | $3.40 \times 10^2$ | $3.39 \times 10^2$ | $3.81 \times 10^2$ | $3.65 \times 10^2$ | $2.86 \times 10^2$    |
|      | Mean    | $1.30 \times 10^3$    | $1.97 \times 10^3$     | $2.37 \times 10^3$ | $2.01 \times 10^3$ | $2.08 \times 10^3$ | $2.04 \times 10^3$ | $2.11 \times 10^3$ | $1.93 \times 10^3$    |
|      | Rank    | 1                     | 4                      | 2                  | 6                  | 5                  | 8                  | 7                  | 3                     |
| F11  | Std     | $4.56 \times 10^0$    | $5.32 \times 10^1$     | $1.44 \times 10^2$ | $9.85 \times 10^2$ | $1.14 \times 10^2$ | $6.57 \times 10^2$ | $1.26 \times 10^3$ | $6.19 \times 10^1$    |
|      | Mean    | $1.11 \times 10^3$    | $1.19 \times 10^3$     | $1.34 \times 10^3$ | $1.36 \times 10^3$ | $1.25 \times 10^3$ | $1.16 \times 10^3$ | $1.64 \times 10^3$ | $1.15 \times 10^3$    |
|      | Rank    | 1                     | 2                      | 5                  | 7                  | 4                  | 6                  | 8                  | 3                     |
| F12  | Std     | $1.58 \times 10^4$    | $1.40 \times 10^4$     | $6.58 \times 10^7$ | $2.50 \times 10^6$ | $2.72 \times 10^6$ | $1.01 \times 10^6$ | $6.27 \times 10^6$ | $4.35 \times 10^6$    |
|      | Mean    | $6.89 \times 10^3$    | $6.98 \times 10^3$     | $4.64 \times 10^6$ | $3.23 \times 10^6$ | $1.67 \times 10^4$ | $2.86 \times 10^5$ | $4.74 \times 10^6$ | $1.15 \times 10^4$    |
|      | Rank    | 2                     | 1                      | 8                  | 4                  | 5                  | 3                  | 7                  | 6                     |
| F13  | Std     | $8.93 \times 10^0$    | $7.01 \times 10^3$     | $1.39 \times 10^4$ | $1.01 \times 10^4$ | $2.49 \times 10^3$ | $8.86 \times 10^3$ | $1.26 \times 10^4$ | $5.36 \times 10^3$    |
|      | Mean    | $1.31 \times 10^3$    | $3.27 \times 10^3$     | $2.32 \times 10^4$ | $8.42 \times 10^3$ | $2.05 \times 10^3$ | $8.04 \times 10^3$ | $1.63 \times 10^4$ | $5.63 \times 10^3$    |
|      | Rank    | 1                     | 4                      | 8                  | 6                  | 2                  | 5                  | 7                  | 3                     |
| F14  | Std     | $9.82 \times 10^0$    | $3.36 \times 10^1$     | $5.10 \times 10^2$ | $5.81 \times 10^2$ | $5.96 \times 10^1$ | $4.67 \times 10^3$ | $1.36 \times 10^3$ | $2.22 \times 10^3$    |
|      | Mean    | $1.42 \times 10^3$    | $1.50 \times 10^3$     | $1.76 \times 10^3$ | $1.53 \times 10^3$ | $1.49 \times 10^3$ | $4.17 \times 10^3$ | $1.82 \times 10^3$ | $2.91 \times 10^3$    |
|      | Rank    | 1                     | 2                      | 4                  | 5                  | 3                  | 8                  | 6                  | 7                     |
| F15  | Std     | $3.28 \times 10^0$    | $3.66 \times 10^2$     | $1.45 \times 10^3$ | $1.71 \times 10^3$ | $1.03 \times 10^3$ | $6.93 \times 10^3$ | $5.78 \times 10^3$ | $3.61 \times 10^3$    |
|      | Mean    | $1.50 \times 10^3$    | $1.74 \times 10^3$     | $3.21 \times 10^3$ | $3.47 \times 10^3$ | $1.75 \times 10^3$ | $4.89 \times 10^3$ | $5.82 \times 10^3$ | $2.26 \times 10^3$    |
|      | Rank    | 1                     | 2                      | 4                  | 5                  | 3                  | 8                  | 7                  | 6                     |
| F16  | Std     | $5.75 \times 10^1$    | $1.46 \times 10^2$     | $1.13 \times 10^2$ | $9.89 \times 10^1$ | $1.10 \times 10^2$ | $1.64 \times 10^2$ | $1.04 \times 10^2$ | $1.53 \times 10^2$    |
|      | Mean    | $1.62 \times 10^3$    | $1.75 \times 10^3$     | $1.76 \times 10^3$ | $1.73 \times 10^3$ | $1.76 \times 10^3$ | $1.97 \times 10^3$ | $1.92 \times 10^3$ | $1.86 \times 10^3$    |
|      | Rank    | 1                     | 6                      | 5                  | 2                  | 4                  | 8                  | 3                  | 7                     |
| F17  | Std     | $2.55 \times 10^1$    | $6.64 \times 10^1$     | $1.34 \times 10^1$ | $1.80 \times 10^1$ | $4.59 \times 10^1$ | $1.06 \times 10^2$ | $3.92 \times 10^1$ | $4.84 \times 10^1$    |
|      | Mean    | $1.71 \times 10^3$    | $1.78 \times 10^3$     | $1.78 \times 10^3$ | $1.76 \times 10^3$ | $1.78 \times 10^3$ | $1.80 \times 10^3$ | $1.79 \times 10^3$ | $1.79 \times 10^3$    |
|      | Rank    | 3                     | 7                      | 1                  | 2                  | 5                  | 8                  | 4                  | 6                     |
| F18  | Std     | $9.55 \times 10^0$    | $1.40 \times 10^4$     | $2.41 \times 10^4$ | $1.32 \times 10^4$ | $1.24 \times 10^4$ | $4.20 \times 10^4$ | $1.25 \times 10^4$ | $1.44 \times 10^4$    |
|      | Mean    | $1.82 \times 10^3$    | $5.95 \times 10^3$     | $4.14 \times 10^4$ | $1.07 \times 10^4$ | $2.19 \times 10^3$ | $1.43 \times 10^4$ | $1.34 \times 10^4$ | $1.25 \times 10^4$    |
|      | Rank    | 1                     | 5                      | 7                  | 4                  | 2                  | 8                  | 3                  | 6                     |
| F19  | Std     | $1.04 \times 10^0$    | $4.55 \times 10^1$     | $2.93 \times 10^3$ | $2.68 \times 10^4$ | $2.10 \times 10^4$ | $1.44 \times 10^5$ | $1.36 \times 10^5$ | $5.33 \times 10^3$    |
|      | Mean    | $1.90 \times 10^3$    | $1.98 \times 10^3$     | $2.80 \times 10^3$ | $1.63 \times 10^4$ | $2.03 \times 10^3$ | $6.03 \times 10^3$ | $1.33 \times 10^4$ | $4.03 \times 10^3$    |
|      | Rank    | 1                     | 2                      | 3                  | 6                  | 5                  | 8                  | 7                  | 4                     |
| F20  | Std     | $3.45 \times 10^1$    | $8.06 \times 10^1$     | $4.78 \times 10^1$ | $5.13 \times 10^1$ | $8.80 \times 10^1$ | $7.69 \times 10^1$ | $8.21 \times 10^1$ | $6.99 \times 10^1$    |
|      | Mean    | $2.01 \times 10^3$    | $2.11 \times 10^3$     | $2.11 \times 10^3$ | $2.08 \times 10^3$ | $2.15 \times 10^3$ | $2.12 \times 10^3$ | $2.19 \times 10^3$ | $2.11 \times 10^3$    |
|      | Rank    | 1                     | 6                      | 2                  | 3                  | 8                  | 5                  | 7                  | 4                     |

Table 1. Cont.

| Func | Measure | ACWMA              | WMA                | DBO                | GGO                | TOC                | IVY                | WOA                | PSO                |
|------|---------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| F21  | Std     | $1.01 \times 10^2$ | $1.96 \times 10^2$ | $2.29 \times 10^1$ | $1.31 \times 10^2$ | $1.82 \times 10^2$ | $1.57 \times 10^2$ | $1.62 \times 10^2$ | $1.45 \times 10^2$ |
|      | Mean    | $2.21 \times 10^3$ | $2.22 \times 10^3$ | $2.25 \times 10^3$ | $2.23 \times 10^3$ | $2.23 \times 10^3$ | $2.22 \times 10^3$ | $2.27 \times 10^3$ | $2.22 \times 10^3$ |
|      | Rank    | 2                  | 8                  | 1                  | 3                  | 7                  | 5                  | 6                  | 4                  |
| F22  | Std     | $6.75 \times 10^2$ | $1.01 \times 10^3$ | $1.16 \times 10^3$ | $1.16 \times 10^3$ | $1.20 \times 10^3$ | $1.29 \times 10^3$ | $1.25 \times 10^3$ | $8.97 \times 10^2$ |
|      | Mean    | $3.29 \times 10^3$ | $5.20 \times 10^3$ | $3.29 \times 10^3$ | $4.71 \times 10^3$ | $5.58 \times 10^3$ | $5.01 \times 10^3$ | $5.52 \times 10^3$ | $4.74 \times 10^3$ |
|      | Rank    | 1                  | 3                  | 5                  | 4                  | 6                  | 8                  | 7                  | 2                  |
| F23  | Std     | $4.15 \times 10^1$ | $1.39 \times 10^2$ | $1.60 \times 10^2$ | $8.37 \times 10^1$ | $2.45 \times 10^2$ | $1.94 \times 10^2$ | $2.22 \times 10^2$ | $1.44 \times 10^2$ |
|      | Mean    | $2.69 \times 10^3$ | $2.82 \times 10^3$ | $3.08 \times 10^3$ | $2.78 \times 10^3$ | $3.00 \times 10^3$ | $2.89 \times 10^3$ | $3.14 \times 10^3$ | $2.92 \times 10^3$ |
|      | Rank    | 1                  | 3                  | 5                  | 2                  | 8                  | 6                  | 7                  | 4                  |
| F24  | Std     | $1.55 \times 10^2$ | $2.71 \times 10^2$ | $6.47 \times 10^1$ | $1.60 \times 10^2$ | $2.75 \times 10^2$ | $3.41 \times 10^2$ | $3.29 \times 10^2$ | $2.12 \times 10^2$ |
|      | Mean    | $2.77 \times 10^3$ | $2.75 \times 10^3$ | $2.72 \times 10^3$ | $2.65 \times 10^3$ | $2.62 \times 10^3$ | $2.50 \times 10^3$ | $2.62 \times 10^3$ | $2.60 \times 10^3$ |
|      | Rank    | 2                  | 5                  | 1                  | 3                  | 6                  | 8                  | 7                  | 4                  |
| F25  | Std     | $2.36 \times 10^1$ | $8.36 \times 10^1$ | $4.68 \times 10^1$ | $2.80 \times 10^1$ | $2.92 \times 10^1$ | $1.52 \times 10^1$ | $2.66 \times 10^1$ | $1.99 \times 10^1$ |
|      | Mean    | $2.93 \times 10^3$ | $2.93 \times 10^3$ | $3.02 \times 10^3$ | $2.94 \times 10^3$ | $2.93 \times 10^3$ | $2.93 \times 10^3$ | $2.94 \times 10^3$ | $2.93 \times 10^3$ |
|      | Rank    | 3                  | 8                  | 7                  | 5                  | 6                  | 1                  | 4                  | 2                  |
| F26  | Std     | $2.91 \times 10^2$ | $5.47 \times 10^2$ | $1.06 \times 10^2$ | $4.12 \times 10^2$ | $5.15 \times 10^2$ | $4.63 \times 10^2$ | $6.20 \times 10^2$ | $4.33 \times 10^2$ |
|      | Mean    | $2.90 \times 10^3$ | $2.94 \times 10^3$ | $3.18 \times 10^3$ | $3.08 \times 10^3$ | $3.10 \times 10^3$ | $3.31 \times 10^3$ | $3.64 \times 10^3$ | $2.99 \times 10^3$ |
|      | Rank    | 2                  | 7                  | 1                  | 3                  | 6                  | 5                  | 8                  | 4                  |
| F27  | Std     | $1.73 \times 10^1$ | $2.84 \times 10^1$ | $7.52 \times 10^0$ | $3.71 \times 10^1$ | $3.08 \times 10^1$ | $2.48 \times 10^1$ | $5.11 \times 10^1$ | $3.45 \times 10^1$ |
|      | Mean    | $3.09 \times 10^3$ | $3.10 \times 10^3$ | $3.11 \times 10^3$ | $3.11 \times 10^3$ | $3.10 \times 10^3$ | $3.11 \times 10^3$ | $3.13 \times 10^3$ | $3.13 \times 10^3$ |
|      | Rank    | 2                  | 4                  | 1                  | 7                  | 5                  | 3                  | 8                  | 6                  |
| F28  | Std     | $1.18 \times 10^2$ | $1.50 \times 10^2$ | $1.08 \times 10^2$ | $1.67 \times 10^2$ | $1.81 \times 10^2$ | $1.14 \times 10^2$ | $1.59 \times 10^2$ | $1.11 \times 10^2$ |
|      | Mean    | $3.41 \times 10^3$ | $3.38 \times 10^3$ | $3.25 \times 10^3$ | $3.42 \times 10^3$ | $3.41 \times 10^3$ | $3.38 \times 10^3$ | $3.41 \times 10^3$ | $3.38 \times 10^3$ |
|      | Rank    | 4                  | 5                  | 1                  | 7                  | 8                  | 3                  | 6                  | 2                  |
| F29  | Std     | $3.75 \times 10^1$ | $7.14 \times 10^1$ | $4.34 \times 10^1$ | $7.40 \times 10^1$ | $9.36 \times 10^1$ | $8.82 \times 10^1$ | $8.61 \times 10^1$ | $6.80 \times 10^1$ |
|      | Mean    | $3.17 \times 10^3$ | $3.26 \times 10^3$ | $3.27 \times 10^3$ | $3.22 \times 10^3$ | $3.28 \times 10^3$ | $3.27 \times 10^3$ | $3.36 \times 10^3$ | $3.25 \times 10^3$ |
|      | Rank    | 1                  | 4                  | 2                  | 5                  | 8                  | 7                  | 6                  | 3                  |
| F30  | Std     | $4.57 \times 10^5$ | $1.06 \times 10^6$ | $1.61 \times 10^6$ | $1.67 \times 10^6$ | $1.08 \times 10^6$ | $8.87 \times 10^5$ | $1.96 \times 10^6$ | $9.37 \times 10^5$ |
|      | Mean    | $4.77 \times 10^3$ | $4.38 \times 10^5$ | $7.80 \times 10^5$ | $7.61 \times 10^5$ | $8.21 \times 10^5$ | $7.42 \times 10^4$ | $4.96 \times 10^5$ | $9.60 \times 10^3$ |
|      | Rank    | 1                  | 4                  | 6                  | 7                  | 5                  | 2                  | 8                  | 3                  |

Table 2. Statistical analysis results on CEC 2017 benchmark function suite at 30 dimensions.

| Func | Measure | ACWMA              | WMA                | DBO                   | GGO                | TOC                | IVY                | WOA                | PSO                |
|------|---------|--------------------|--------------------|-----------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| F1   | Std     | $7.15 \times 10^3$ | $5.50 \times 10^3$ | $2.64 \times 10^9$    | $3.16 \times 10^9$ | $3.89 \times 10^8$ | $5.92 \times 10^3$ | $4.19 \times 10^7$ | $7.90 \times 10^8$ |
|      | Mean    | $3.07 \times 10^3$ | $1.64 \times 10^3$ | $2.09 \times 10^{10}$ | $6.16 \times 10^9$ | $6.71 \times 10^7$ | $2.04 \times 10^3$ | $8.26 \times 10^7$ | $1.18 \times 10^4$ |
|      | Rank    | 3                  | 1                  | 7                     | 8                  | 5                  | 2                  | 4                  | 6                  |
| F3   | Std     | $4.97 \times 10^2$ | $2.03 \times 10^3$ | $7.73 \times 10^3$    | $1.12 \times 10^4$ | $4.17 \times 10^4$ | $5.46 \times 10^3$ | $6.22 \times 10^4$ | $3.23 \times 10^3$ |
|      | Mean    | $1.31 \times 10^3$ | $5.25 \times 10^3$ | $6.13 \times 10^4$    | $6.45 \times 10^4$ | $9.28 \times 10^3$ | $4.14 \times 10^4$ | $1.88 \times 10^5$ | $3.91 \times 10^3$ |
|      | Rank    | 1                  | 2                  | 5                     | 6                  | 7                  | 4                  | 8                  | 3                  |
| F4   | Std     | $2.34 \times 10^1$ | $3.26 \times 10^1$ | $1.41 \times 10^3$    | $2.98 \times 10^2$ | $6.46 \times 10^1$ | $2.47 \times 10^1$ | $4.90 \times 10^1$ | $4.19 \times 10^1$ |
|      | Mean    | $4.84 \times 10^2$ | $4.74 \times 10^2$ | $3.07 \times 10^3$    | $1.02 \times 10^3$ | $5.47 \times 10^2$ | $5.09 \times 10^2$ | $5.97 \times 10^2$ | $5.18 \times 10^2$ |
|      | Rank    | 1                  | 3                  | 8                     | 7                  | 6                  | 2                  | 5                  | 4                  |
| F5   | Std     | $4.08 \times 10^1$ | $5.46 \times 10^1$ | $2.92 \times 10^1$    | $3.53 \times 10^1$ | $3.42 \times 10^1$ | $5.61 \times 10^1$ | $4.93 \times 10^1$ | $3.03 \times 10^1$ |
|      | Mean    | $5.84 \times 10^2$ | $6.97 \times 10^2$ | $7.75 \times 10^2$    | $6.94 \times 10^2$ | $7.04 \times 10^2$ | $6.98 \times 10^2$ | $7.50 \times 10^2$ | $6.27 \times 10^2$ |
|      | Rank    | 5                  | 7                  | 1                     | 4                  | 3                  | 8                  | 6                  | 2                  |
| F6   | Std     | $1.10 \times 10^0$ | $1.24 \times 10^1$ | $5.62 \times 10^0$    | $8.03 \times 10^0$ | $7.29 \times 10^0$ | $1.11 \times 10^1$ | $1.25 \times 10^1$ | $7.50 \times 10^0$ |
|      | Mean    | $6.01 \times 10^2$ | $6.50 \times 10^2$ | $6.57 \times 10^2$    | $6.27 \times 10^2$ | $6.55 \times 10^2$ | $6.00 \times 10^2$ | $6.68 \times 10^2$ | $6.25 \times 10^2$ |
|      | Rank    | 1                  | 7                  | 2                     | 5                  | 3                  | 6                  | 8                  | 4                  |
| F7   | Std     | $5.33 \times 10^1$ | $6.38 \times 10^1$ | $3.76 \times 10^1$    | $4.40 \times 10^1$ | $1.27 \times 10^2$ | $1.09 \times 10^2$ | $7.54 \times 10^1$ | $3.49 \times 10^1$ |
|      | Mean    | $8.15 \times 10^2$ | $9.48 \times 10^2$ | $1.14 \times 10^3$    | $9.95 \times 10^2$ | $1.20 \times 10^3$ | $1.23 \times 10^3$ | $1.20 \times 10^3$ | $8.55 \times 10^2$ |
|      | Rank    | 4                  | 5                  | 2                     | 3                  | 8                  | 7                  | 6                  | 1                  |
| F8   | Std     | $3.86 \times 10^1$ | $4.95 \times 10^1$ | $2.11 \times 10^1$    | $2.39 \times 10^1$ | $4.81 \times 10^1$ | $2.91 \times 10^1$ | $6.45 \times 10^1$ | $2.03 \times 10^1$ |
|      | Mean    | $8.84 \times 10^2$ | $9.79 \times 10^2$ | $1.03 \times 10^3$    | $9.56 \times 10^2$ | $9.64 \times 10^2$ | $9.52 \times 10^2$ | $9.94 \times 10^2$ | $9.05 \times 10^2$ |
|      | Rank    | 5                  | 7                  | 2                     | 3                  | 6                  | 4                  | 8                  | 1                  |



Table 2. Cont.

| Func | Measure | ACWMA                 | WMA                | DBO                | GGO                | TOC                | IVY                | WOA                | PSO                |
|------|---------|-----------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| F9   | Std     | $2.27 \times 10^1$    | $2.87 \times 10^3$ | $1.09 \times 10^3$ | $8.70 \times 10^2$ | $9.20 \times 10^2$ | $3.69 \times 10^2$ | $2.92 \times 10^3$ | $1.24 \times 10^3$ |
|      | Mean    | $9.14 \times 10^2$    | $6.01 \times 10^3$ | $6.54 \times 10^3$ | $3.23 \times 10^3$ | $3.22 \times 10^3$ | $5.42 \times 10^3$ | $8.76 \times 10^3$ | $2.59 \times 10^3$ |
|      | Rank    | 1                     | 7                  | 5                  | 3                  | 4                  | 2                  | 8                  | 6                  |
| F10  | Std     | $2.90 \times 10^2$    | $8.40 \times 10^2$ | $8.14 \times 10^2$ | $8.65 \times 10^2$ | $1.04 \times 10^3$ | $5.13 \times 10^2$ | $8.26 \times 10^2$ | $6.29 \times 10^2$ |
|      | Mean    | $6.10 \times 10^3$    | $5.17 \times 10^3$ | $7.37 \times 10^3$ | $6.42 \times 10^3$ | $6.53 \times 10^3$ | $5.27 \times 10^3$ | $6.56 \times 10^3$ | $4.78 \times 10^3$ |
|      | Rank    | 1                     | 6                  | 4                  | 7                  | 8                  | 2                  | 5                  | 3                  |
| F11  | Std     | $5.47 \times 10^1$    | $6.39 \times 10^1$ | $1.11 \times 10^3$ | $1.24 \times 10^3$ | $2.60 \times 10^3$ | $1.92 \times 10^2$ | $6.33 \times 10^2$ | $5.05 \times 10^1$ |
|      | Mean    | $1.26 \times 10^3$    | $1.31 \times 10^3$ | $4.52 \times 10^3$ | $4.02 \times 10^3$ | $1.47 \times 10^3$ | $1.35 \times 10^3$ | $1.73 \times 10^3$ | $1.22 \times 10^3$ |
|      | Rank    | 2                     | 3                  | 6                  | 7                  | 8                  | 4                  | 5                  | 1                  |
| F12  | Std     | $1.21 \times 10^5$    | $1.18 \times 10^5$ | $1.43 \times 10^9$ | $2.27 \times 10^8$ | $3.42 \times 10^7$ | $9.41 \times 10^5$ | $4.98 \times 10^7$ | $9.57 \times 10^7$ |
|      | Mean    | $5.78 \times 10^4$    | $8.08 \times 10^4$ | $3.14 \times 10^9$ | $2.43 \times 10^8$ | $4.38 \times 10^6$ | $1.45 \times 10^6$ | $6.91 \times 10^7$ | $3.48 \times 10^5$ |
|      | Rank    | 2                     | 1                  | 8                  | 7                  | 4                  | 3                  | 5                  | 6                  |
| F13  | Std     | $1.88 \times 10^4$    | $3.05 \times 10^4$ | $1.13 \times 10^9$ | $1.76 \times 10^6$ | $1.11 \times 10^6$ | $1.51 \times 10^4$ | $1.13 \times 10^5$ | $1.12 \times 10^6$ |
|      | Mean    | $8.84 \times 10^3$    | $2.47 \times 10^4$ | $5.10 \times 10^8$ | $1.10 \times 10^6$ | $3.72 \times 10^4$ | $1.25 \times 10^4$ | $1.78 \times 10^5$ | $2.31 \times 10^4$ |
|      | Rank    | 2                     | 3                  | 8                  | 7                  | 5                  | 1                  | 4                  | 6                  |
| F14  | Std     | $3.99 \times 10^1$    | $2.21 \times 10^3$ | $3.07 \times 10^5$ | $8.07 \times 10^5$ | $1.91 \times 10^6$ | $4.05 \times 10^5$ | $2.03 \times 10^6$ | $1.45 \times 10^4$ |
|      | Mean    | $1.49 \times 10^3$    | $3.99 \times 10^3$ | $1.83 \times 10^5$ | $2.36 \times 10^5$ | $1.91 \times 10^3$ | $3.52 \times 10^5$ | $1.71 \times 10^6$ | $1.28 \times 10^4$ |
|      | Rank    | 1                     | 2                  | 4                  | 6                  | 7                  | 5                  | 8                  | 3                  |
| F15  | Std     | $1.17 \times 10^4$    | $3.82 \times 10^4$ | $2.57 \times 10^6$ | $1.98 \times 10^6$ | $1.20 \times 10^4$ | $3.69 \times 10^3$ | $5.86 \times 10^4$ | $1.06 \times 10^4$ |
|      | Mean    | $1.65 \times 10^3$    | $2.00 \times 10^4$ | $8.50 \times 10^5$ | $4.20 \times 10^5$ | $2.57 \times 10^3$ | $3.94 \times 10^3$ | $9.58 \times 10^4$ | $9.23 \times 10^3$ |
|      | Rank    | 3                     | 5                  | 8                  | 7                  | 4                  | 1                  | 6                  | 2                  |
| F16  | Std     | $2.80 \times 10^2$    | $2.90 \times 10^2$ | $3.30 \times 10^2$ | $3.38 \times 10^2$ | $5.90 \times 10^2$ | $2.48 \times 10^2$ | $4.34 \times 10^2$ | $3.31 \times 10^2$ |
|      | Mean    | $2.24 \times 10^3$    | $2.61 \times 10^3$ | $3.46 \times 10^3$ | $2.92 \times 10^3$ | $3.03 \times 10^3$ | $2.92 \times 10^3$ | $3.65 \times 10^3$ | $2.56 \times 10^3$ |
|      | Rank    | 2                     | 3                  | 4                  | 6                  | 8                  | 1                  | 7                  | 5                  |
| F17  | Std     | $7.43 \times 10^1$    | $2.07 \times 10^2$ | $2.32 \times 10^2$ | $1.61 \times 10^2$ | $2.73 \times 10^2$ | $2.94 \times 10^2$ | $2.62 \times 10^2$ | $2.39 \times 10^2$ |
|      | Mean    | $1.86 \times 10^3$    | $2.37 \times 10^3$ | $2.61 \times 10^3$ | $2.14 \times 10^3$ | $2.55 \times 10^3$ | $2.57 \times 10^3$ | $2.52 \times 10^3$ | $2.21 \times 10^3$ |
|      | Rank    | 1                     | 3                  | 4                  | 2                  | 7                  | 8                  | 6                  | 5                  |
| F18  | Std     | $1.21 \times 10^2$    | $1.97 \times 10^5$ | $2.32 \times 10^6$ | $4.43 \times 10^6$ | $3.53 \times 10^6$ | $1.35 \times 10^6$ | $4.75 \times 10^6$ | $2.56 \times 10^5$ |
|      | Mean    | $1.90 \times 10^3$    | $1.84 \times 10^5$ | $1.52 \times 10^6$ | $1.05 \times 10^6$ | $5.49 \times 10^4$ | $8.63 \times 10^5$ | $2.16 \times 10^6$ | $1.88 \times 10^5$ |
|      | Rank    | 1                     | 2                  | 5                  | 7                  | 6                  | 4                  | 8                  | 3                  |
| F19  | Std     | $8.71 \times 10^3$    | $1.80 \times 10^4$ | $9.52 \times 10^7$ | $5.13 \times 10^6$ | $2.21 \times 10^4$ | $6.29 \times 10^3$ | $5.62 \times 10^6$ | $1.38 \times 10^4$ |
|      | Mean    | $1.95 \times 10^3$    | $5.63 \times 10^3$ | $7.40 \times 10^7$ | $2.57 \times 10^6$ | $3.08 \times 10^3$ | $4.22 \times 10^3$ | $4.12 \times 10^6$ | $7.77 \times 10^3$ |
|      | Rank    | 2                     | 4                  | 8                  | 6                  | 5                  | 1                  | 7                  | 3                  |
| F20  | Std     | $9.03 \times 10^1$    | $1.92 \times 10^2$ | $1.37 \times 10^2$ | $1.28 \times 10^2$ | $1.91 \times 10^2$ | $2.16 \times 10^2$ | $1.81 \times 10^2$ | $1.78 \times 10^2$ |
|      | Mean    | $2.23 \times 10^3$    | $2.68 \times 10^3$ | $2.49 \times 10^3$ | $2.49 \times 10^3$ | $2.68 \times 10^3$ | $2.58 \times 10^3$ | $2.80 \times 10^3$ | $2.51 \times 10^3$ |
|      | Rank    | 1                     | 7                  | 3                  | 2                  | 6                  | 8                  | 5                  | 4                  |
| F21  | Std     | $5.00 \times 10^2$    | $9.02 \times 10^2$ | $1.33 \times 10^2$ | $8.33 \times 10^2$ | $1.00 \times 10^3$ | $9.09 \times 10^2$ | $8.80 \times 10^2$ | $5.53 \times 10^2$ |
|      | Mean    | $2.89 \times 10^3$    | $3.88 \times 10^3$ | $2.81 \times 10^3$ | $3.44 \times 10^3$ | $3.83 \times 10^3$ | $4.18 \times 10^3$ | $4.52 \times 10^3$ | $3.18 \times 10^3$ |
|      | Rank    | 2                     | 6                  | 1                  | 4                  | 8                  | 7                  | 5                  | 3                  |
| F22  | Std     | $1.69 \times 10^3$    | $2.17 \times 10^3$ | $1.40 \times 10^3$ | $1.97 \times 10^3$ | $1.99 \times 10^3$ | $2.22 \times 10^3$ | $1.94 \times 10^3$ | $2.13 \times 10^3$ |
|      | Mean    | $1.50 \times 10^4$    | $1.37 \times 10^4$ | $1.99 \times 10^4$ | $1.78 \times 10^4$ | $1.79 \times 10^4$ | $1.41 \times 10^4$ | $1.67 \times 10^4$ | $1.27 \times 10^4$ |
|      | Rank    | 2                     | 7                  | 1                  | 4                  | 5                  | 8                  | 3                  | 6                  |
| F23  | Std     | $3.92 \times 10^2$    | $4.74 \times 10^2$ | $8.46 \times 10^2$ | $4.31 \times 10^2$ | $8.96 \times 10^2$ | $4.54 \times 10^2$ | $9.99 \times 10^2$ | $5.65 \times 10^2$ |
|      | Mean    | $3.45 \times 10^3$    | $4.80 \times 10^3$ | $5.73 \times 10^3$ | $4.61 \times 10^3$ | $5.52 \times 10^3$ | $4.46 \times 10^3$ | $6.16 \times 10^3$ | $4.41 \times 10^3$ |
|      | Rank    | 1                     | 4                  | 6                  | 2                  | 7                  | 3                  | 8                  | 5                  |
| F24  | Std     | $3.92 \times 10^2$    | $4.28 \times 10^2$ | $6.49 \times 10^2$ | $9.90 \times 10^2$ | $6.15 \times 10^2$ | $6.49 \times 10^2$ | $8.16 \times 10^2$ | $7.30 \times 10^2$ |
|      | Mean    | $3.89 \times 10^3$    | $4.81 \times 10^3$ | $5.39 \times 10^3$ | $4.51 \times 10^3$ | $5.14 \times 10^3$ | $5.78 \times 10^3$ | $5.70 \times 10^3$ | $4.13 \times 10^3$ |
|      | Rank    | 1                     | 2                  | 4                  | 8                  | 3                  | 5                  | 7                  | 6                  |
| F25  | Std     | $3.93 \times 10^{-1}$ | $2.08 \times 10^0$ | $1.77 \times 10^2$ | $1.10 \times 10^2$ | $5.14 \times 10^1$ | $1.05 \times 10^1$ | $4.01 \times 10^1$ | $2.19 \times 10^1$ |
|      | Mean    | $2.88 \times 10^3$    | $2.88 \times 10^3$ | $3.50 \times 10^3$ | $3.15 \times 10^3$ | $2.94 \times 10^3$ | $2.88 \times 10^3$ | $2.98 \times 10^3$ | $2.92 \times 10^3$ |
|      | Rank    | 1                     | 2                  | 8                  | 7                  | 6                  | 3                  | 5                  | 4                  |
| F26  | Std     | $4.64 \times 10^2$    | $9.28 \times 10^2$ | $6.49 \times 10^2$ | $4.91 \times 10^2$ | $1.82 \times 10^3$ | $2.11 \times 10^3$ | $1.07 \times 10^3$ | $1.27 \times 10^3$ |
|      | Mean    | $4.45 \times 10^3$    | $5.64 \times 10^3$ | $6.87 \times 10^3$ | $5.51 \times 10^3$ | $7.50 \times 10^3$ | $6.06 \times 10^3$ | $7.86 \times 10^3$ | $5.44 \times 10^3$ |
|      | Rank    | 1                     | 4                  | 3                  | 2                  | 7                  | 8                  | 5                  | 6                  |

Table 2. Cont.

| Func | Measure | ACWMA              | WMA                | DBO                | GGO                | TOC                | IVY                | WOA                | PSO                |
|------|---------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| F27  | Std     | $1.08 \times 10^1$ | $2.94 \times 10^1$ | $1.15 \times 10^2$ | $6.95 \times 10^1$ | $7.03 \times 10^1$ | $5.56 \times 10^1$ | $9.56 \times 10^1$ | $7.64 \times 10^1$ |
|      | Mean    | $3.21 \times 10^3$ | $3.24 \times 10^3$ | $3.53 \times 10^3$ | $3.35 \times 10^3$ | $3.34 \times 10^3$ | $3.25 \times 10^3$ | $3.39 \times 10^3$ | $3.28 \times 10^3$ |
|      | Rank    | 1                  | 2                  | 8                  | 4                  | 5                  | 3                  | 7                  | 6                  |
| F28  | Std     | $5.19 \times 10^1$ | $5.30 \times 10^1$ | $4.22 \times 10^2$ | $2.53 \times 10^2$ | $8.88 \times 10^1$ | $1.75 \times 10^1$ | $5.95 \times 10^1$ | $5.27 \times 10^1$ |
|      | Mean    | $3.21 \times 10^3$ | $3.21 \times 10^3$ | $4.33 \times 10^3$ | $3.66 \times 10^3$ | $3.29 \times 10^3$ | $3.21 \times 10^3$ | $3.38 \times 10^3$ | $3.28 \times 10^3$ |
|      | Rank    | 2                  | 4                  | 8                  | 7                  | 6                  | 1                  | 5                  | 3                  |
| F29  | Std     | $1.50 \times 10^2$ | $2.43 \times 10^2$ | $2.51 \times 10^2$ | $4.56 \times 10^2$ | $3.44 \times 10^2$ | $3.48 \times 10^2$ | $5.35 \times 10^2$ | $2.18 \times 10^2$ |
|      | Mean    | $3.58 \times 10^3$ | $3.99 \times 10^3$ | $4.49 \times 10^3$ | $4.31 \times 10^3$ | $4.77 \times 10^3$ | $4.05 \times 10^3$ | $5.09 \times 10^3$ | $4.04 \times 10^3$ |
|      | Rank    | 1                  | 3                  | 4                  | 7                  | 5                  | 6                  | 8                  | 2                  |
| F30  | Std     | $3.86 \times 10^3$ | $1.71 \times 10^5$ | $2.35 \times 10^7$ | $1.82 \times 10^7$ | $3.30 \times 10^5$ | $3.27 \times 10^3$ | $1.36 \times 10^7$ | $6.84 \times 10^4$ |
|      | Mean    | $8.52 \times 10^3$ | $1.85 \times 10^4$ | $4.17 \times 10^7$ | $1.64 \times 10^7$ | $5.16 \times 10^4$ | $1.02 \times 10^4$ | $9.93 \times 10^6$ | $1.82 \times 10^4$ |
|      | Rank    | 2                  | 4                  | 8                  | 7                  | 5                  | 1                  | 6                  | 3                  |

Table 3. Statistical analysis results on CEC 2017 benchmark function suite at 50 dimensions.

| Func | Measure | ACWMA              | WMA                | DBO                   | GGO                   | TOC                | IVY                | WOA                | PSO                |
|------|---------|--------------------|--------------------|-----------------------|-----------------------|--------------------|--------------------|--------------------|--------------------|
| F1   | Std     | $8.57 \times 10^3$ | $8.07 \times 10^3$ | $5.99 \times 10^9$    | $7.28 \times 10^9$    | $2.80 \times 10^9$ | $1.42 \times 10^3$ | $2.64 \times 10^8$ | $2.19 \times 10^9$ |
|      | Mean    | $5.54 \times 10^3$ | $8.28 \times 10^3$ | $6.02 \times 10^{10}$ | $2.63 \times 10^{10}$ | $9.18 \times 10^8$ | $2.84 \times 10^3$ | $4.76 \times 10^8$ | $1.52 \times 10^9$ |
|      | Rank    | 3                  | 2                  | 7                     | 8                     | 6                  | 1                  | 4                  | 5                  |
| F3   | Std     | $7.12 \times 10^3$ | $1.59 \times 10^4$ | $1.91 \times 10^4$    | $2.58 \times 10^4$    | $1.01 \times 10^5$ | $6.62 \times 10^4$ | $6.27 \times 10^4$ | $1.72 \times 10^4$ |
|      | Mean    | $2.70 \times 10^4$ | $6.65 \times 10^4$ | $1.65 \times 10^5$    | $1.56 \times 10^5$    | $7.30 \times 10^4$ | $2.14 \times 10^5$ | $1.56 \times 10^5$ | $6.39 \times 10^4$ |
|      | Rank    | 1                  | 2                  | 4                     | 5                     | 8                  | 7                  | 6                  | 3                  |
| F4   | Std     | $4.25 \times 10^1$ | $5.03 \times 10^1$ | $1.41 \times 10^3$    | $1.52 \times 10^3$    | $4.49 \times 10^2$ | $5.71 \times 10^1$ | $1.85 \times 10^2$ | $1.90 \times 10^2$ |
|      | Mean    | $5.27 \times 10^2$ | $5.48 \times 10^2$ | $1.26 \times 10^4$    | $4.40 \times 10^3$    | $8.68 \times 10^2$ | $5.41 \times 10^2$ | $1.03 \times 10^3$ | $7.32 \times 10^2$ |
|      | Rank    | 1                  | 2                  | 7                     | 8                     | 6                  | 3                  | 4                  | 5                  |
| F5   | Std     | $9.17 \times 10^1$ | $8.19 \times 10^1$ | $3.97 \times 10^1$    | $4.04 \times 10^1$    | $6.85 \times 10^1$ | $3.10 \times 10^1$ | $1.00 \times 10^2$ | $5.44 \times 10^1$ |
|      | Mean    | $6.67 \times 10^2$ | $8.76 \times 10^2$ | $1.04 \times 10^3$    | $9.31 \times 10^2$    | $9.31 \times 10^2$ | $8.72 \times 10^2$ | $9.85 \times 10^2$ | $7.44 \times 10^2$ |
|      | Rank    | 7                  | 6                  | 2                     | 3                     | 5                  | 1                  | 8                  | 4                  |
| F6   | Std     | $4.65 \times 10^0$ | $1.22 \times 10^1$ | $5.15 \times 10^0$    | $8.89 \times 10^0$    | $6.91 \times 10^0$ | $2.07 \times 10^1$ | $9.50 \times 10^0$ | $1.02 \times 10^1$ |
|      | Mean    | $6.06 \times 10^2$ | $6.58 \times 10^2$ | $6.75 \times 10^2$    | $6.51 \times 10^2$    | $6.69 \times 10^2$ | $6.12 \times 10^2$ | $6.82 \times 10^2$ | $6.39 \times 10^2$ |
|      | Rank    | 1                  | 7                  | 2                     | 4                     | 3                  | 8                  | 5                  | 6                  |
| F7   | Std     | $9.58 \times 10^1$ | $1.69 \times 10^2$ | $5.28 \times 10^1$    | $1.16 \times 10^2$    | $2.09 \times 10^2$ | $1.36 \times 10^2$ | $1.21 \times 10^2$ | $7.28 \times 10^1$ |
|      | Mean    | $1.00 \times 10^3$ | $1.27 \times 10^3$ | $1.64 \times 10^3$    | $1.43 \times 10^3$    | $1.82 \times 10^3$ | $1.69 \times 10^3$ | $1.77 \times 10^3$ | $1.09 \times 10^3$ |
|      | Rank    | 3                  | 7                  | 1                     | 4                     | 8                  | 6                  | 5                  | 2                  |
| F8   | Std     | $9.11 \times 10^1$ | $8.00 \times 10^1$ | $3.08 \times 10^1$    | $4.03 \times 10^1$    | $7.82 \times 10^1$ | $3.90 \times 10^1$ | $5.70 \times 10^1$ | $4.51 \times 10^1$ |
|      | Mean    | $1.00 \times 10^3$ | $1.21 \times 10^3$ | $1.34 \times 10^3$    | $1.21 \times 10^3$    | $1.25 \times 10^3$ | $1.18 \times 10^3$ | $1.25 \times 10^3$ | $1.06 \times 10^3$ |
|      | Rank    | 8                  | 7                  | 1                     | 3                     | 6                  | 2                  | 5                  | 4                  |
| F9   | Std     | $7.20 \times 10^2$ | $6.97 \times 10^3$ | $3.63 \times 10^3$    | $3.42 \times 10^3$    | $3.47 \times 10^3$ | $6.50 \times 10^2$ | $6.21 \times 10^3$ | $2.46 \times 10^3$ |
|      | Mean    | $1.26 \times 10^3$ | $2.63 \times 10^4$ | $2.69 \times 10^4$    | $1.55 \times 10^4$    | $1.07 \times 10^4$ | $1.33 \times 10^4$ | $2.53 \times 10^4$ | $8.44 \times 10^3$ |
|      | Rank    | 2                  | 8                  | 6                     | 4                     | 5                  | 1                  | 7                  | 3                  |
| F10  | Std     | $1.03 \times 10^3$ | $1.01 \times 10^3$ | $8.69 \times 10^2$    | $1.15 \times 10^3$    | $1.71 \times 10^3$ | $7.95 \times 10^2$ | $1.19 \times 10^3$ | $9.64 \times 10^2$ |
|      | Mean    | $1.16 \times 10^4$ | $8.25 \times 10^3$ | $1.37 \times 10^4$    | $1.00 \times 10^4$    | $1.16 \times 10^4$ | $8.09 \times 10^3$ | $1.14 \times 10^4$ | $7.34 \times 10^3$ |
|      | Rank    | 5                  | 4                  | 2                     | 6                     | 8                  | 1                  | 7                  | 3                  |
| F11  | Std     | $1.31 \times 10^2$ | $1.11 \times 10^2$ | $3.00 \times 10^3$    | $5.20 \times 10^3$    | $3.64 \times 10^3$ | $1.78 \times 10^3$ | $1.22 \times 10^4$ | $7.42 \times 10^1$ |
|      | Mean    | $1.64 \times 10^3$ | $1.62 \times 10^3$ | $1.75 \times 10^4$    | $2.24 \times 10^4$    | $2.02 \times 10^3$ | $3.79 \times 10^3$ | $2.19 \times 10^4$ | $1.37 \times 10^3$ |
|      | Rank    | 3                  | 2                  | 5                     | 7                     | 6                  | 4                  | 8                  | 1                  |
| F12  | Std     | $1.11 \times 10^6$ | $1.39 \times 10^6$ | $6.51 \times 10^9$    | $1.72 \times 10^9$    | $1.29 \times 10^9$ | $1.41 \times 10^6$ | $2.24 \times 10^8$ | $6.58 \times 10^8$ |
|      | Mean    | $1.87 \times 10^6$ | $1.48 \times 10^6$ | $2.26 \times 10^{10}$ | $2.99 \times 10^9$    | $3.27 \times 10^8$ | $3.20 \times 10^6$ | $3.97 \times 10^8$ | $5.67 \times 10^7$ |
|      | Rank    | 1                  | 2                  | 8                     | 7                     | 6                  | 3                  | 4                  | 5                  |
| F13  | Std     | $9.20 \times 10^3$ | $2.04 \times 10^4$ | $3.26 \times 10^9$    | $2.17 \times 10^8$    | $2.51 \times 10^8$ | $6.29 \times 10^3$ | $3.53 \times 10^6$ | $2.41 \times 10^8$ |
|      | Mean    | $6.26 \times 10^3$ | $2.42 \times 10^4$ | $4.87 \times 10^9$    | $2.93 \times 10^8$    | $3.07 \times 10^5$ | $5.08 \times 10^3$ | $4.14 \times 10^6$ | $3.99 \times 10^4$ |
|      | Rank    | 2                  | 3                  | 8                     | 5                     | 7                  | 1                  | 4                  | 6                  |
| F14  | Std     | $1.17 \times 10^3$ | $3.77 \times 10^4$ | $3.18 \times 10^6$    | $4.85 \times 10^6$    | $1.75 \times 10^6$ | $8.38 \times 10^5$ | $1.06 \times 10^6$ | $1.00 \times 10^5$ |
|      | Mean    | $1.70 \times 10^3$ | $4.27 \times 10^4$ | $3.61 \times 10^6$    | $1.48 \times 10^6$    | $9.46 \times 10^4$ | $9.75 \times 10^5$ | $1.18 \times 10^6$ | $4.20 \times 10^4$ |
|      | Rank    | 1                  | 2                  | 7                     | 8                     | 6                  | 4                  | 5                  | 3                  |

Table 3. Cont.

| Func | Measure | ACWMA              | WMA                | DBO                | GGO                | TOC                | IVY                | WOA                | PSO                |
|------|---------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| F15  | Std     | $5.65 \times 10^3$ | $1.67 \times 10^4$ | $6.34 \times 10^8$ | $3.55 \times 10^8$ | $3.20 \times 10^8$ | $6.19 \times 10^3$ | $1.11 \times 10^6$ | $9.69 \times 10^3$ |
|      | Mean    | $7.42 \times 10^3$ | $1.96 \times 10^4$ | $3.81 \times 10^8$ | $2.83 \times 10^7$ | $1.87 \times 10^4$ | $1.90 \times 10^4$ | $2.04 \times 10^5$ | $1.34 \times 10^4$ |
|      | Rank    | 1                  | 4                  | 8                  | 7                  | 6                  | 2                  | 5                  | 3                  |
| F16  | Std     | $4.88 \times 10^2$ | $4.63 \times 10^2$ | $4.13 \times 10^2$ | $4.30 \times 10^2$ | $8.15 \times 10^2$ | $4.41 \times 10^2$ | $8.79 \times 10^2$ | $5.18 \times 10^2$ |
|      | Mean    | $2.99 \times 10^3$ | $3.42 \times 10^3$ | $5.09 \times 10^3$ | $4.27 \times 10^3$ | $4.22 \times 10^3$ | $3.73 \times 10^3$ | $5.41 \times 10^3$ | $3.35 \times 10^3$ |
|      | Rank    | 5                  | 4                  | 1                  | 2                  | 7                  | 3                  | 8                  | 6                  |
| F17  | Std     | $3.01 \times 10^2$ | $4.13 \times 10^2$ | $3.76 \times 10^2$ | $2.93 \times 10^2$ | $3.91 \times 10^2$ | $4.00 \times 10^2$ | $4.50 \times 10^2$ | $2.58 \times 10^2$ |
|      | Mean    | $2.76 \times 10^3$ | $3.65 \times 10^3$ | $4.09 \times 10^3$ | $3.36 \times 10^3$ | $3.74 \times 10^3$ | $3.63 \times 10^3$ | $4.04 \times 10^3$ | $3.17 \times 10^3$ |
|      | Rank    | 3                  | 7                  | 4                  | 2                  | 5                  | 6                  | 8                  | 1                  |
| F18  | Std     | $4.08 \times 10^4$ | $3.33 \times 10^5$ | $6.06 \times 10^6$ | $1.16 \times 10^7$ | $8.94 \times 10^6$ | $1.10 \times 10^6$ | $1.15 \times 10^7$ | $8.75 \times 10^5$ |
|      | Mean    | $3.58 \times 10^4$ | $4.75 \times 10^5$ | $6.13 \times 10^6$ | $8.46 \times 10^6$ | $2.86 \times 10^5$ | $1.72 \times 10^6$ | $1.40 \times 10^7$ | $2.06 \times 10^5$ |
|      | Rank    | 1                  | 2                  | 5                  | 8                  | 6                  | 4                  | 7                  | 3                  |
| F19  | Std     | $1.15 \times 10^4$ | $1.48 \times 10^4$ | $3.65 \times 10^8$ | $7.80 \times 10^6$ | $1.03 \times 10^6$ | $1.16 \times 10^4$ | $4.51 \times 10^6$ | $8.79 \times 10^5$ |
|      | Mean    | $1.39 \times 10^4$ | $1.82 \times 10^4$ | $2.49 \times 10^8$ | $3.94 \times 10^6$ | $4.74 \times 10^4$ | $1.91 \times 10^4$ | $4.17 \times 10^6$ | $2.43 \times 10^4$ |
|      | Rank    | 1                  | 3                  | 8                  | 7                  | 5                  | 2                  | 6                  | 4                  |
| F20  | Std     | $3.06 \times 10^2$ | $3.53 \times 10^2$ | $3.16 \times 10^2$ | $3.24 \times 10^2$ | $4.30 \times 10^2$ | $3.29 \times 10^2$ | $2.94 \times 10^2$ | $2.86 \times 10^2$ |
|      | Mean    | $3.00 \times 10^3$ | $3.56 \times 10^3$ | $3.60 \times 10^3$ | $3.19 \times 10^3$ | $3.59 \times 10^3$ | $3.43 \times 10^3$ | $3.65 \times 10^3$ | $3.14 \times 10^3$ |
|      | Rank    | 3                  | 7                  | 4                  | 5                  | 8                  | 6                  | 2                  | 1                  |
| F21  | Std     | $9.18 \times 10^2$ | $1.03 \times 10^3$ | $9.96 \times 10^2$ | $1.20 \times 10^3$ | $5.22 \times 10^2$ | $3.61 \times 10^2$ | $1.08 \times 10^3$ | $6.71 \times 10^2$ |
|      | Mean    | $3.86 \times 10^3$ | $6.63 \times 10^3$ | $5.49 \times 10^3$ | $6.66 \times 10^3$ | $6.38 \times 10^3$ | $5.92 \times 10^3$ | $6.88 \times 10^3$ | $4.85 \times 10^3$ |
|      | Rank    | 4                  | 6                  | 5                  | 8                  | 2                  | 1                  | 7                  | 3                  |
| F22  | Std     | $3.22 \times 10^3$ | $2.43 \times 10^3$ | $2.35 \times 10^3$ | $2.68 \times 10^3$ | $3.36 \times 10^3$ | $2.28 \times 10^3$ | $3.59 \times 10^3$ | $2.38 \times 10^3$ |
|      | Mean    | $2.92 \times 10^4$ | $2.18 \times 10^4$ | $3.52 \times 10^4$ | $3.37 \times 10^4$ | $3.16 \times 10^4$ | $2.15 \times 10^4$ | $3.11 \times 10^4$ | $2.05 \times 10^4$ |
|      | Rank    | 6                  | 4                  | 2                  | 5                  | 7                  | 1                  | 8                  | 3                  |
| F23  | Std     | $7.14 \times 10^2$ | $1.25 \times 10^3$ | $4.83 \times 10^2$ | $5.03 \times 10^2$ | $1.16 \times 10^3$ | $1.02 \times 10^3$ | $1.21 \times 10^3$ | $6.63 \times 10^2$ |
|      | Mean    | $4.68 \times 10^3$ | $7.31 \times 10^3$ | $1.00 \times 10^4$ | $7.74 \times 10^3$ | $9.98 \times 10^3$ | $6.44 \times 10^3$ | $1.03 \times 10^4$ | $6.96 \times 10^3$ |
|      | Rank    | 4                  | 8                  | 1                  | 2                  | 6                  | 5                  | 7                  | 3                  |
| F24  | Std     | $9.23 \times 10^2$ | $1.13 \times 10^3$ | $1.84 \times 10^2$ | $9.53 \times 10^2$ | $7.20 \times 10^2$ | $2.60 \times 10^2$ | $7.98 \times 10^2$ | $9.22 \times 10^2$ |
|      | Mean    | $4.66 \times 10^3$ | $6.99 \times 10^3$ | $9.28 \times 10^3$ | $8.55 \times 10^3$ | $7.94 \times 10^3$ | $8.00 \times 10^3$ | $8.53 \times 10^3$ | $6.05 \times 10^3$ |
|      | Rank    | 6                  | 8                  | 1                  | 7                  | 3                  | 2                  | 4                  | 5                  |
| F25  | Std     | $1.97 \times 10^1$ | $2.92 \times 10^1$ | $8.66 \times 10^2$ | $7.91 \times 10^2$ | $1.32 \times 10^2$ | $1.43 \times 10^1$ | $1.29 \times 10^2$ | $1.34 \times 10^2$ |
|      | Mean    | $3.02 \times 10^3$ | $3.03 \times 10^3$ | $8.71 \times 10^3$ | $5.43 \times 10^3$ | $3.27 \times 10^3$ | $3.09 \times 10^3$ | $3.38 \times 10^3$ | $3.19 \times 10^3$ |
|      | Rank    | 2                  | 3                  | 8                  | 7                  | 5                  | 1                  | 4                  | 6                  |
| F26  | Std     | $5.70 \times 10^2$ | $1.31 \times 10^3$ | $8.95 \times 10^2$ | $1.45 \times 10^3$ | $1.71 \times 10^3$ | $2.98 \times 10^3$ | $1.73 \times 10^3$ | $9.44 \times 10^2$ |
|      | Mean    | $5.54 \times 10^3$ | $7.91 \times 10^3$ | $1.26 \times 10^4$ | $1.03 \times 10^4$ | $1.29 \times 10^4$ | $9.70 \times 10^3$ | $1.34 \times 10^4$ | $8.55 \times 10^3$ |
|      | Rank    | 1                  | 4                  | 2                  | 5                  | 6                  | 8                  | 7                  | 3                  |
| F27  | Std     | $1.03 \times 10^2$ | $2.16 \times 10^2$ | $2.99 \times 10^2$ | $4.27 \times 10^2$ | $2.24 \times 10^2$ | $1.68 \times 10^2$ | $3.18 \times 10^2$ | $1.44 \times 10^2$ |
|      | Mean    | $3.41 \times 10^3$ | $3.52 \times 10^3$ | $4.57 \times 10^3$ | $4.27 \times 10^3$ | $4.02 \times 10^3$ | $3.58 \times 10^3$ | $4.43 \times 10^3$ | $3.64 \times 10^3$ |
|      | Rank    | 1                  | 4                  | 6                  | 8                  | 5                  | 3                  | 7                  | 2                  |
| F28  | Std     | $1.97 \times 10^1$ | $3.12 \times 10^1$ | $5.35 \times 10^2$ | $5.68 \times 10^2$ | $5.58 \times 10^2$ | $3.06 \times 10^1$ | $2.18 \times 10^2$ | $3.07 \times 10^2$ |
|      | Mean    | $3.30 \times 10^3$ | $3.31 \times 10^3$ | $7.23 \times 10^3$ | $5.92 \times 10^3$ | $3.56 \times 10^3$ | $3.34 \times 10^3$ | $4.06 \times 10^3$ | $3.60 \times 10^3$ |
|      | Rank    | 1                  | 3                  | 6                  | 8                  | 7                  | 2                  | 4                  | 5                  |
| F29  | Std     | $3.22 \times 10^2$ | $4.16 \times 10^2$ | $1.30 \times 10^3$ | $1.32 \times 10^3$ | $1.06 \times 10^3$ | $3.52 \times 10^2$ | $1.29 \times 10^3$ | $5.13 \times 10^2$ |
|      | Mean    | $4.36 \times 10^3$ | $4.93 \times 10^3$ | $7.92 \times 10^3$ | $6.65 \times 10^3$ | $6.99 \times 10^3$ | $4.74 \times 10^3$ | $8.01 \times 10^3$ | $4.86 \times 10^3$ |
|      | Rank    | 1                  | 3                  | 7                  | 8                  | 5                  | 2                  | 6                  | 4                  |
| F30  | Std     | $6.46 \times 10^5$ | $1.09 \times 10^6$ | $3.71 \times 10^8$ | $1.35 \times 10^8$ | $1.36 \times 10^7$ | $1.85 \times 10^5$ | $6.46 \times 10^7$ | $5.27 \times 10^6$ |
|      | Mean    | $1.36 \times 10^6$ | $2.11 \times 10^6$ | $7.88 \times 10^8$ | $2.69 \times 10^8$ | $1.32 \times 10^7$ | $1.04 \times 10^6$ | $1.47 \times 10^8$ | $6.01 \times 10^6$ |
|      | Rank    | 2                  | 3                  | 8                  | 7                  | 5                  | 1                  | 6                  | 4                  |

### 1. Unimodal function result analysis

The experimental results demonstrate a consistent conclusion at the quantitative level: the proposed ACWMA exhibits superior comprehensive performance over all compared algorithms in CEC 2017 unimodal function optimization tasks. Specifically, across the full-dimensional tests of 10 and 50 dimensions, the ACWMA achieved top-ranked optimization performance for functions f1 and f3, comprehensively outperforming seven mainstream algorithms including WMA, DBO, and GGO. Even in the 30-dimensional optimization

scenario for  $f_1$ , the algorithm maintained a position within the top three, while preserving optimal performance for  $f_3$  at 30 dimensions. Of particular significance, as the problem dimensionality increased from 10 to 50 dimensions, the ACWMA exhibited no performance degradation; conversely, it further widened the performance gap relative to comparative algorithms in high-dimensional scenarios. This characteristic distinctly differentiates the proposed algorithm from the common deficiencies observed in most algorithms, namely accuracy deterioration and stability reduction under high-dimensional conditions.

## 2. Multimodal function result analysis

To address the core scientific challenges in multimodal optimization problems—namely, the imbalance between exploration and exploitation, limited convergence efficiency, and attenuated population search capability—the proposed ACWMA achieves synergistic breakthroughs in optimization accuracy, convergence speed, and high-dimensional robustness across systematic tests on multimodal functions F4–F10 of the CEC 2017 benchmark suite at 10, 30, and 50 dimensions. Its comprehensive performance significantly surpasses that of mainstream algorithms such as WOA, PSO, and DBO. The core innovation of this algorithm lies in the construction of a three-layer collaborative optimization framework. First, through the coupling of an adaptive parameter switching mechanism with Lévy flight perturbations, dynamic adaptation between global exploration and local exploitation is realized, fundamentally mitigating local optima entrapment. Second, an optimal-value leadership strategy is introduced to enhance the directional guidance of elite individuals, substantially compressing ineffective iterative processes and improving convergence efficiency. Third, a team-based collaborative learning mechanism is designed based on a three-level fitness-gap model, establishing differentiated adaptive learning rules for the population to facilitate efficient information transfer among individuals and enhance overall population search capability. The organic integration of these three core strategies resolves the common dilemma faced by traditional metaheuristic algorithms in multimodal optimization, where it is difficult to simultaneously achieve precision, speed, and robustness, thereby providing a novel algorithmic paradigm for solving high-dimensional, multimodal complex optimization problems.

## 3. Hybrid function result analysis

The CEC 2017 hybrid functions construct integrated optimization problems through a multi-function coupling mechanism, accurately reproducing the inherent nonlinear, multi-constrained, and dimensionality-dependent complexity characteristics of real-world optimization problems, thereby establishing a scientific experimental framework for performance validation and practical value verification of various optimization algorithms. In multi-dimensional gradient experiments within this benchmark testing system, the ACWMA demonstrates significant performance advantages and cross-dimensional robustness. Under low-to-medium dimensional scenarios of 10 and 30 dimensions, the algorithm achieved optimal comprehensive performance across all benchmark test functions, with optimization accuracy and solution stability significantly surpassing those of comparative algorithms, validating its efficient solving capability for medium-low complexity optimization problems. For complex 50-dimensional scenarios, the performance advantages of the ACWMA become further pronounced: compared with seven mainstream competing algorithms represented by WMA, it maintains performance leadership across all benchmark functions, and the magnitude of performance degradation during dimensional scaling from low to high dimensions is substantially lower than that of peer algorithms. This fully substantiates its strong adaptability to high-dimensional complex optimization problems, providing an effective technical solution for solving high-dimensional optimization problems in engineering applications.

#### 4. Composition function result analysis

The composition functions in the CEC 2017 test suite serve as classical benchmarks for evaluating the comprehensive performance of optimization algorithms. Their complex characteristics of multimodality, strong nonlinearity, and high-dimensional coupling pose stringent challenges to the global exploration and local exploitation capabilities of algorithms. Comparative experiments were conducted on composition functions F20–F30 from this suite at 10, 30, and 50 dimensions. The results indicate that the optimization performance of the ACWMA significantly outperforms that of comparative algorithms, ranking first across all dimensional tests and demonstrating superior adaptability in high-dimensional scenarios. The algorithm entered the top three across all 11 test functions and achieved optimal solutions in nearly half of them, validating its strong adaptability and robustness to composition functions with diverse structures. These findings also demonstrate its effectiveness in handling high-dimensional complex composition function optimization problems, providing high-performance algorithmic support for solving complex optimization scenarios in practical engineering applications.

##### 4.2.2. Convergence Analysis of CEC 2017 Functions

Figures 4–6 present the convergence characteristic curves of selected benchmark functions (F1, F3, F5, F6, F17, F18, F26, and F27) from the CEC 2017 test suite under 10-, 30-, and 50-dimensional optimization spaces, respectively. Through theoretical analysis and experimental validation of the ACWMA convergence properties, it is evident that the algorithm demonstrates significantly superior capability in rapid approximation of global optimal solutions compared with contemporary comparative optimization algorithms. The superiority of its optimization performance is directly corroborated by the convergence rate and approximation accuracy observed in the convergence curves. From a mechanistic perspective, the exceptional global exploration capability and precise local search efficacy of the ACWMA fundamentally originate from its innovative optimization mechanism design: the organic coupling of an adaptive parameter switching mechanism with Lévy flight perturbation strategies achieves dynamic equilibrium between exploration and exploitation processes, effectively breaking through local optima constraints and theoretically suppressing premature convergence issues during the optimization process, thereby ensuring the integrity and stability of the search process.

The substantial improvement in convergence speed of the ACWMA relative to existing metaheuristic algorithms further validates the theoretical effectiveness of the optimal-value leadership strategy—this strategy theoretically shortens the algorithm's convergence cycle and accelerates the search process for global optimal solutions by enhancing the guiding role of elite individuals. Simultaneously, the team-based collaborative learning mechanism constructed upon a three-level fitness-gap model improves the overall population search efficiency from the perspective of population evolution theory through the design of adaptive learning rules, providing robust theoretical support for efficient algorithmic optimization.

Experimental results demonstrate that the ACWMA is an efficient, stable, and robust optimization method, particularly suitable for solving high-dimensional complex engineering optimization problems, exhibiting prominent engineering application advantages in hybrid function optimization scenarios. It possesses significant theoretical research value and broad prospects for practical engineering implementation.

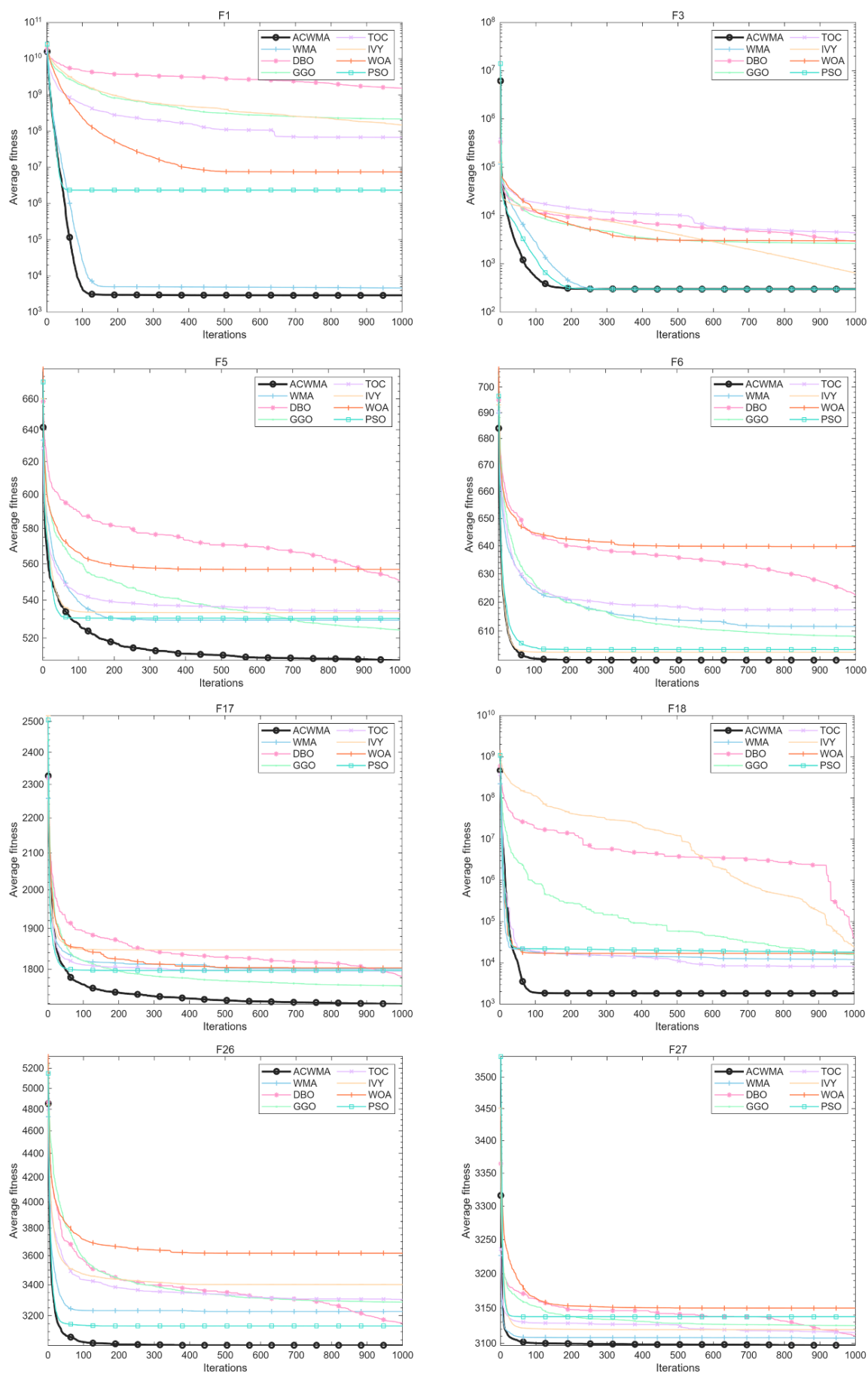


Figure 4. Convergence plots of selected test functions at dimension Dim = 10.



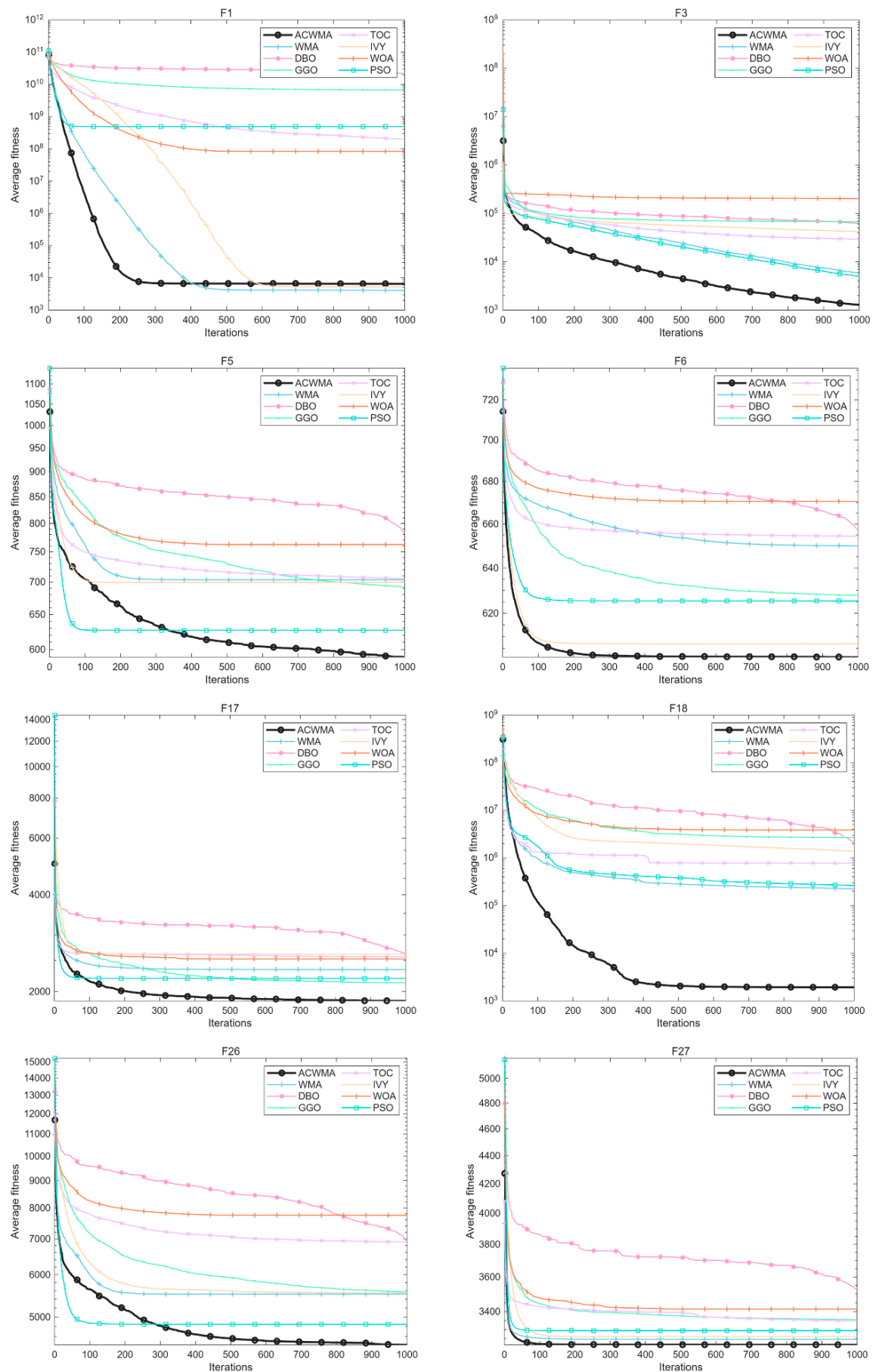


Figure 5. Convergence plots of selected test functions at dimension Dim = 30.

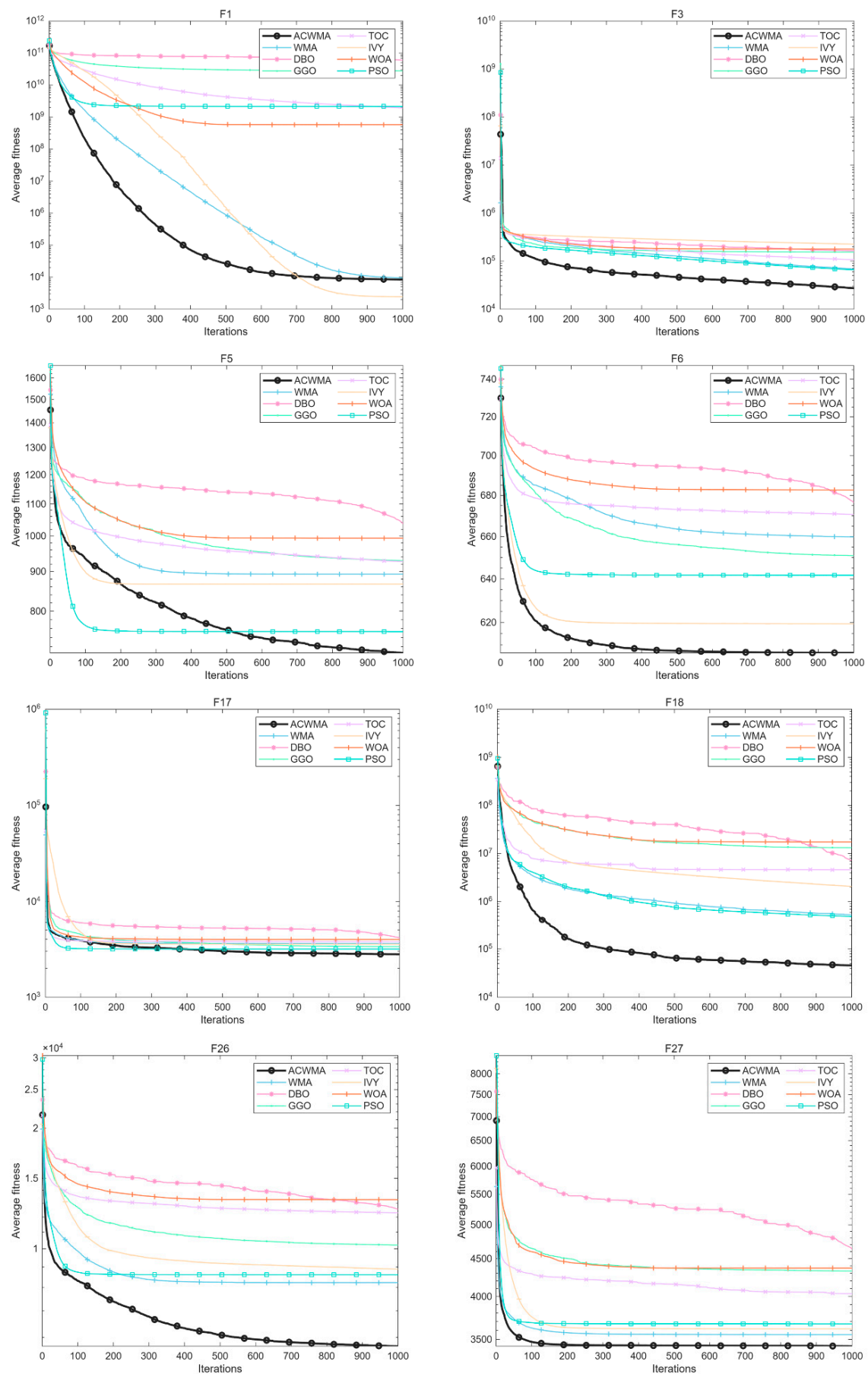


Figure 6. Convergence plots of selected test functions at dimension Dim = 50.

#### 4.2.3. Statistical Test Analysis of CEC 2017 Functions

To validate the optimization performance and statistical validity of the proposed ACWMA, this study employed the Wilcoxon rank-sum test to conduct cross-dimensional performance quantitative verification at 10, 30, and 50 dimensions against comparative algorithms including WMA, DBO, GGO, TOC, IVY, WOA, and PSO. This testing method requires no prior assumptions regarding data distribution characteristics, effectively circumventing the applicability limitations of parametric tests and providing objective and reliable statistical evidence for performance comparisons among different optimization algorithms. The research systematically analyzed the  $p$ -values and statistical significance results across all dimensional configurations, utilizing three categorical symbols ( $\uparrow$  means better,  $\downarrow$  means worse, and  $\approx$  means equal) to denote that the ACWMA achieved statistically significantly superior, significantly inferior, or statistically non-significant performance relative to comparative algorithms, respectively.

As evidenced by the test results presented in Tables 4–6, the ACWMA demonstrated significant optimization advantages across the 10-, 30-, and 50-dimensional benchmark function suites. In multi-dimensional testing scenarios, its performance achieved statistically significantly superior levels compared with all comparative algorithms, fully substantiating the algorithm's solution effectiveness, dimensional adaptability, and performance stability across complex optimization problems of varying dimensionality.

**Table 4.** Wilcoxon rank-sum test results (10 Dim).

| Func | ACWMA vs.<br>WMA                     | ACWMA vs.<br>DBO                     | ACWMA vs.<br>GGO                     | ACWMA vs.<br>TOC                     | ACWMA vs.<br>IVY                     | ACWMA vs.<br>WOA                     | ACWMA vs.<br>PSO                     |
|------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| F1   | $9.07 \times 10^{-3}$<br>$\uparrow$  | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $1.44 \times 10^{-2}$<br>$\uparrow$  | $3.02 \times 10^{-11}$<br>$\uparrow$ | $6.10 \times 10^{-1}$<br>$\approx$   |
| F3   | $1.33 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $1.31 \times 10^{-10}$<br>$\uparrow$ |
| F4   | $2.06 \times 10^{-1}$<br>$\approx$   | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $9.76 \times 10^{-10}$<br>$\uparrow$ | $5.49 \times 10^{-11}$<br>$\uparrow$ | $4.74 \times 10^{-10}$<br>$\uparrow$ |
| F5   | $1.86 \times 10^{-9}$<br>$\uparrow$  | $3.02 \times 10^{-11}$<br>$\uparrow$ | $1.96 \times 10^{-10}$<br>$\uparrow$ | $5.07 \times 10^{-10}$<br>$\uparrow$ | $4.61 \times 10^{-10}$<br>$\uparrow$ | $3.34 \times 10^{-11}$<br>$\uparrow$ | $3.16 \times 10^{-10}$<br>$\uparrow$ |
| F6   | $4.08 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $4.04 \times 10^{-1}$<br>$\approx$   | $3.02 \times 10^{-11}$<br>$\uparrow$ | $5.49 \times 10^{-11}$<br>$\uparrow$ |
| F7   | $4.62 \times 10^{-10}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $4.50 \times 10^{-11}$<br>$\uparrow$ | $3.82 \times 10^{-9}$<br>$\uparrow$  | $3.02 \times 10^{-11}$<br>$\uparrow$ | $8.99 \times 10^{-11}$<br>$\uparrow$ | $4.86 \times 10^{-3}$<br>$\uparrow$  |
| F8   | $8.14 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $1.61 \times 10^{-10}$<br>$\uparrow$ | $1.17 \times 10^{-9}$<br>$\uparrow$  | $1.69 \times 10^{-9}$<br>$\uparrow$  | $3.69 \times 10^{-11}$<br>$\uparrow$ | $6.50 \times 10^{-8}$<br>$\uparrow$  |
| F9   | $5.03 \times 10^{-10}$<br>$\uparrow$ | $2.84 \times 10^{-11}$<br>$\uparrow$ | $2.84 \times 10^{-11}$<br>$\uparrow$ | $2.84 \times 10^{-11}$<br>$\uparrow$ | $2.84 \times 10^{-11}$<br>$\uparrow$ | $2.84 \times 10^{-11}$<br>$\uparrow$ | $1.32 \times 10^{-9}$<br>$\uparrow$  |
| F10  | $1.96 \times 10^{-10}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $8.89 \times 10^{-10}$<br>$\uparrow$ | $6.70 \times 10^{-11}$<br>$\uparrow$ | $1.78 \times 10^{-10}$<br>$\uparrow$ | $8.15 \times 10^{-11}$<br>$\uparrow$ | $1.86 \times 10^{-9}$<br>$\uparrow$  |
| F11  | $5.49 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $4.08 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $7.38 \times 10^{-10}$<br>$\uparrow$ |
| F12  | $4.73 \times 10^{-1}$<br>$\approx$   | $3.02 \times 10^{-11}$<br>$\uparrow$ | $5.49 \times 10^{-11}$<br>$\uparrow$ | $1.27 \times 10^{-2}$<br>$\uparrow$  | $1.86 \times 10^{-9}$<br>$\uparrow$  | $5.57 \times 10^{-10}$<br>$\uparrow$ | $4.06 \times 10^{-2}$<br>$\uparrow$  |
| F13  | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ |
| F14  | $8.15 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ |
| F15  | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ |
| F16  | $5.19 \times 10^{-7}$<br>$\uparrow$  | $2.32 \times 10^{-6}$<br>$\uparrow$  | $1.32 \times 10^{-4}$<br>$\uparrow$  | $2.78 \times 10^{-7}$<br>$\uparrow$  | $8.89 \times 10^{-10}$<br>$\uparrow$ | $3.02 \times 10^{-11}$<br>$\uparrow$ | $5.86 \times 10^{-6}$<br>$\uparrow$  |

Table 4. Cont.

| Func | ACWMA vs.<br>WMA            | ACWMA vs.<br>DBO            | ACWMA vs.<br>GGO            | ACWMA vs.<br>TOC            | ACWMA vs.<br>IVY            | ACWMA vs.<br>WOA            | ACWMA vs.<br>PSO            |
|------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| F17  | $5.00 \times 10^{-9}$<br>↑  | $5.57 \times 10^{-10}$<br>↑ | $9.76 \times 10^{-10}$<br>↑ | $7.38 \times 10^{-10}$<br>↑ | $3.82 \times 10^{-10}$<br>↑ | $3.16 \times 10^{-10}$<br>↑ | $1.07 \times 10^{-9}$<br>↑  |
| F18  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ |
| F19  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ |
| F20  | $1.86 \times 10^{-9}$<br>↑  | $1.17 \times 10^{-9}$<br>↑  | $2.03 \times 10^{-9}$<br>↑  | $4.62 \times 10^{-10}$<br>↑ | $1.10 \times 10^{-8}$<br>↑  | $1.78 \times 10^{-10}$<br>↑ | $7.38 \times 10^{-10}$<br>↑ |
| F21  | $4.59 \times 10^{-2}$<br>↑  | $2.71 \times 10^{-2}$<br>↑  | $1.44 \times 10^{-2}$<br>↑  | $2.16 \times 10^{-3}$<br>↑  | $1.76 \times 10^{-1}$<br>≈  | $2.16 \times 10^{-3}$<br>↑  | $3.33 \times 10^{-2}$<br>↑  |
| F22  | $3.08 \times 10^{-8}$<br>↑  | $3.55 \times 10^{-1}$<br>≈  | $4.35 \times 10^{-5}$<br>↑  | $7.77 \times 10^{-9}$<br>↑  | $2.20 \times 10^{-7}$<br>↑  | $1.49 \times 10^{-6}$<br>↑  | $3.35 \times 10^{-8}$<br>↑  |
| F23  | $3.01 \times 10^{-7}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $1.29 \times 10^{-6}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $2.67 \times 10^{-9}$<br>↑  | $4.50 \times 10^{-11}$<br>↑ | $1.41 \times 10^{-9}$<br>↑  |
| F24  | $1.54 \times 10^{-1}$<br>≈  | $9.71 \times 10^{-1}$<br>≈  | $6.95 \times 10^{-1}$<br>≈  | $6.00 \times 10^{-1}$<br>≈  | $1.41 \times 10^{-1}$<br>≈  | $4.73 \times 10^{-1}$<br>≈  | $4.20 \times 10^{-2}$<br>↑  |
| F25  | $4.40 \times 10^{-1}$<br>≈  | $6.41 \times 10^{-10}$<br>↑ | $1.63 \times 10^{-8}$<br>↑  | $1.35 \times 10^{-3}$<br>↑  | $5.51 \times 10^{-3}$<br>↑  | $4.01 \times 10^{-6}$<br>↑  | $2.10 \times 10^{-1}$<br>≈  |
| F26  | $8.30 \times 10^{-1}$<br>≈  | $8.83 \times 10^{-7}$<br>↑  | $6.52 \times 10^{-7}$<br>↑  | $1.58 \times 10^{-4}$<br>↑  | $1.24 \times 10^{-3}$<br>↑  | $5.46 \times 10^{-6}$<br>↑  | $7.84 \times 10^{-1}$<br>≈  |
| F27  | $6.20 \times 10^{-4}$<br>↑  | $1.07 \times 10^{-7}$<br>↑  | $2.83 \times 10^{-8}$<br>↑  | $7.08 \times 10^{-8}$<br>↑  | $7.08 \times 10^{-8}$<br>↑  | $4.99 \times 10^{-9}$<br>↑  | $1.01 \times 10^{-8}$<br>↑  |
| F28  | $3.97 \times 10^{-1}$<br>≈  | $7.08 \times 10^{-2}$<br>≈  | $2.79 \times 10^{-5}$<br>↑  | $4.70 \times 10^{-3}$<br>↑  | $7.93 \times 10^{-1}$<br>≈  | $8.51 \times 10^{-3}$<br>↑  | $9.64 \times 10^{-1}$<br>≈  |
| F29  | $2.15 \times 10^{-6}$<br>↑  | $1.85 \times 10^{-8}$<br>↑  | $5.56 \times 10^{-4}$<br>↑  | $1.60 \times 10^{-7}$<br>↑  | $7.09 \times 10^{-8}$<br>↑  | $1.21 \times 10^{-10}$<br>↑ | $1.53 \times 10^{-5}$<br>↑  |
| F30  | $1.53 \times 10^{-2}$<br>↑  | $6.90 \times 10^{-4}$<br>↑  | $6.20 \times 10^{-4}$<br>↑  | $1.95 \times 10^{-3}$<br>↑  | $6.57 \times 10^{-2}$<br>≈  | $3.34 \times 10^{-3}$<br>↑  | $4.84 \times 10^{-2}$<br>↑  |

Table 5. Wilcoxon rank-sum test results (30 Dim).

| Func | ACWMA vs.<br>WMA            | ACWMA vs.<br>DBO            | ACWMA vs.<br>GGO            | ACWMA vs.<br>TOC            | ACWMA vs.<br>IVY            | ACWMA vs.<br>WOA            | ACWMA vs.<br>PSO            |
|------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| F1   | $4.29 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $7.28 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $1.30 \times 10^{-3}$<br>↑  |
| F3   | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $2.15 \times 10^{-10}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $2.44 \times 10^{-9}$<br>↑  |
| F4   | $8.42 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $1.41 \times 10^{-9}$<br>↑  | $2.84 \times 10^{-4}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.65 \times 10^{-8}$<br>↑  |
| F5   | $3.82 \times 10^{-9}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.82 \times 10^{-10}$<br>↑ | $1.46 \times 10^{-10}$<br>↑ | $4.18 \times 10^{-9}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $1.06 \times 10^{-3}$<br>↑  |
| F6   | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $2.84 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ |
| F7   | $3.96 \times 10^{-8}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $7.39 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $8.24 \times 10^{-2}$<br>≈  |
| F8   | $1.25 \times 10^{-7}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $7.77 \times 10^{-9}$<br>↑  | $1.31 \times 10^{-8}$<br>↑  | $7.69 \times 10^{-8}$<br>↑  | $5.57 \times 10^{-10}$<br>↑ | $5.37 \times 10^{-2}$<br>≈  |
| F9   | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ |
| F10  | $6.74 \times 10^{-6}$<br>↑  | $4.31 \times 10^{-8}$<br>↑  | $1.27 \times 10^{-2}$<br>↑  | $2.75 \times 10^{-3}$<br>↑  | $7.77 \times 10^{-9}$<br>↑  | $6.38 \times 10^{-3}$<br>↑  | $3.16 \times 10^{-10}$<br>↑ |

Table 5. Cont.

| Func | ACWMA vs.<br>WMA            | ACWMA vs.<br>DBO            | ACWMA vs.<br>GGO            | ACWMA vs.<br>TOC            | ACWMA vs.<br>IVY            | ACWMA vs.<br>WOA            | ACWMA vs.<br>PSO            |
|------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| F11  | $2.60 \times 10^{-5}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $1.07 \times 10^{-9}$<br>↑  | $7.30 \times 10^{-4}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $4.21 \times 10^{-2}$<br>↑  |
| F12  | $9.93 \times 10^{-2}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.69 \times 10^{-11}$<br>↑ | $5.49 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $5.97 \times 10^{-5}$<br>↑  |
| F13  | $3.50 \times 10^{-3}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $1.64 \times 10^{-5}$<br>↑  | $7.84 \times 10^{-1}$<br>≈  | $4.08 \times 10^{-11}$<br>↑ | $2.50 \times 10^{-3}$<br>↑  |
| F14  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $4.98 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $4.50 \times 10^{-11}$<br>↑ |
| F15  | $9.83 \times 10^{-8}$<br>↑  | $3.69 \times 10^{-11}$<br>↑ | $3.69 \times 10^{-11}$<br>↑ | $2.88 \times 10^{-6}$<br>↑  | $1.53 \times 10^{-5}$<br>↑  | $8.99 \times 10^{-11}$<br>↑ | $1.19 \times 10^{-6}$<br>↑  |
| F16  | $3.09 \times 10^{-6}$<br>↑  | $3.69 \times 10^{-11}$<br>↑ | $4.69 \times 10^{-8}$<br>↑  | $2.44 \times 10^{-9}$<br>↑  | $6.72 \times 10^{-10}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $1.89 \times 10^{-4}$<br>↑  |
| F17  | $2.37 \times 10^{-10}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $5.46 \times 10^{-9}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.34 \times 10^{-11}$<br>↑ | $2.38 \times 10^{-7}$<br>↑  |
| F18  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.34 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ |
| F19  | $3.01 \times 10^{-7}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.34 \times 10^{-11}$<br>↑ | $3.83 \times 10^{-6}$<br>↑  | $2.88 \times 10^{-6}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $2.03 \times 10^{-7}$<br>↑  |
| F20  | $1.46 \times 10^{-10}$<br>↑ | $2.37 \times 10^{-10}$<br>↑ | $6.12 \times 10^{-10}$<br>↑ | $3.34 \times 10^{-11}$<br>↑ | $1.33 \times 10^{-10}$<br>↑ | $4.08 \times 10^{-11}$<br>↑ | $1.70 \times 10^{-8}$<br>↑  |
| F21  | $4.64 \times 10^{-3}$<br>↑  | $2.77 \times 10^{-1}$<br>≈  | $8.77 \times 10^{-2}$<br>≈  | $2.81 \times 10^{-2}$<br>↑  | $9.52 \times 10^{-4}$<br>↑  | $1.73 \times 10^{-7}$<br>↑  | $7.17 \times 10^{-1}$<br>≈  |
| F22  | $5.75 \times 10^{-2}$<br>≈  | $3.69 \times 10^{-11}$<br>↑ | $6.01 \times 10^{-8}$<br>↑  | $9.83 \times 10^{-8}$<br>↑  | $7.01 \times 10^{-2}$<br>≈  | $9.21 \times 10^{-5}$<br>↑  | $2.16 \times 10^{-3}$<br>↑  |
| F23  | $8.99 \times 10^{-11}$<br>↑ | $1.86 \times 10^{-9}$<br>↑  | $6.72 \times 10^{-10}$<br>↑ | $1.78 \times 10^{-10}$<br>↑ | $2.67 \times 10^{-9}$<br>↑  | $4.20 \times 10^{-10}$<br>↑ | $1.87 \times 10^{-7}$<br>↑  |
| F24  | $6.70 \times 10^{-11}$<br>↑ | $2.44 \times 10^{-9}$<br>↑  | $9.03 \times 10^{-4}$<br>↑  | $4.62 \times 10^{-10}$<br>↑ | $5.07 \times 10^{-10}$<br>↑ | $3.20 \times 10^{-9}$<br>↑  | $4.43 \times 10^{-3}$<br>↑  |
| F25  | $3.78 \times 10^{-2}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $6.53 \times 10^{-7}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ |
| F26  | $1.36 \times 10^{-7}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.82 \times 10^{-10}$<br>↑ | $4.74 \times 10^{-6}$<br>↑  | $2.61 \times 10^{-2}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $5.55 \times 10^{-2}$<br>≈  |
| F27  | $1.43 \times 10^{-5}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $5.49 \times 10^{-11}$<br>↑ | $1.70 \times 10^{-8}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $1.70 \times 10^{-8}$<br>↑  |
| F28  | $1.33 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $1.16 \times 10^{-7}$<br>↑  | $5.20 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $6.01 \times 10^{-8}$<br>↑  |
| F29  | $3.65 \times 10^{-8}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $1.09 \times 10^{-10}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $4.18 \times 10^{-9}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $9.26 \times 10^{-9}$<br>↑  |
| F30  | $3.56 \times 10^{-4}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $1.21 \times 10^{-10}$<br>↑ | $4.64 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $7.74 \times 10^{-6}$<br>↑  |

Table 6. Wilcoxon rank-sum test results (50 Dim).

| Func | ACWMA vs.<br>WMA            | ACWMA vs.<br>DBO            | ACWMA vs.<br>GGO            | ACWMA vs.<br>TOC            | ACWMA vs.<br>IVY            | ACWMA vs.<br>WOA            | ACWMA vs.<br>PSO            |
|------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| F1   | $3.87 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $2.25 \times 10^{-4}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ |
| F3   | $3.69 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $2.60 \times 10^{-8}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $6.07 \times 10^{-11}$<br>↑ |
| F4   | $6.35 \times 10^{-2}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $4.64 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $6.72 \times 10^{-10}$<br>↑ |

Table 6. Cont.

| Func | ACWMA vs.<br>WMA            | ACWMA vs.<br>DBO            | ACWMA vs.<br>GGO            | ACWMA vs.<br>TOC            | ACWMA vs.<br>IVY            | ACWMA vs.<br>WOA            | ACWMA vs.<br>PSO            |
|------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| F5   | $3.08 \times 10^{-8}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.34 \times 10^{-11}$<br>↑ | $6.12 \times 10^{-10}$<br>↑ | $4.62 \times 10^{-10}$<br>↑ | $3.34 \times 10^{-11}$<br>↑ | $1.22 \times 10^{-2}$<br>↑  |
| F6   | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $5.49 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.34 \times 10^{-11}$<br>↑ |
| F7   | $2.60 \times 10^{-8}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $6.38 \times 10^{-3}$<br>↑  |
| F8   | $2.61 \times 10^{-10}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $2.37 \times 10^{-10}$<br>↑ | $4.20 \times 10^{-10}$<br>↑ | $3.20 \times 10^{-9}$<br>↑  | $1.78 \times 10^{-10}$<br>↑ | $3.11 \times 10^{-1}$<br>≈  |
| F9   | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ |
| F10  | $2.23 \times 10^{-9}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $1.17 \times 10^{-4}$<br>↑  | $7.62 \times 10^{-1}$<br>≈  | $1.41 \times 10^{-9}$<br>↑  | $5.59 \times 10^{-1}$<br>≈  | $2.37 \times 10^{-10}$<br>↑ |
| F11  | $8.65 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $1.87 \times 10^{-7}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $4.57 \times 10^{-9}$<br>↑  |
| F12  | $8.88 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $4.98 \times 10^{-4}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $4.50 \times 10^{-11}$<br>↑ |
| F13  | $1.17 \times 10^{-5}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $9.93 \times 10^{-2}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $2.03 \times 10^{-9}$<br>↑  |
| F14  | $3.69 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $6.70 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.34 \times 10^{-11}$<br>↑ |
| F15  | $5.46 \times 10^{-6}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $1.11 \times 10^{-3}$<br>↑  | $3.99 \times 10^{-4}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $7.30 \times 10^{-4}$<br>↑  |
| F16  | $2.13 \times 10^{-4}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $1.86 \times 10^{-9}$<br>↑  | $2.60 \times 10^{-8}$<br>↑  | $6.36 \times 10^{-5}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $5.94 \times 10^{-2}$<br>≈  |
| F17  | $4.57 \times 10^{-9}$<br>↑  | $3.69 \times 10^{-11}$<br>↑ | $7.09 \times 10^{-8}$<br>↑  | $5.07 \times 10^{-10}$<br>↑ | $2.60 \times 10^{-8}$<br>↑  | $1.09 \times 10^{-10}$<br>↑ | $1.53 \times 10^{-5}$<br>↑  |
| F18  | $3.69 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.47 \times 10^{-10}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $1.41 \times 10^{-9}$<br>↑  |
| F19  | $3.18 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $5.61 \times 10^{-5}$<br>↑  | $2.77 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $6.10 \times 10^{-3}$<br>↑  |
| F20  | $1.85 \times 10^{-8}$<br>↑  | $7.12 \times 10^{-9}$<br>↑  | $1.44 \times 10^{-3}$<br>↑  | $3.35 \times 10^{-8}$<br>↑  | $2.88 \times 10^{-6}$<br>↑  | $1.86 \times 10^{-9}$<br>↑  | $3.51 \times 10^{-2}$<br>↑  |
| F21  | $3.20 \times 10^{-9}$<br>↑  | $2.00 \times 10^{-5}$<br>↑  | $4.11 \times 10^{-7}$<br>↑  | $9.92 \times 10^{-11}$<br>↑ | $5.97 \times 10^{-9}$<br>↑  | $1.46 \times 10^{-10}$<br>↑ | $5.57 \times 10^{-3}$<br>↑  |
| F22  | $2.39 \times 10^{-8}$<br>↑  | $1.78 \times 10^{-10}$<br>↑ | $8.89 \times 10^{-10}$<br>↑ | $5.32 \times 10^{-3}$<br>↑  | $8.48 \times 10^{-9}$<br>↑  | $1.38 \times 10^{-2}$<br>↑  | $5.97 \times 10^{-9}$<br>↑  |
| F23  | $6.12 \times 10^{-10}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $5.53 \times 10^{-8}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $6.12 \times 10^{-10}$<br>↑ |
| F24  | $1.43 \times 10^{-8}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $1.33 \times 10^{-10}$<br>↑ | $8.99 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $4.71 \times 10^{-4}$<br>↑  |
| F25  | $6.10 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $6.07 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $1.21 \times 10^{-10}$<br>↑ |
| F26  | $1.46 \times 10^{-10}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.69 \times 10^{-11}$<br>↑ | $7.66 \times 10^{-5}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $5.49 \times 10^{-11}$<br>↑ |
| F27  | $1.60 \times 10^{-3}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.34 \times 10^{-11}$<br>↑ | $4.98 \times 10^{-11}$<br>↑ | $4.42 \times 10^{-6}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $7.12 \times 10^{-9}$<br>↑  |
| F28  | $5.79 \times 10^{-1}$<br>≈  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $4.44 \times 10^{-7}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.34 \times 10^{-11}$<br>↑ |
| F29  | $1.86 \times 10^{-6}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $4.08 \times 10^{-5}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $4.11 \times 10^{-7}$<br>↑  |
| F30  | $3.83 \times 10^{-6}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $3.02 \times 10^{-11}$<br>↑ | $3.34 \times 10^{-11}$<br>↑ | $6.38 \times 10^{-3}$<br>↑  | $3.02 \times 10^{-11}$<br>↑ | $4.50 \times 10^{-11}$<br>↑ |



#### 4.3. Ablation Study of Improvement Strategies

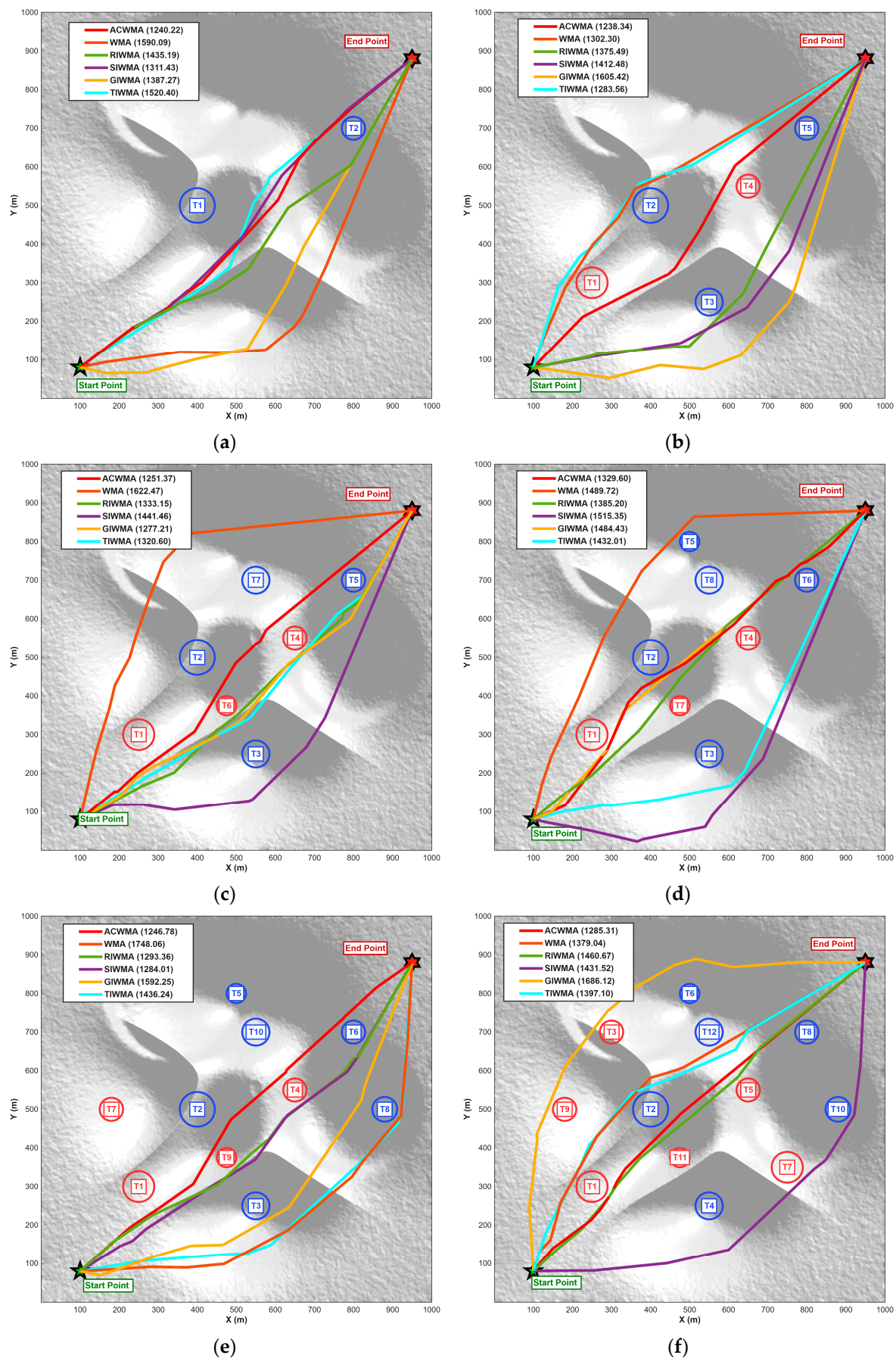
To validate the effectiveness of the proposed ACWMA, comparative experiments were conducted against four strategic variants: the random walk strategy (RIWMA), spiral flight strategy (TIWMA), Gaussian random walk strategy (GIWMA), and triangular walk strategy (SIWMA). All comparative algorithms uniformly employed the team-based collaborative learning mechanism and an identical path planning framework, ensuring experimental fairness and result reliability.

##### 4.3.1. Statistical Results Analysis of Cost Functions for Improvement Strategies

Under the unified team-based collaborative learning mechanism and identical path planning framework, comparative experiments were conducted among the original WMA, four single-strategy improved algorithms, and the ACWMA, as illustrated in Figure 7. All algorithms maintained consistent mechanisms with variations existing solely in search strategies, enabling objective assessment of the actual impact of different strategies on path planning performance. The experimental results in Table 7 demonstrate that ACWMA outperforms WMA, RIWMA, TIWMA, GIWMA, and SIWMA in terms of path cost, convergence speed, and operational stability, exhibiting stronger potential for practical application. Specifically, RIWMA can expand the search range but lacks smoothness; TIWMA fails to enhance planning effectiveness; GIWMA improves path stability yet is prone to local optima entrapment; SIWMA demonstrates certain adaptability to complex environments but with limited generalization capability. In contrast, ACWMA achieves comprehensive improvements in exploration, exploitation, and adaptability through synergistic fusion of multiple strategies under the unified mechanism, demonstrating more reliable and superior path planning performance. These findings establish a solid foundation for subsequent research and application of 3D UUV path planning in typical marine operating conditions.

**Table 7.** Cost function results analysis for path planning of all algorithms.

| Scenario |      | ACWMA    | WMA      | RIWMA    | TIWMA    | SIWMA    | GIWMA    |
|----------|------|----------|----------|----------|----------|----------|----------|
| 1        | Mean | 1302.857 | 1598.749 | 1477.720 | 1419.435 | 1433.686 | 1549.575 |
|          | Std  | 144.696  | 32.316   | 97.055   | 173.451  | 125.602  | 76.102   |
|          | Rank | 1        | 4        | 3        | 6        | 2        | 5        |
| 2        | Mean | 1301.298 | 1319.472 | 1427.261 | 1578.204 | 1639.666 | 1326.433 |
|          | Std  | 131.624  | 73.951   | 154.463  | 196.543  | 84.597   | 119.025  |
|          | Rank | 1        | 3        | 2        | 5        | 4        | 6        |
| 3        | Mean | 1346.800 | 1641.731 | 1413.708 | 1524.616 | 1375.699 | 1368.068 |
|          | Std  | 205.959  | 91.718   | 115.114  | 124.217  | 167.784  | 135.954  |
|          | Rank | 1        | 4        | 2        | 6        | 5        | 3        |
| 4        | Mean | 1447.842 | 1515.808 | 1436.501 | 1572.934 | 1530.751 | 1463.281 |
|          | Std  | 150.214  | 116.792  | 134.694  | 90.584   | 133.721  | 135.798  |
|          | Rank | 1        | 3        | 2        | 5        | 4        | 6        |
| 5        | Mean | 1356.333 | 1757.279 | 1405.739 | 1398.241 | 1655.753 | 1478.722 |
|          | Std  | 217.340  | 44.863   | 190.523  | 157.736  | 125.749  | 91.451   |
|          | Rank | 1        | 5        | 3        | 4        | 2        | 6        |
| 6        | Mean | 1370.319 | 1393.271 | 1496.407 | 1464.716 | 1727.658 | 1485.268 |
|          | Std  | 128.657  | 62.170   | 101.673  | 91.739   | 144.368  | 215.867  |
|          | Rank | 1        | 2        | 3        | 5        | 4        | 6        |



**Figure 7.** 3D top views of planned paths for each algorithm: (a) Scenario 1; (b) Scenario 2; (c) Scenario 3; (d) Scenario 4; (e) Scenario 5; (f) Scenario 6.

#### 4.3.2. Convergence Analysis of Improvement Strategies

Quantitative analysis of experimental data and convergence curves in Figure 8 reveals that the core advantages of ACWMA are manifested in the synergistic enhancement of convergence speed, solution accuracy, and robustness, demonstrating the rationality and innovation of its mechanism design. From the perspective of convergence characteristics, ACWMA can guide the population to rapidly converge to high-quality solution spaces at the early iteration stage, entering the efficient exploitation phase earlier than the original WMA and its variants. Throughout the entire iterative cycle, fitness values decrease steadily without significant stagnation, fluctuation, or premature convergence, exhibiting excellent stability. Local magnification analysis of curves in the later iteration stage indicates that ACWMA continues to exploit superior solutions, whereas comparative algorithms exhibit varying degrees of convergence stagnation, highlighting its advantage in escaping local optima and achieving global optimization. Quantitative evaluation demonstrates that ACWMA achieves the lowest optimal fitness values across all six scenarios; even in Scenario 5 where it approaches RIWMA, it forms an advantage through shorter convergence time, corroborating its global optimization capability. These results validate the effectiveness of ACWMA's adaptive collaborative weighting mechanism, indicating that it is not a simple superposition of strategies but rather achieves strategy complementarity through weight regulation, balancing exploration and exploitation capabilities.

In summary, ACWMA breaks through the performance limitations of traditional WMA and its variants, exhibiting strong practicality and robustness in complex optimization problems such as path planning, and provides feasible technical pathways and theoretical support for metaheuristic algorithm improvement and engineering applications.

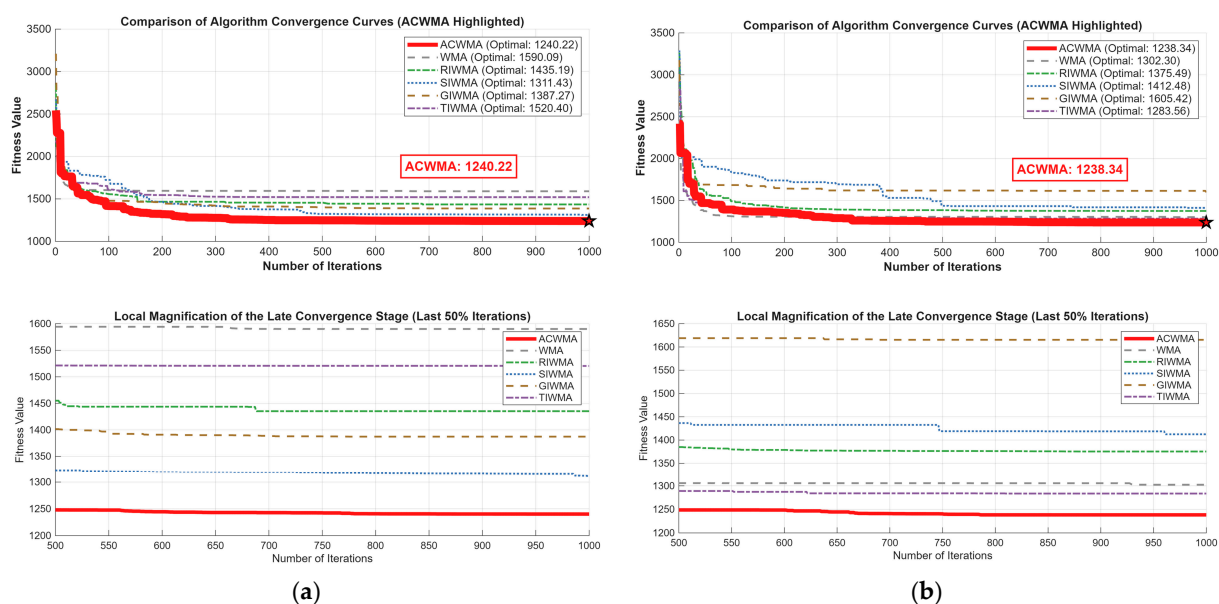
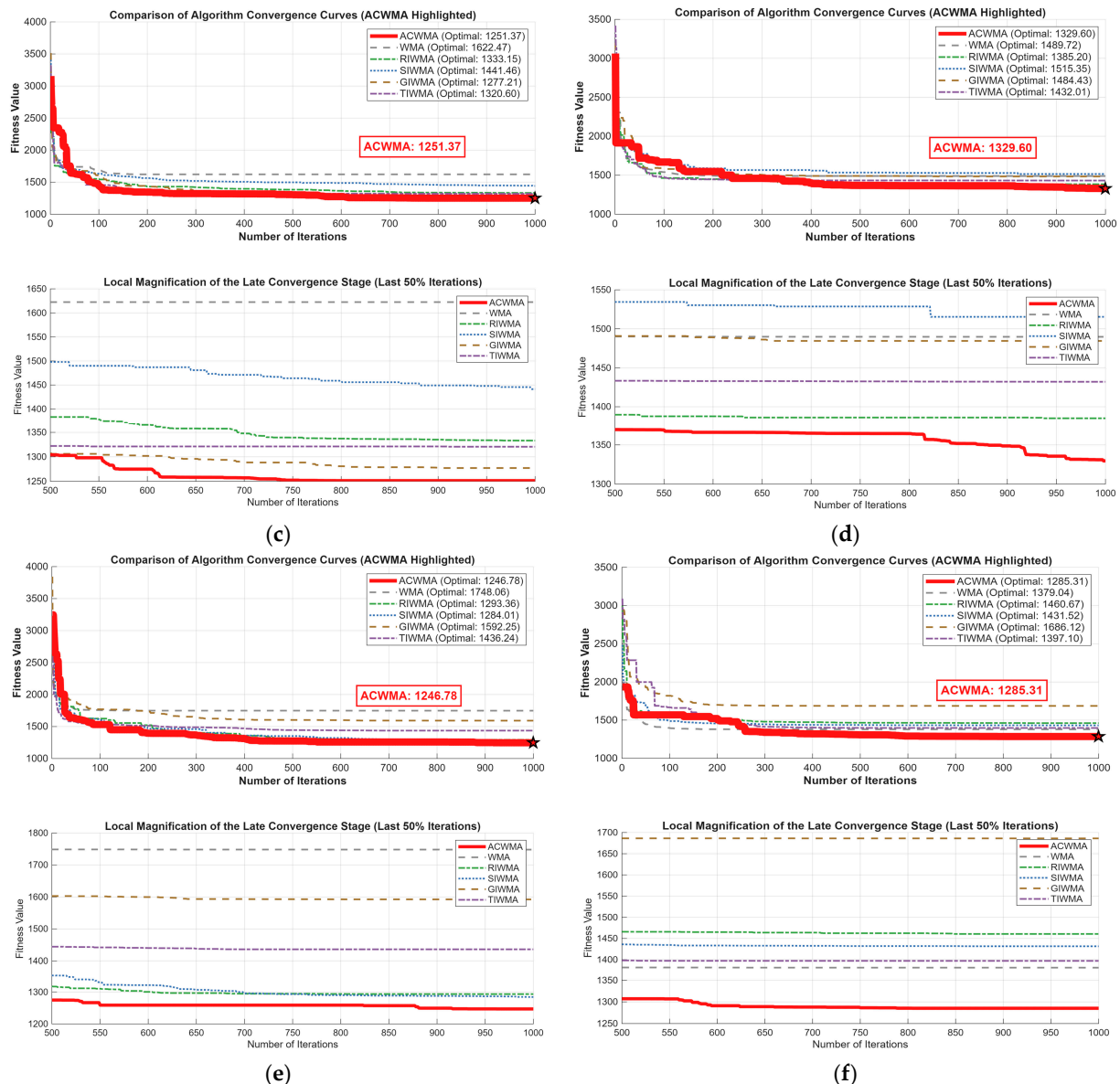


Figure 8. Cont.



**Figure 8.** Convergence curve comparison of improved strategy algorithms: (a) Scenario 1; (b) Scenario 2; (c) Scenario 3; (d) Scenario 4; (e) Scenario 5; (f) Scenario 6.

#### 4.4. 3D Path Planning Simulation Experiments for UUVs in Typical Marine Operating Conditions

To address the severe challenges posed by typical marine operating conditions to UUV path planning, in this paper, three types of differentiated experimental scenarios (odd obstacles, even obstacles, and dynamically moving obstacle scenarios) are designed to systematically verify the global search capability and optimization performance of the ACWMA in 3D UUV path planning, objectively revealing its technical advantages relative to comparative algorithms. During the path search phase, all candidate paths were uniformly encoded as parameter vectors comprising magnitude, elevation angle, and azimuth angle, with global optimization conducted within the configuration space to ultimately obtain the navigation path that minimizes the total cost function. To ensure the rigor and reliability of experimental conclusions, the mean and standard deviation of the total cost function are adopted as the core quantitative evaluation indicators for statistical result analysis, thereby validating the effectiveness and superiority of the ACWMA in 3D UUV path planning tasks from multiple dimensions.



#### 4.4.1. Experiments in Scenario A

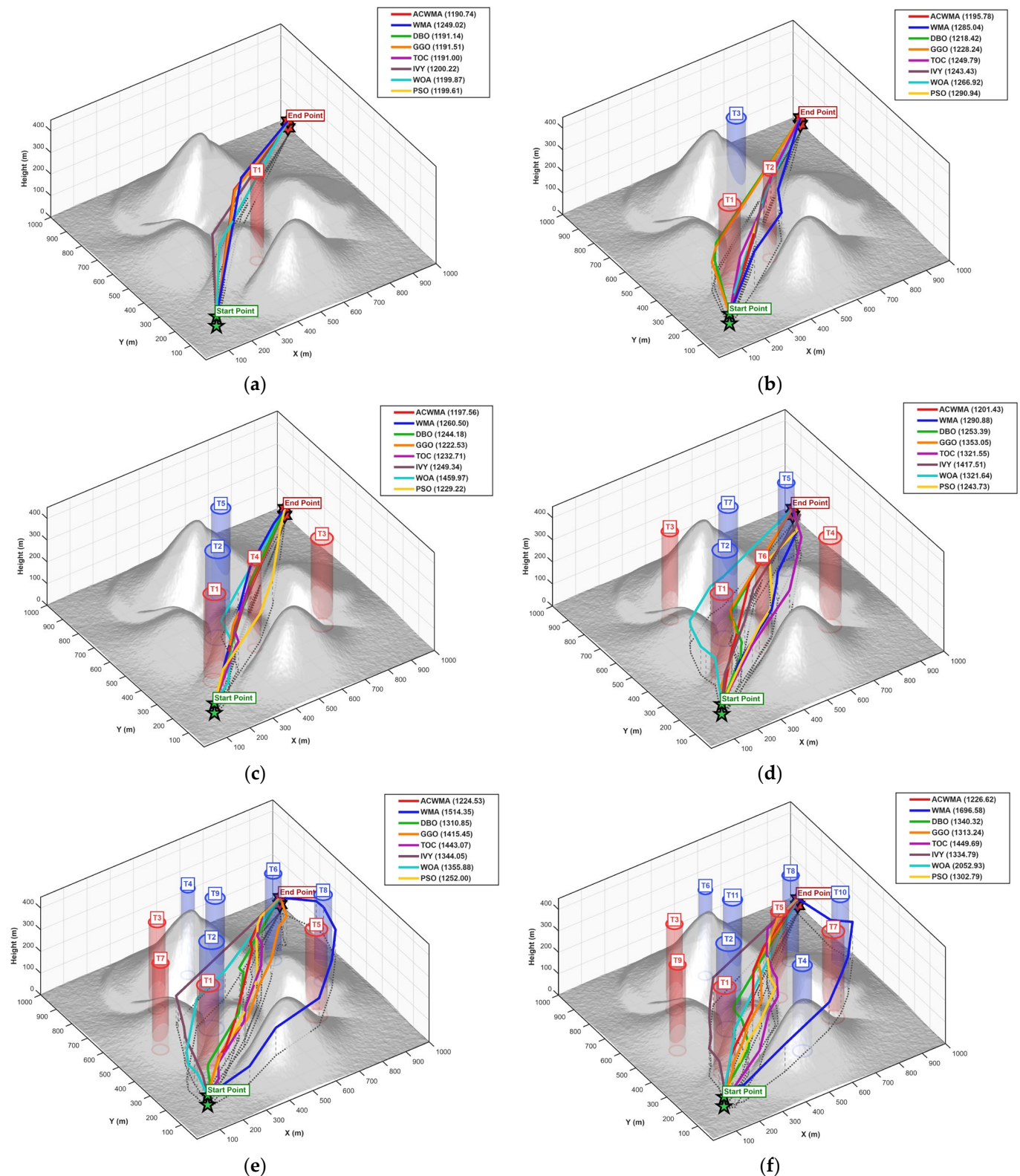
Figure 9 illustrates the path planning performance of different algorithms across various scenarios. It can be observed that in simple Scenario 1, the performance differences among all tested algorithms are minimal and can be considered functionally equivalent. In Scenarios 2 and 3, the ACWMA demonstrates significantly superior path quality compared with traditional WOA and PSO algorithms. Specifically, traditional WOA and PSO algorithms are susceptible to local optima influence, resulting in limited search capabilities confined to the neighborhood of local optima and an inability to achieve global optimal path search. In complex Scenarios 4–6, the ACWMA exhibits exceptional global exploration capability and precise local search performance, an advantage attributable to its improved optimization mechanism. In contrast, algorithms including WMA, DBO, GGO, TOC, and IVY possess inherent deficiencies: WMA suffers from limited global search capability and premature convergence issues; DBO and GGO algorithms exhibit slow convergence speeds and weak robustness in complex environments; TOC and IVY algorithms struggle to balance global exploration and local exploitation due to insufficient local search precision. Experimental results confirm that the ACWMA can generate superior-quality path planning solutions, with its core advantages in global exploration and local fine-grained search fully validated.

Figure 10 presents the top views of navigation paths generated by each algorithm under six application scenarios, intuitively demonstrating the performance differences between the ACWMA and other comparative algorithms in path planning. Analysis indicates that all algorithms satisfy the safety and engineering feasibility requirements of collision-free paths and navigation depths within threshold constraints, whereas the ACWMA exhibits distinctly superior comprehensive performance. This algorithm efficiently generates globally optimal navigation paths, demonstrating significant advantages in planning precision and global optimization capability, further validating its superiority in path planning and is consistent with the analytical results regarding global exploration and local fine-grained search performance presented previously.

Figure 11 displays the best convergence curves of each algorithm during the iterative process across different application scenarios. The convergence curves of the ACWMA clearly demonstrate superior convergence performance, indicating that the designed adaptive parameter switching mechanism, coupled with Lévy flight perturbations, effectively balances the exploration and exploitation capabilities of the algorithm and circumvents local optima traps. Except for Scenario 1, observation of the locally magnified convergence curves in the later stages (last 50% iterations) reveals that the ACWMA exhibits significantly enhanced convergence speed compared with other metaheuristic algorithms. This effectively validates that the introduction of the optimal-value leadership strategy strengthens the guiding role of elite individuals and accelerates algorithmic convergence speed. Furthermore, the constructed team-based collaborative learning mechanism, which designs adaptive learning rules based on a three-level fitness-gap model, effectively enhances population search efficiency.

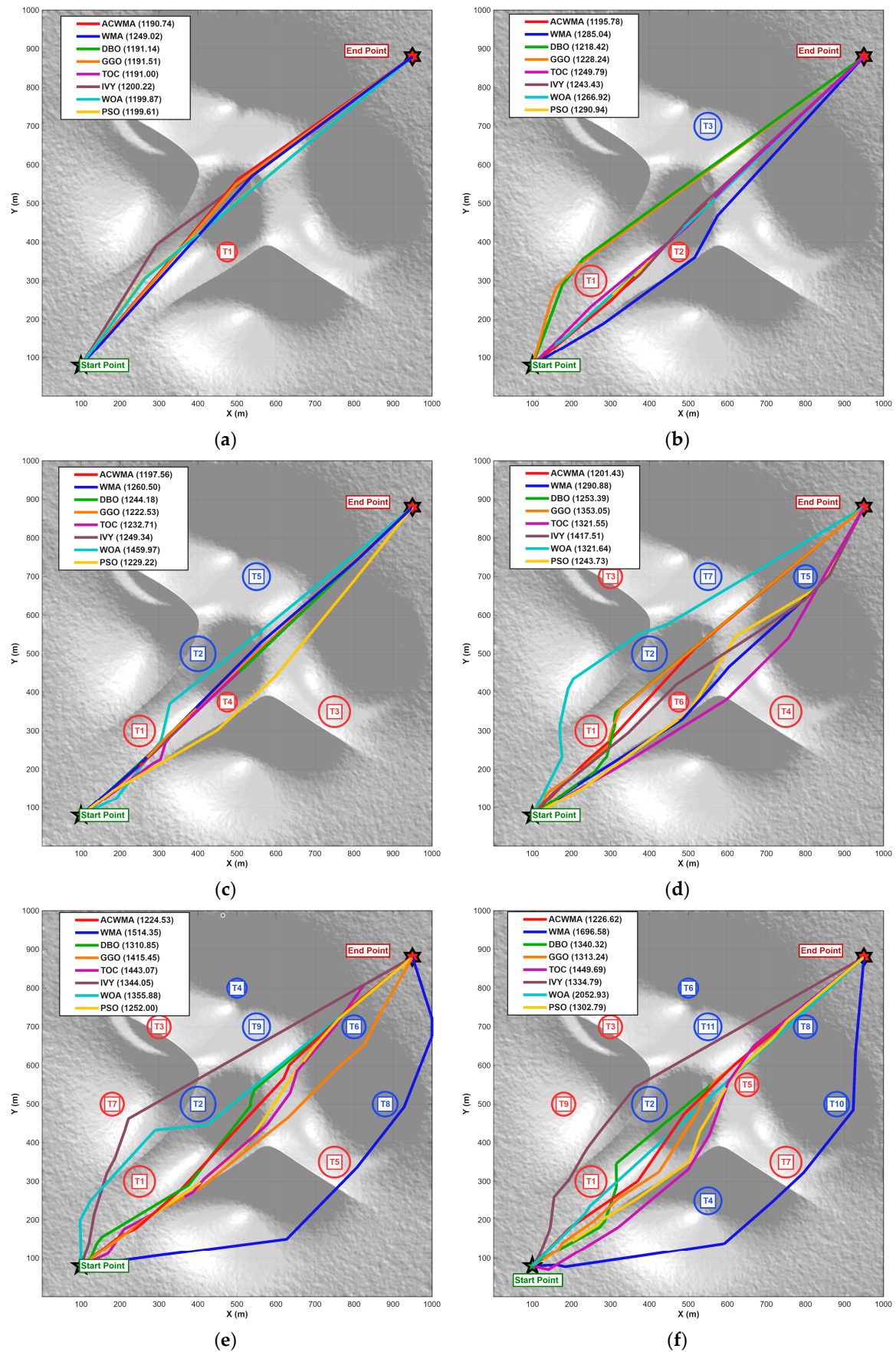
Statistical data from Table 8 reveal that in simple application Scenario 1, all comparative algorithms can generate navigation paths of relatively high quality, with no statistically significant performance differences between the ACWMA and other comparative algorithms. In application Scenarios 2 and 3, the comprehensive performance of the ACWMA is significantly superior to that of novel algorithms (GGO, TOC, IVY, etc.) and traditional metaheuristic algorithms (WOA, PSO), demonstrating certain performance advantages. In complex application Scenarios 4, 5, and 6, the performance differences between the ACWMA and the WMA are 89.45, 289.82, and 469.96, respectively, with the differences exhibiting a significant increasing trend as scenario complexity escalates. This fully in-

indicates that the performance advantages of the ACWMA become progressively more pronounced with increasing environmental complexity, enabling effective planning of navigation paths closer to global optimal solutions in typical marine operating conditions and further validating its superiority under complex constraint scenarios.

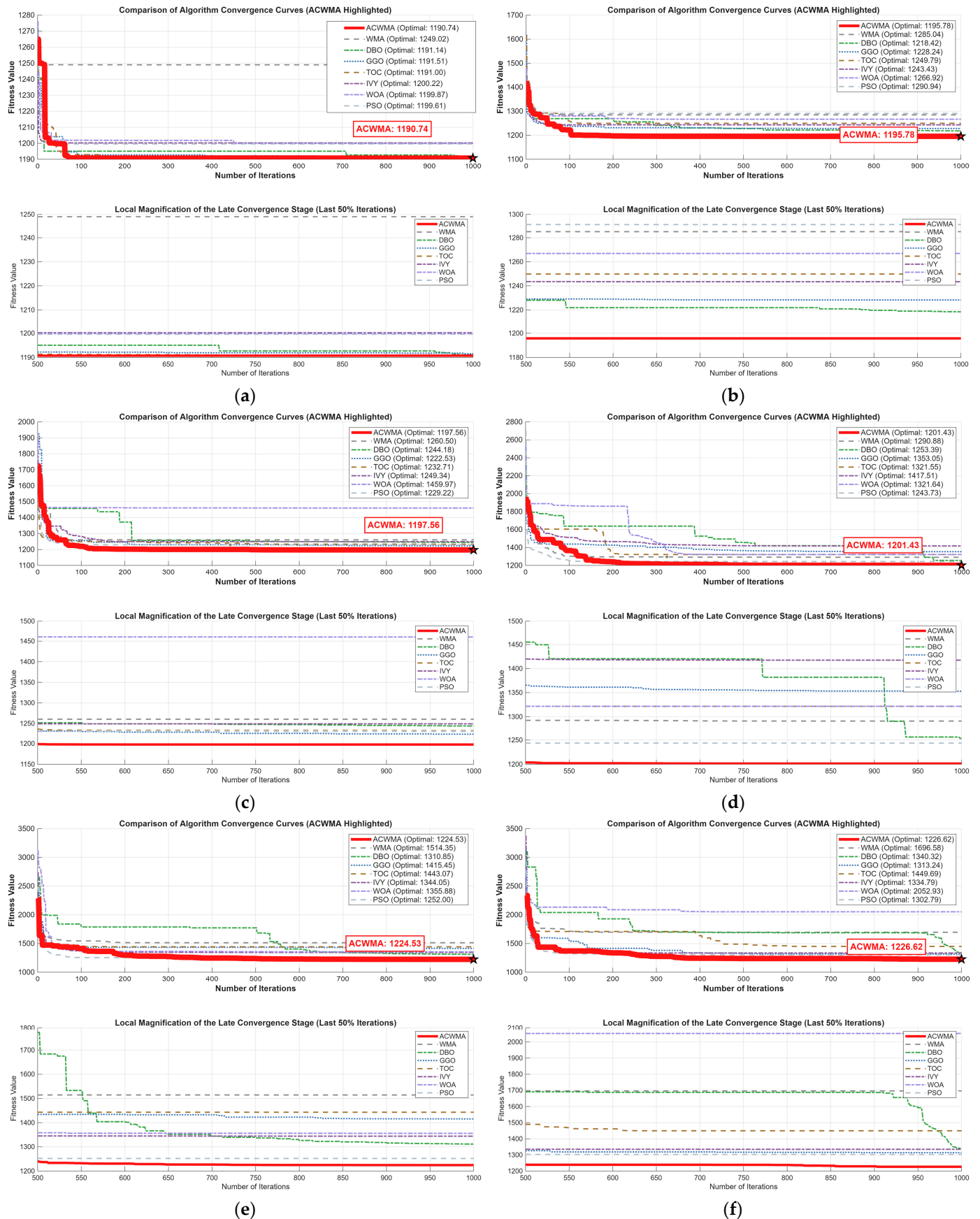


**Figure 9.** 3D views of planned paths for each algorithm: (a) Scenario 1; (b) Scenario 2; (c) Scenario 3; (d) Scenario 4; (e) Scenario 5; (f) Scenario 6.





**Figure 10.** Top views of planned paths for each algorithm: (a) Scenario 1; (b) Scenario 2; (c) Scenario 3; (d) Scenario 4; (e) Scenario 5; (f) Scenario 6.

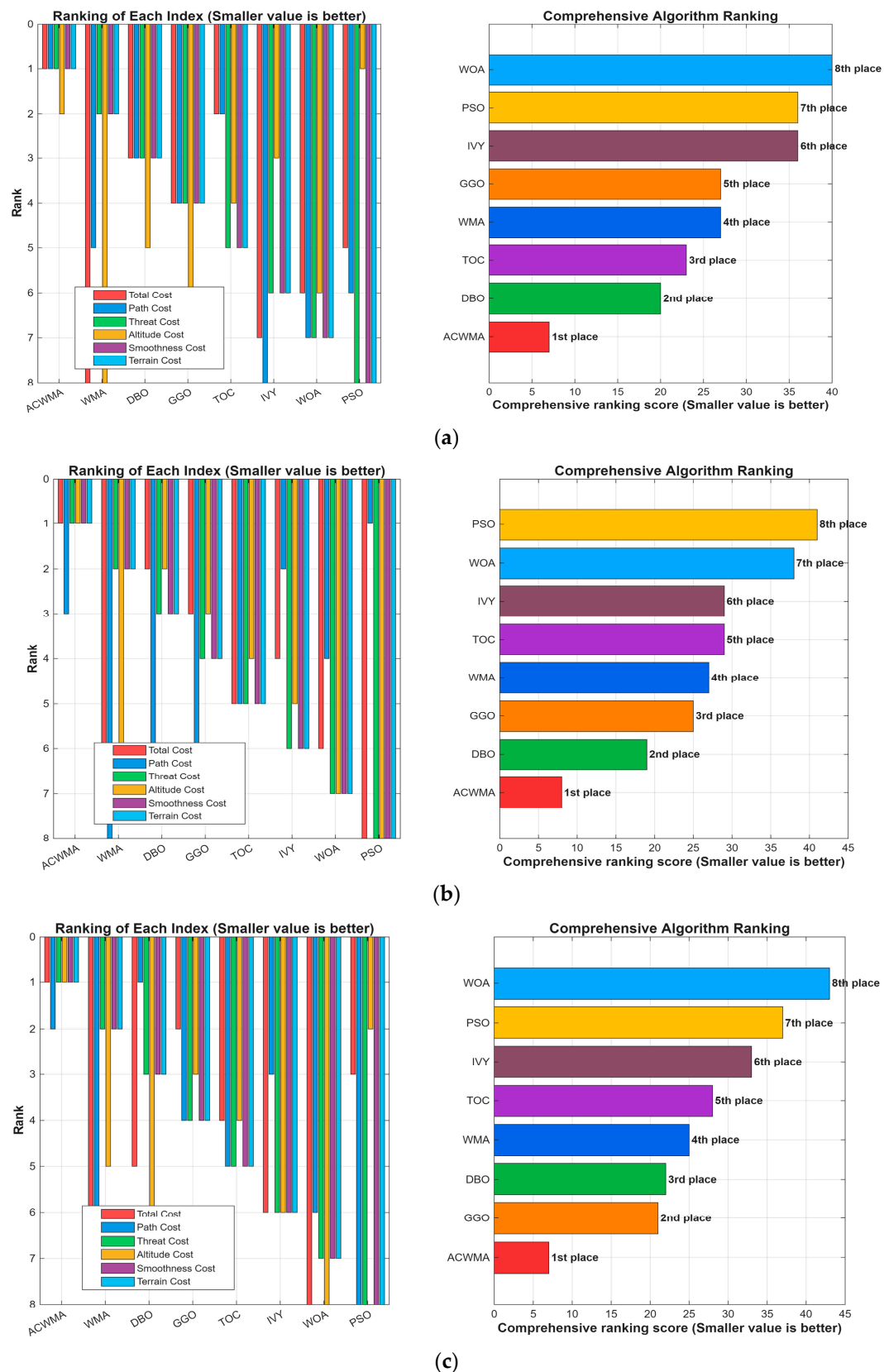


**Figure 11.** Convergence curve comparison of all algorithms: (a) Scenario 1; (b) Scenario 2; (c) Scenario 3; (d) Scenario 4; (e) Scenario 5; (f) Scenario 6.

**Table 8.** Statistical result analysis of the cost function for path planning of all algorithms.

| Scenario |      | ACWMA    | WMA      | DBO      | GGO      | TOC      | IVY      | WOA      | PSO      |
|----------|------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1        | Mean | 1192.199 | 1249.054 | 1194.962 | 1193.365 | 1192.814 | 1200.301 | 1201.121 | 1200.027 |
|          | Std  | 7.912    | 0.315    | 5.768    | 5.068    | 6.664    | 1.303    | 3.730    | 3.943    |
|          | Rank | 1        | 4        | 2        | 5        | 3        | 6        | 8        | 7        |
| 2        | Mean | 1203.465 | 1286.742 | 1238.195 | 1232.707 | 1254.671 | 1244.773 | 1270.670 | 1291.565 |
|          | Std  | 25.434   | 10.607   | 25.948   | 14.759   | 18.714   | 8.376    | 11.391   | 3.366    |
|          | Rank | 1        | 4        | 2        | 3        | 5        | 6        | 7        | 8        |
| 3        | Mean | 1210.575 | 1268.467 | 1292.090 | 1239.496 | 1241.569 | 1259.286 | 1461.517 | 1232.646 |
|          | Std  | 50.776   | 38.733   | 80.021   | 62.913   | 16.181   | 31.535   | 15.234   | 25.084   |
|          | Rank | 1        | 4        | 3        | 2        | 5        | 6        | 8        | 7        |
| 4        | Mean | 1245.410 | 1306.319 | 1508.769 | 1386.529 | 1377.163 | 1446.912 | 1467.534 | 1254.225 |
|          | Std  | 105.642  | 55.153   | 149.476  | 44.108   | 118.403  | 57.316   | 231.345  | 60.233   |
|          | Rank | 1        | 2        | 3        | 4        | 5        | 8        | 7        | 6        |
| 5        | Mean | 1276.014 | 1523.855 | 1594.499 | 1439.905 | 1448.489 | 1363.158 | 1391.940 | 1261.808 |
|          | Std  | 91.094   | 29.820   | 244.012  | 82.062   | 22.715   | 89.186   | 173.528  | 63.710   |
|          | Rank | 1        | 3        | 2        | 4        | 6        | 7        | 8        | 5        |
| 6        | Mean | 1288.479 | 1710.153 | 1776.234 | 1380.960 | 1565.872 | 1356.396 | 2077.489 | 1321.146 |
|          | Std  | 120.457  | 62.964   | 231.957  | 132.704  | 127.153  | 111.923  | 47.684   | 89.373   |
|          | Rank | 1        | 4        | 3        | 2        | 7        | 6        | 8        | 5        |

Based on the Friedman analysis results presented in Figures 12 and 13, all comparative algorithms are capable of generating feasible paths satisfying constraints on path length, threat cost, flight altitude, smoothness, and terrain collision, thereby validating the fundamental feasibility and effectiveness of these algorithms in path planning tasks. However, when confronted with scenarios exhibiting substantial variations in environmental complexity, constraint intensity, and obstacle distribution, significant differences emerge among algorithms in terms of optimization accuracy, convergence performance, and comprehensive cost. In higher-complexity Scenarios 4, 5, and 6, the classical WOA and PSO algorithms, constrained by their inherent optimization mechanisms, are prone to premature convergence and local optima entrapment in complex high-dimensional search spaces. Their comprehensive performance rankings generally fall within the 5th–8th positions, demonstrating evident performance bottlenecks in 3D path planning tasks with multiple constraints and strong interference and failing to stably output high-quality trajectories meeting practical application requirements. In contrast, WMA achieved comprehensive rankings of 2nd, 3rd, and 4th in the aforementioned scenarios, with its optimization accuracy and operational stability significantly improved relative to traditional baseline algorithms. The proposed ACWMA demonstrates particularly outstanding comprehensive performance, maintaining a stable 1st-place ranking across all test scenarios. Compared with other comparative algorithms, the ACWMA not only possesses stronger global search capability and higher convergence precision, effectively circumventing local optima issues, but also exhibits excellent robustness and reliability under varying complexity scenarios, further validating its significant superiority and engineering applicability in multi-constrained 3D path planning problems from a statistical perspective.



**Figure 12.** Overall ranking comparison of different algorithms: (a) Scene 1; (b) Scene 2; (c) Scene 3.

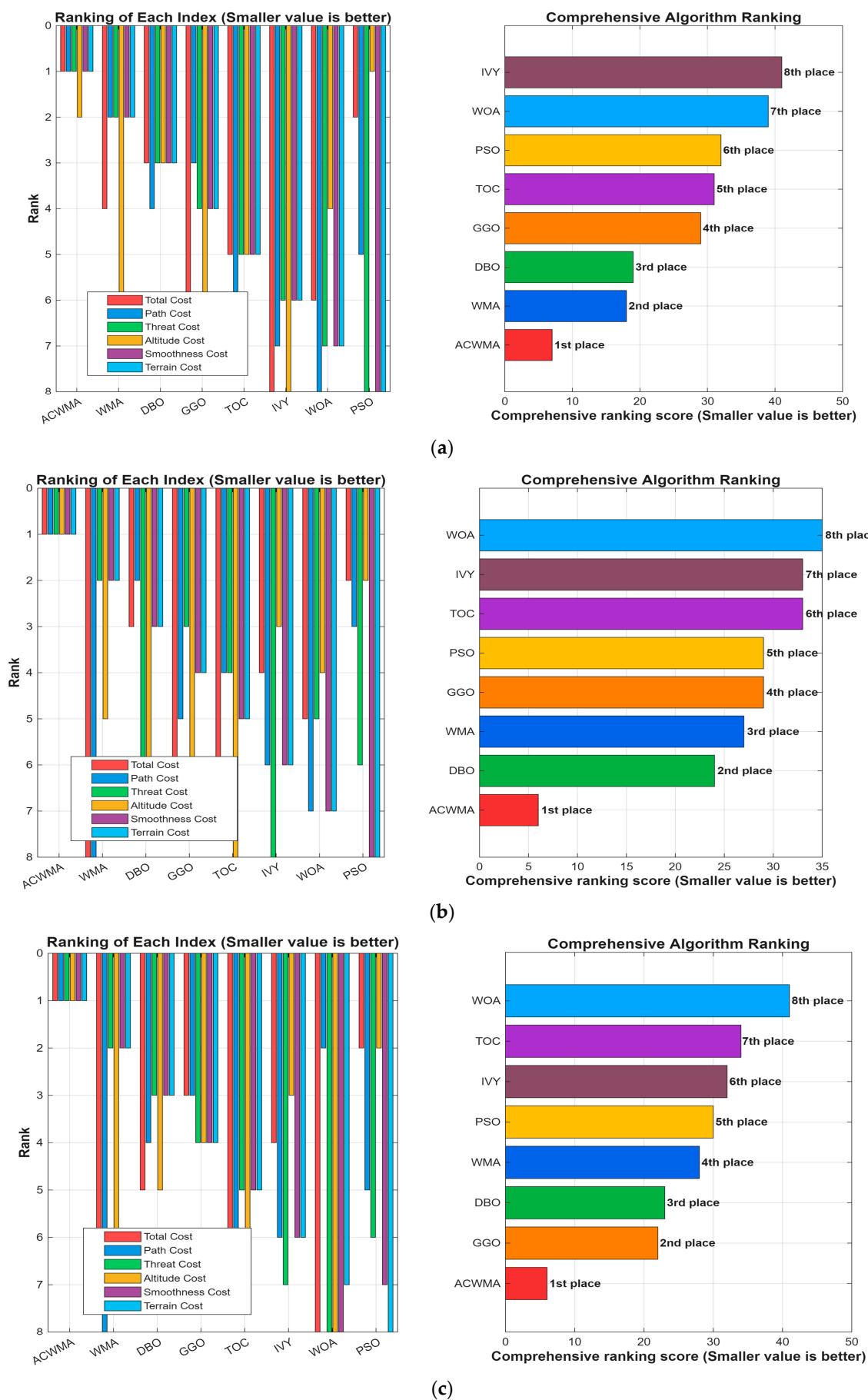


Figure 13. Overall ranking comparison of different algorithms: (a) Scene 4; (b) Scene 5; (c) Scene 6.



#### 4.4.2. Experiments in Scenario B

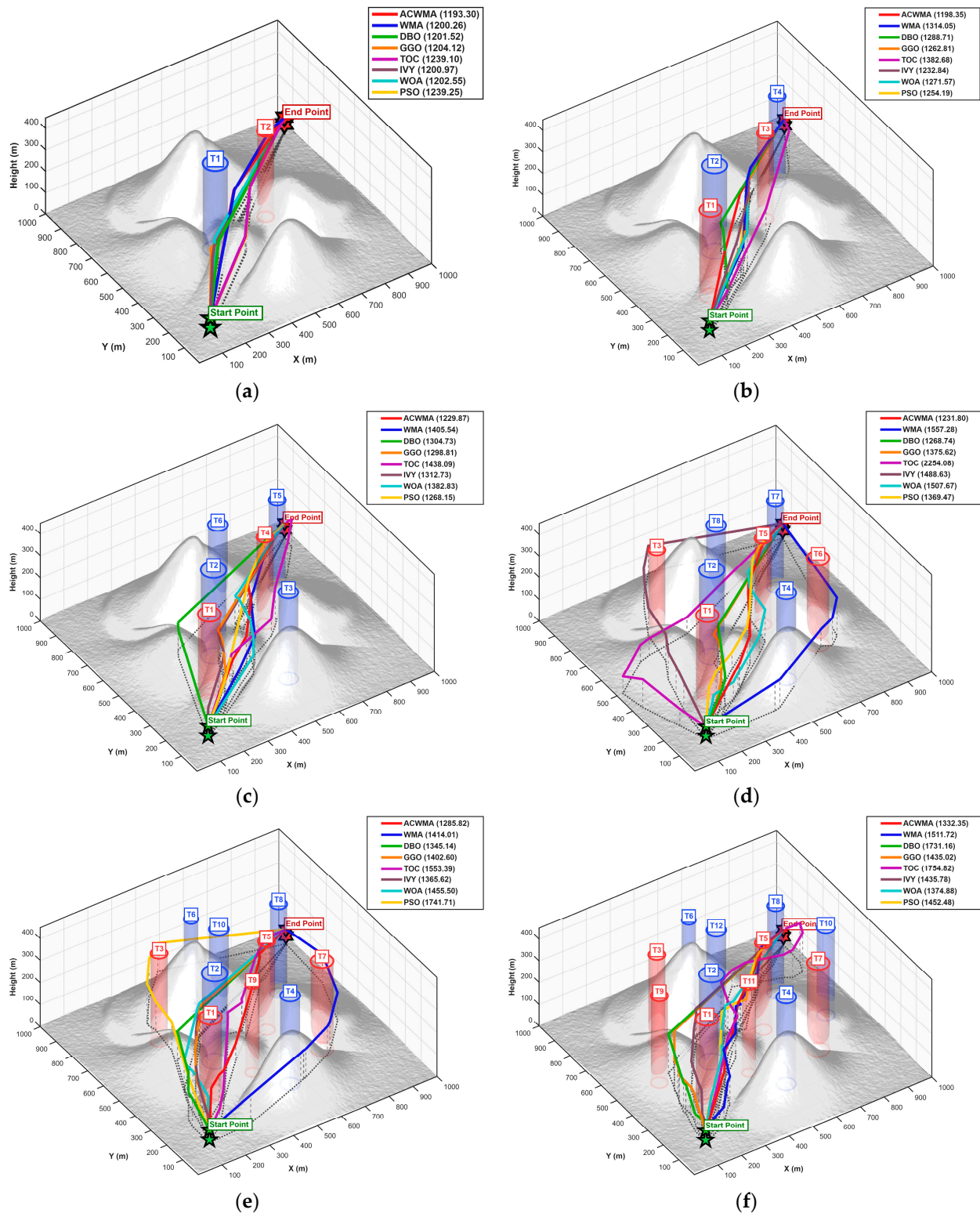
As illustrated in Figures 14 and 15, in simple Scenario 1, the performance differences among comparative algorithms in path planning are marginal and lack statistical significance. In Scenarios 2–6 with progressively increasing complexity, the performance advantages of the ACWMA relative to traditional WOA and PSO algorithms exhibit a significantly increasing trend as environmental complexity escalates. Traditional WOA and PSO algorithms are prone to entrapment in local optima regions, with limited global search efficiency; WMA suffers from an insufficient global exploration range and susceptibility to premature convergence; DBO and GGO algorithms demonstrate slow convergence rates and suboptimal adaptability and robustness in complex environments; TOC and IVY algorithms exhibit low local exploitation precision, struggling to establish efficient dynamic equilibrium between global exploration and local exploitation. In contrast, the ACWMA, leveraging its improved optimization mechanism, demonstrates superior global exploration capability and precise local exploitation performance in complex environments. Experimental results fully validate that the ACWMA can obtain higher-quality path planning solutions, with its superiority in balancing global exploration and local fine-grained exploitation effectively confirmed.

Figure 16 compares the convergence characteristics of each algorithm during the iterative optimization process across different scenarios, systematically revealing the performance differences between the ACWMA and traditional metaheuristic algorithms. From the overall convergence curves, the ACWMA significantly outperforms comparative algorithms in three dimensions: convergence precision, convergence speed, and convergence stability. The essence of this performance advantage lies in the synergistic effect of the proposed adaptive parameter adjustment mechanism and Lévy flight perturbation strategy: the former achieves dynamic adaptive allocation of algorithmic weights between global exploration and local exploitation, while the latter effectively breaks through local optima constraints; together, they suppress premature convergence phenomena and enhance optimization process stability.

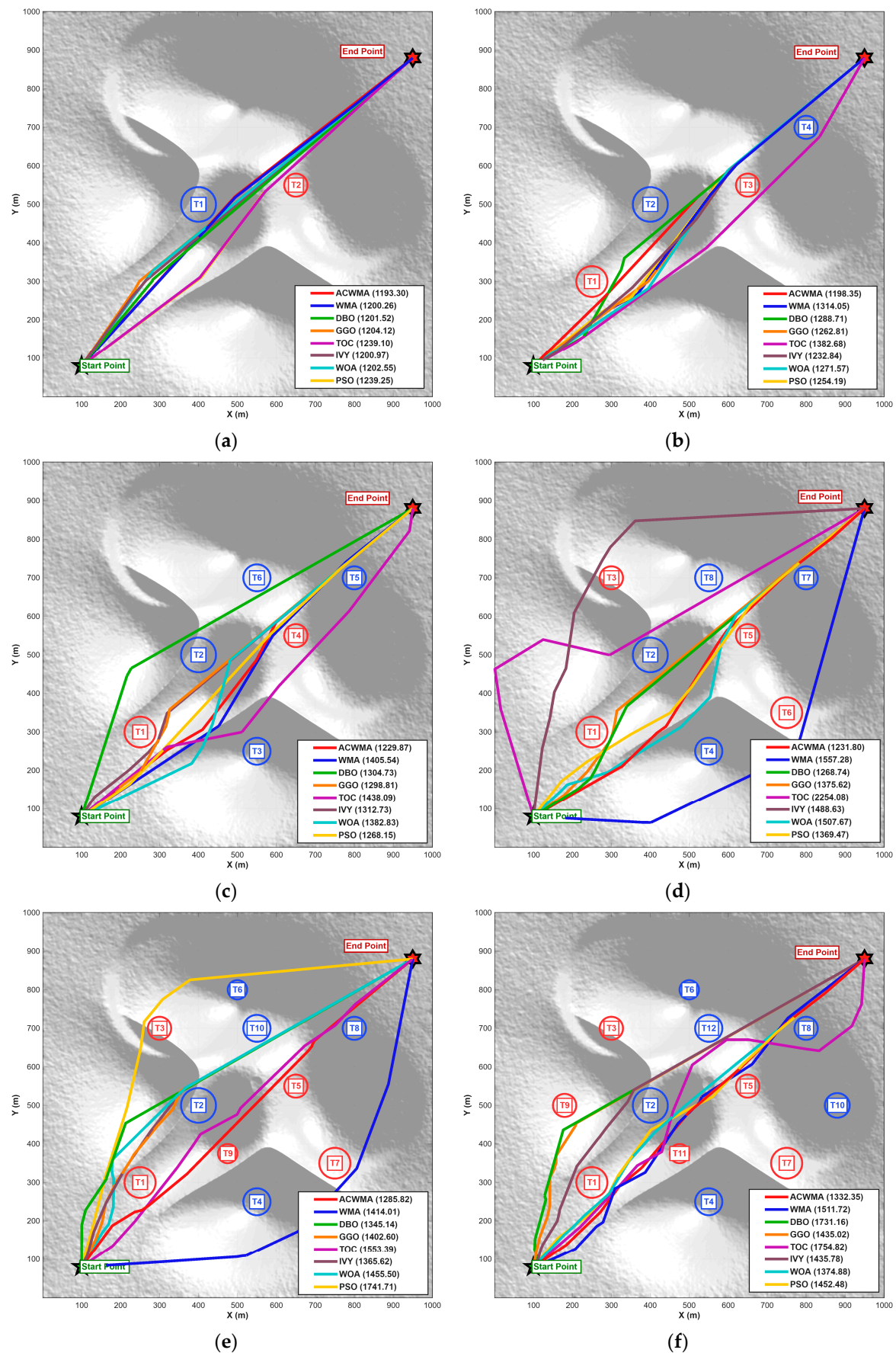
To further quantify the convergence advantages of the ACWMA, local magnification analysis was conducted on convergence curves in later iteration stages. Results indicate that in complex Scenarios 2–6, the convergence speed of the ACWMA is substantially improved relative to other metaheuristic algorithms, validating the effectiveness of the optimal-value guidance strategy: this strategy enhances the guiding role of elite individuals in population evolution direction, shortening the convergence path of algorithmic optimization and improving convergence efficiency. Furthermore, the team-based collaborative learning mechanism constructed upon a three-level fitness difference model, combined with adaptive update rules, further enhances the global search capability and local optimization precision of the population through differentiated learning guidance for population individuals, effectively resolving the core problem of “exploration-exploitation imbalance” inherent in traditional algorithms. In summary, experimental results fully confirm that the multiple improvement strategies proposed in this study form synergistic effects, endowing the ACWMA with superior comprehensive performance and stronger adaptability to complex environments in path planning optimization problems, thereby providing effective algorithmic improvement insights for similar optimization problems.

Statistical data from Table 9 reveal that across all six scenarios, the mean cost function values of the proposed ACWMA are superior to those of all comparative algorithms, with performance rankings consistently maintaining first place. It is the sole method achieving optimal performance across all scenarios, fully validating the core competitiveness and superiority of this algorithm in path planning tasks.

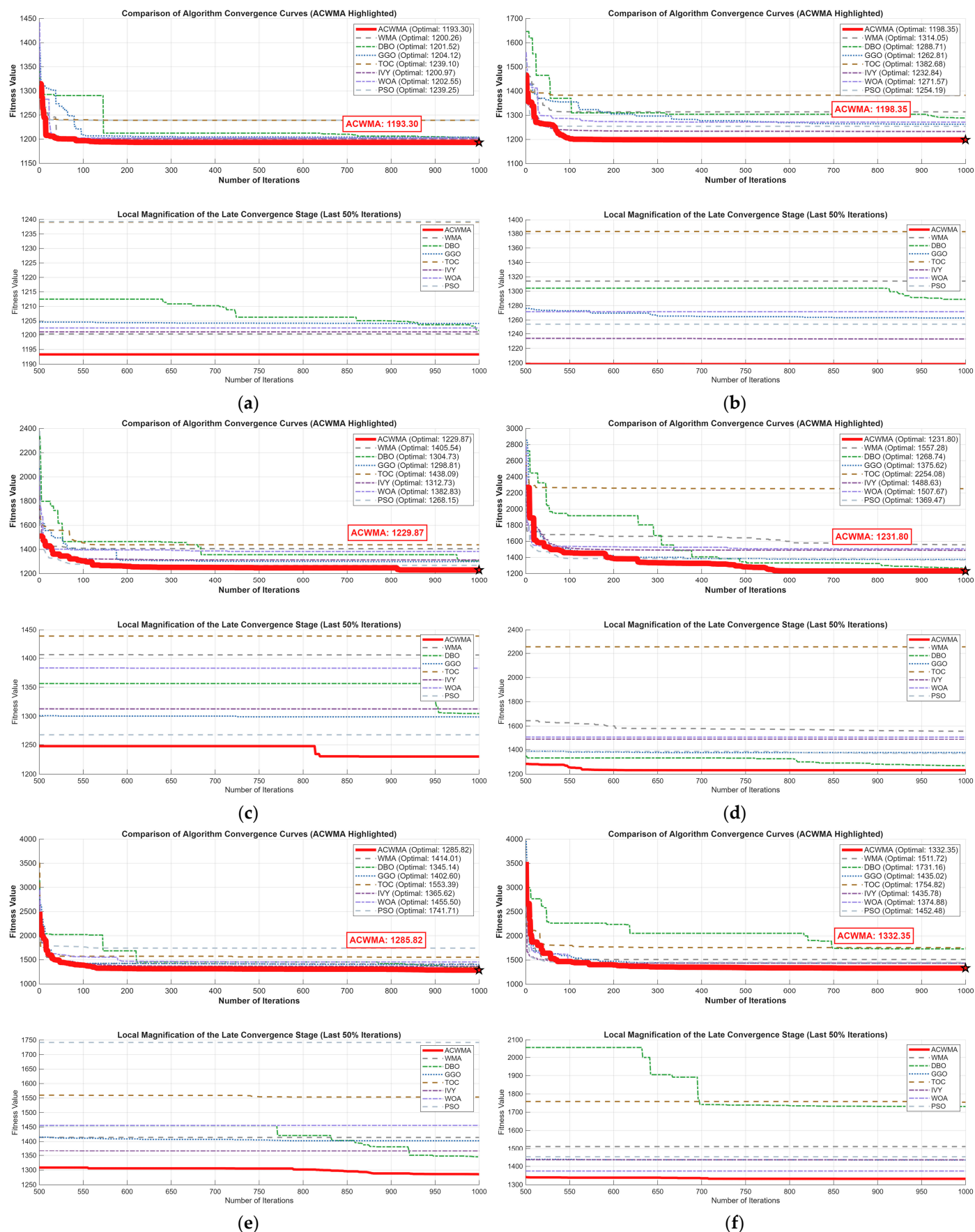




**Figure 14.** 3D views of planned paths for each algorithm: (a) Scenario 1; (b) Scenario 2; (c) Scenario 3; (d) Scenario 4; (e) Scenario 5; (f) Scenario 6.



**Figure 15.** Top views of planned paths for each algorithm: (a) Scenario 1; (b) Scenario 2; (c) Scenario 3; (d) Scenario 4; (e) Scenario 5; (f) Scenario 6.



**Figure 16.** Convergence curve comparison of all algorithms: (a) Scenario 1; (b) Scenario 2; (c) Scenario 3; (d) Scenario 4; (e) Scenario 5; (f) Scenario 6.



**Table 9.** Statistical result analysis of the cost function for path planning of all algorithms.

| Scenario |      | ACWMA    | WMA      | DBO      | GGO      | TOC      | IVY      | WOA      | PSO      |
|----------|------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1        | Mean | 1195.336 | 1202.650 | 1221.771 | 1211.323 | 1239.498 | 1202.004 | 1204.672 | 1239.694 |
|          | Std  | 10.853   | 12.414   | 29.008   | 22.991   | 3.276    | 7.660    | 14.144   | 4.458    |
|          | Rank | 1        | 2        | 3        | 4        | 6        | 5        | 7        | 8        |
| 2        | Mean | 1205.649 | 1318.248 | 1319.578 | 1289.498 | 1383.825 | 1237.633 | 1278.315 | 1255.933 |
|          | Std  | 27.343   | 20.507   | 54.088   | 35.149   | 6.188    | 17.006   | 27.390   | 14.451   |
|          | Rank | 1        | 2        | 5        | 3        | 8        | 4        | 7        | 6        |
| 3        | Mean | 1260.251 | 1408.809 | 1408.382 | 1327.869 | 1448.922 | 1319.962 | 1391.413 | 1275.278 |
|          | Std  | 43.175   | 11.806   | 105.193  | 67.092   | 38.478   | 29.897   | 35.343   | 43.099   |
|          | Rank | 1        | 2        | 4        | 3        | 7        | 5        | 8        | 6        |
| 4        | Mean | 1324.971 | 1625.141 | 1540.912 | 1422.122 | 2258.986 | 1508.081 | 1523.472 | 1393.233 |
|          | Std  | 139.062  | 49.338   | 320.792  | 136.619  | 17.932   | 64.118   | 55.346   | 59.573   |
|          | Rank | 1        | 4        | 2        | 3        | 8        | 5        | 7        | 6        |
| 5        | Mean | 1332.808 | 1426.580 | 1539.780 | 1434.372 | 1569.796 | 1390.285 | 1487.289 | 1747.769 |
|          | Std  | 103.858  | 58.936   | 219.686  | 125.995  | 69.890   | 90.935   | 93.736   | 22.471   |
|          | Rank | 1        | 2        | 3        | 4        | 6        | 5        | 7        | 8        |
| 6        | Mean | 1391.083 | 1523.863 | 2020.081 | 1477.467 | 1777.050 | 1448.143 | 1417.188 | 1463.205 |
|          | Std  | 165.992  | 68.199   | 254.096  | 158.903  | 74.159   | 71.470   | 137.022  | 105.590  |
|          | Rank | 1        | 2        | 6        | 4        | 8        | 5        | 3        | 7        |

In Scenario 1, the mean cost of the ACWMA is 1195.336, representing improvements of 0.55% and 0.61% over the second-ranked IVY (1202.004) and WMA (1202.65), respectively, and already demonstrating superior optimization performance in simple environments. As scenario complexity progressively increases from Scenario 2 to Scenario 6, the mean cost function values of various comparative algorithms exhibit significant upward trends, whereas the ACWMA consistently maintains the lowest mean cost, with the performance gap relative to the second-best algorithm continuously expanding as complexity escalates.

Quantitative results indicate that in Scenario 4, the mean cost of the ACWMA is 1324.971, representing a 6.83% improvement over the second-best GGO algorithm (1422.122), while the worst-performing TOC (2258.986) achieves only 58.66% of its performance. In the highest-complexity Scenario 6, the mean cost of the ACWMA is 1391.083, representing an 8.71% improvement over the second-best WMA (1523.863).

In summary, the ACWMA possesses excellent and stable global optimization capabilities across path planning scenarios of varying complexity, with more pronounced performance gains as complexity increases, demonstrating exceptional robustness, scenario adaptability, and engineering application potential.

#### 4.4.3. Experiments in Scenario C

To verify the dynamic optimization performance and environmental adaptability of the ACWMA in time-varying environments and to overcome the limitations of using static test scenarios for algorithm performance validation, the experimental setup is designed to fully consider the key influencing factors of real marine environments. Ocean current disturbance effects and dynamic obstacle height variations are introduced, and six independent and distinct motion patterns are designed to drive obstacles to move dynamically. Among them, ocean current parameters adopt a standardized configuration: current intensity is set to 0.6, turbulence coefficient is 0.25, and the main current direction is defined as [0.7, 0.7], which accurately simulates the characteristics of real ocean flow fields. In total, 12 threat regions are established in the experiment. The six distinct motion patterns are independent and non-interfering, which can effectively drive each obstacle to move separately and avoid spatial aggregation of obstacles. These patterns fully cover typical time-varying scenarios such as

regular motion, periodic trajectory motion, and highly dynamic fluctuations, providing a more realistic marine environment for algorithm performance testing.

The specific motion modes of each obstacle are defined as follows:

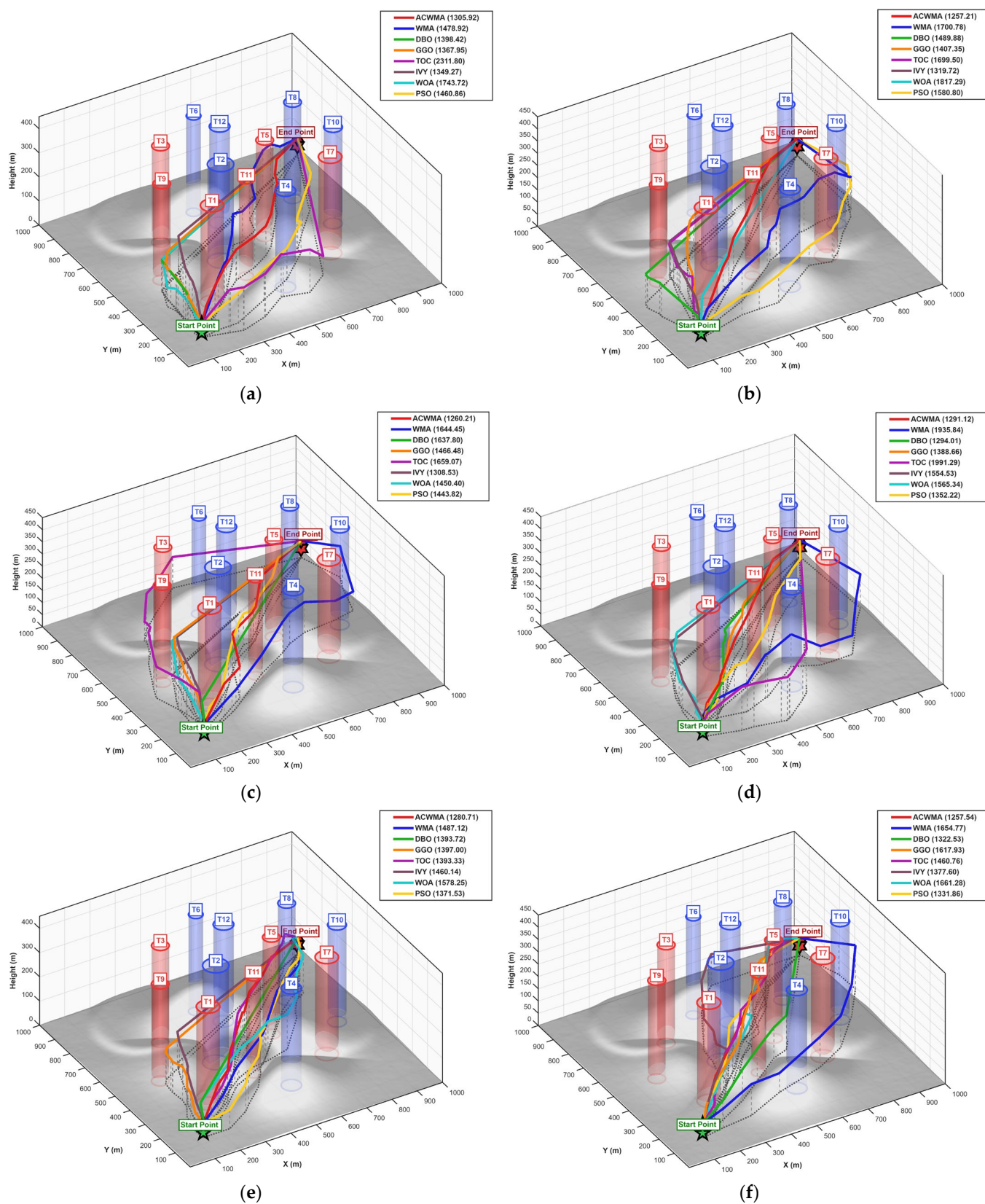
1. Motion Mode 1 (X-direction sinusoidal oscillation): The obstacle performs periodic reciprocating motion along the horizontal X-axis.
2. Motion Mode 2 (Ocean Drift Motion): Simulates the dynamic motion mechanism of obstacles disturbed by ocean currents, inertial drag, and spatial boundary constraints in real marine environments. A time-varying motion model combining smooth random walk and dynamic boundary rebound is established.
3. Motion Mode 3 (Oblique uniform drift): The obstacle drifts slowly and continuously along a fixed oblique direction.
4. Motion Mode 4 (Vertical height dynamic variation): The Z-coordinate of the obstacle fluctuates slightly with time.
5. Motion Mode 5 (figure-eight closed trajectory motion): The obstacle moves periodically along a closed figure-eight curve with a complex trajectory and strong time-varying characteristics.
6. Motion Mode 6 (Elliptical circular motion): The obstacle moves cyclically along a closed elliptical trajectory.

It can be clearly concluded from the experimental results shown in Figures 17 and 18 that the ACWMA demonstrates significant superiority in all core performance evaluation metrics in the dynamic obstacle movement experimental scenario, which simulates the characteristics of real marine environments, introduces ocean current disturbance effects and dynamic height variations of obstacles, and is constructed using six independent motion modes with distinct dynamic characteristics. Especially under two types of complex dynamic conditions with stronger environmental non-stationarity, namely ocean drift motion and figure-eight closed trajectory motion, the ACWMA can achieve rapid response to environmental mutations, accurate tracking of time-varying optimal paths, and a stable and continuous convergence process. It fully exhibits its tracking robustness, environmental adaptability, and fast response advantages under strong dynamic disturbances, which effectively alleviates the common problems of traditional optimization algorithms in complex dynamic scenarios, such as tracking lag, decreased optimization accuracy, and premature convergence.

The underlying mechanism is as follows: The adaptive parameter switching mechanism can balance the intensity of global exploration and local exploitation in real time according to the evolutionary state of the population; the optimal leader strategy can rapidly drive the population to converge directionally toward high-quality solution regions; and the collaborative learning mechanism enables efficient information interaction between levels and complementary advantages among individuals. The synergy of these three components endows the ACWMA with faster response speed, higher dynamic tracking accuracy, and stronger anti-disturbance performance under dynamic environmental perturbations. It can quickly reconstruct and approach the optimal path, reliably avoid dynamic obstacles, and is less prone to becoming trapped in local optima.

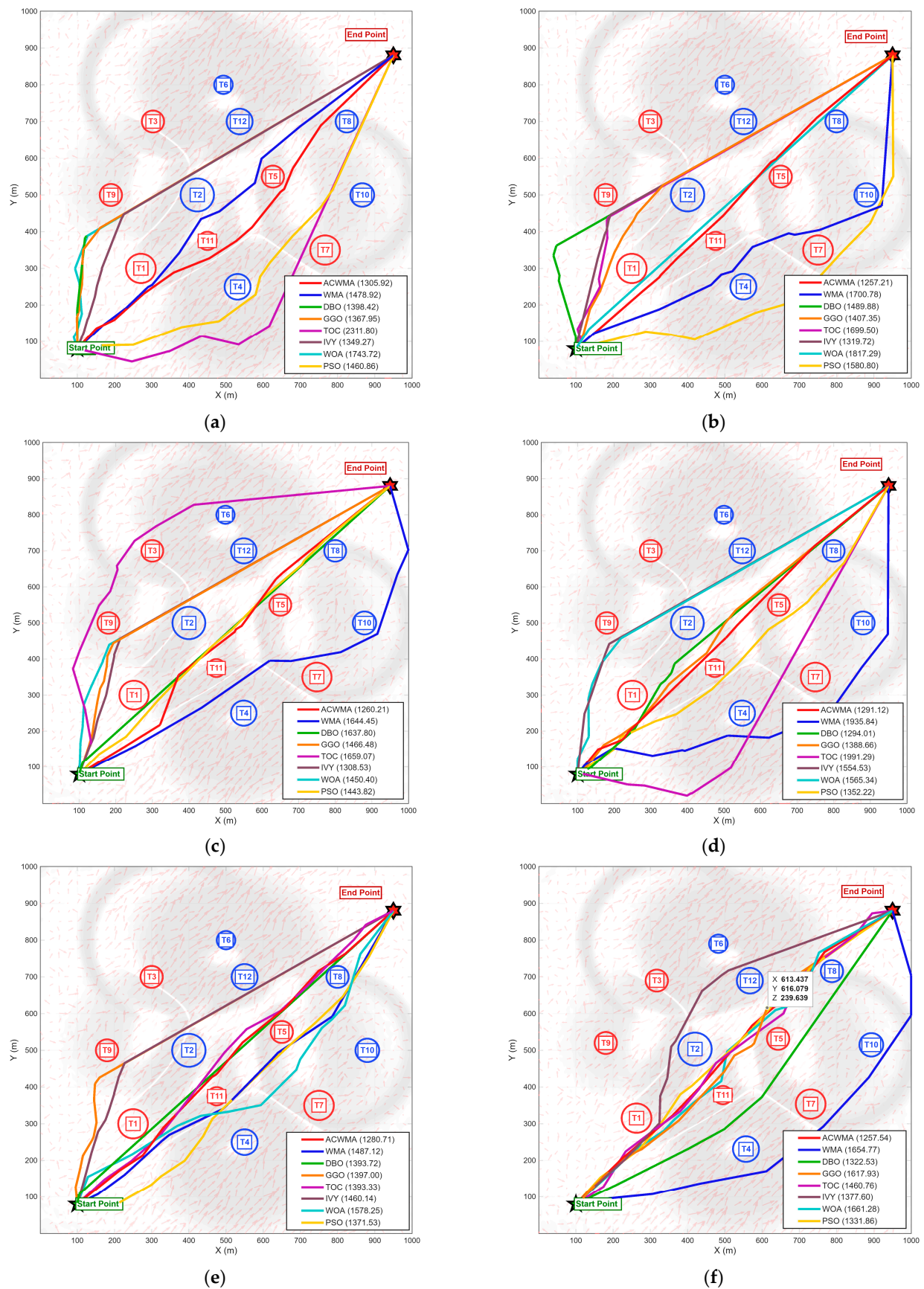
To intuitively compare the optimization processes of different algorithms in dynamic environments, Figure 19 presents the optimal fitness convergence curves of each algorithm under six types of dynamic obstacle motion scenarios. From the overall convergence trend, it can be observed that the ACWMA outperforms the other comparison algorithms in terms of faster convergence speed and higher convergence accuracy under most dynamic conditions, demonstrating stronger dynamic optimization advantages. The results confirm that the introduction of the adaptive parameter adjustment mechanism and the Lévy flight perturbation strategy can reasonably allocate the intensity of global exploration and local exploitation during the iteration process, enabling the algorithm to maintain stable

search in time-varying environments and effectively reduce the probability of falling into local optima.

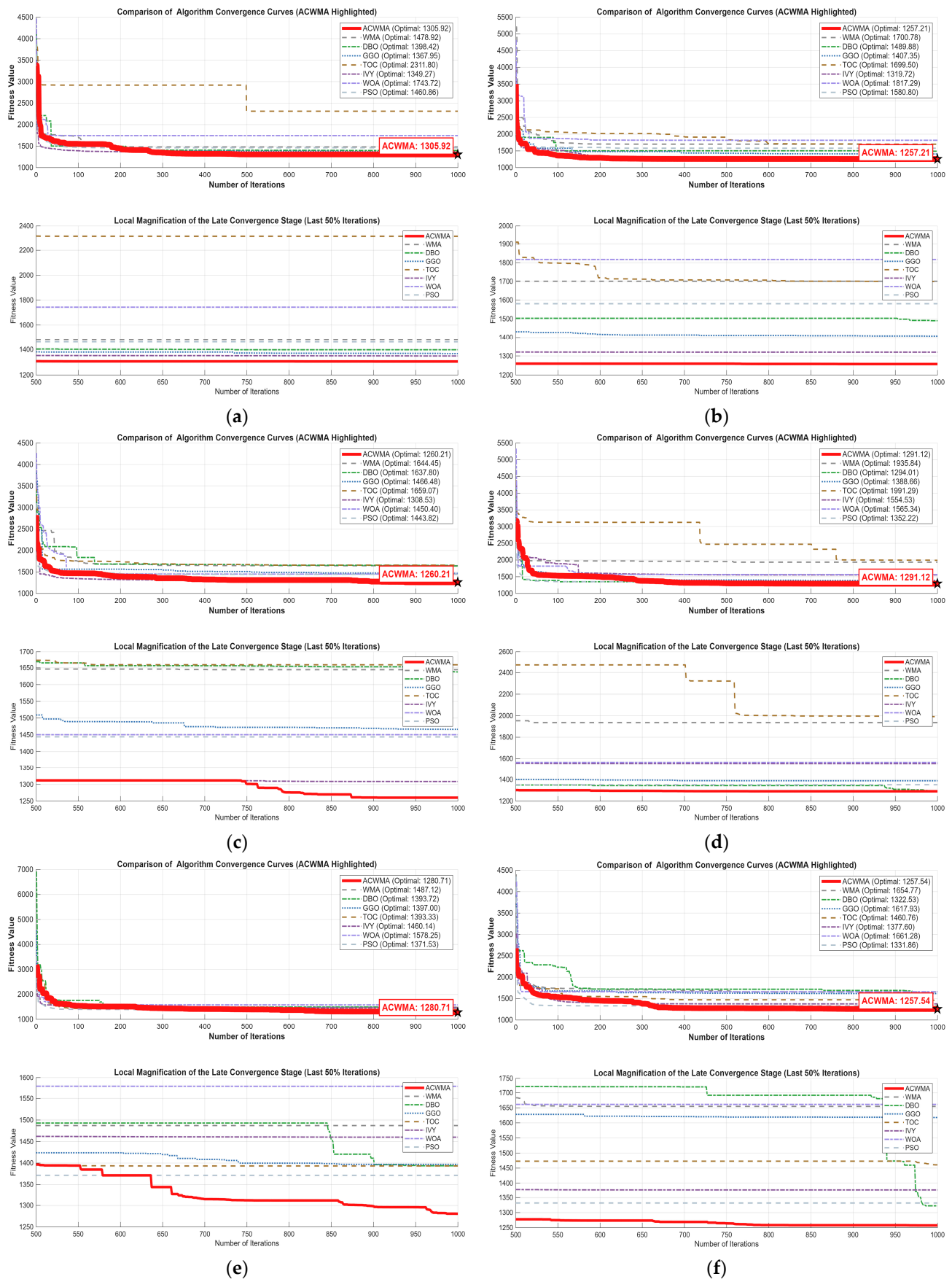


**Figure 17.** 3D diagrams of six dynamic obstacle motion modes in time-varying marine environments: (a) Motion Mode 1; (b) Motion Mode 2; (c) Motion Mode 3; (d) Motion Mode 4; (e) Motion Mode 5; (f) Motion Mode 6.





**Figure 18.** Top view of six dynamic obstacle motion modes in time-varying marine environments: (a) Motion Mode 1; (b) Motion Mode 2; (c) Motion Mode 3; (d) Motion Mode 4; (e) Motion Mode 5; (f) Motion Mode 6.



**Figure 19.** Comparison of convergence curves of various algorithms for six types of dynamic obstacle motion modes in time-varying marine environments: (a) Motion Mode 1; (b) Motion Mode 2; (c) Motion Mode 3; (d) Motion Mode 4; (e) Motion Mode 5; (f) Motion Mode 6.

Further local magnification analysis is performed on the convergence curves in the latter 50% of iterations. With the exception of the oblique uniform drift scenario, the ACWMA exhibits significantly better convergence stability and late-stage optimization performance than conventional metaheuristic algorithms under the other five complex dynamic modes. The above results fully demonstrate that the proposed collaborative optimization framework can effectively improve the environmental adaptability, search stability, and practical application capability of the algorithm in dynamic path planning problems.

As shown in Table 10, quantitative comparisons based on six types of dynamic obstacle scenarios with ocean current disturbance environments demonstrate that the ACWMA achieves the best performance under all tested conditions, with an average rank steadily at 1.00. Its overall performance is significantly superior to existing meta-heuristic algorithms such as PSO, IVY, GGO, DBO, WMA, WOA, and TOC. The average fitness value of ACWMA is only 1348.07, which is 6.4%~7.1% lower than that of the second-echelon algorithms, showing remarkable improvements in optimization accuracy and stability. In contrast, algorithms including DBO, WMA, and WOA have fitness values 14.2%~25.3% higher and average ranks of  $\geq 6.17$ , and are prone to defects such as premature convergence, large search fluctuations, and insufficient robustness in dynamic environments. In typical dynamic scenarios including strong ocean current disturbances, non-stationary trajectories, and large-scale drift, the advantages of ACWMA become even more prominent. This verifies its excellent ability to maintain a balance between exploration and exploitation under strong dynamic interference, with high robustness, strong generalization, and favorable engineering practical value.

**Table 10.** Statistical result analysis of the cost function for path planning of all algorithms.

| Motion Mode |      | ACWMA    | WMA      | DBO      | GGO      | TOC      | IVY      | WOA      | PSO      |
|-------------|------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1           | Mean | 1368.871 | 1519.858 | 1453.044 | 1418.646 | 2618.112 | 1382.888 | 1756.297 | 1474.941 |
|             | Std  | 87.707   | 145.117  | 192.330  | 124.067  | 312.306  | 182.981  | 104.432  | 97.511   |
|             | Rank | 1        | 6        | 4        | 3        | 8        | 2        | 7        | 5        |
| 2           | Mean | 1297.497 | 1745.909 | 1543.980 | 1475.375 | 1871.191 | 1360.471 | 1856.276 | 1595.731 |
|             | Std  | 127.578  | 224.376  | 140.121  | 168.940  | 161.016  | 123.515  | 204.801  | 95.771   |
|             | Rank | 1        | 6        | 4        | 3        | 8        | 2        | 7        | 5        |
| 3           | Mean | 1326.063 | 1702.425 | 1722.896 | 1528.864 | 1700.677 | 1353.038 | 1510.202 | 1455.850 |
|             | Std  | 63.703   | 180.046  | 176.058  | 174.003  | 115.591  | 115.846  | 244.773  | 59.375   |
|             | Rank | 1        | 7        |          | 5        | 6        | 2        | 4        | 3        |
| 4           | Mean | 1362.509 | 1956.226 | 1370.293 | 1446.447 | 2642.050 | 1632.376 | 1608.173 | 1390.460 |
|             | Std  | 84.610   | 52.735   | 181.894  | 151.147  | 475.291  | 198.960  | 179.351  | 205.046  |
|             | Rank | 1        | 8        | 2        | 4        | 7        | 6        | 5        | 3        |
| 5           | Mean | 1389.250 | 1512.609 | 1561.687 | 1468.468 | 1440.476 | 1478.177 | 1595.116 | 1429.633 |
|             | Std  | 96.005   | 119.381  | 311.179  | 151.579  | 191.267  | 77.693   | 102.419  | 191.655  |
|             | Rank | 1        | 6        | 7        | 4        | 3        | 5        | 8        | 2        |
| 6           | Mean | 1344.229 | 1696.984 | 1776.517 | 1659.669 | 1524.091 | 1425.477 | 1673.342 | 1355.611 |
|             | Std  | 110.787  | 119.671  | 231.149  | 152.850  | 105.826  | 160.881  | 155.968  | 157.114  |
|             | Rank | 1        | 7        | 8        | 5        | 4        | 3        | 6        | 2        |

## 5. Conclusions

Aiming at the high-dimensional, non-convex, and multi-constrained optimization challenge in 3D path planning for UUVs in typical marine operating conditions, this paper proposes an ACWMA. The environmental safety and motion feasibility of the generated path are defined as the core objectives of algorithm design and the key criteria for performance evaluation. First, a comprehensively constrained and coupled cost function is established from three perspectives: path optimality, environmental safety, and motion feasibility, thereby formulating a standardized mathematical model for the planning prob-

lem. Meanwhile, according to the inherent mapping relationship between the kinematic characteristics of unmanned vehicles and the path search space, the mechanism design and model implementation of the ACWMA are completed. To systematically verify the comprehensive performance of the algorithm, a high-precision 3D terrain model is constructed based on real measured marine topographic data, and 12 groups of benchmark test scenarios are generated for simulation experiments. The experimental results demonstrate that the ACWMA can obtain the optimal-quality path solutions in most test scenarios and exhibits excellent convergence stability. Comparative validation with state-of-the-art metaheuristic algorithms, including WMA, GGO, TOC, and PSO, further confirms that the ACWMA achieves significant performance superiority in UUV 3D path planning. The paths generated by the proposed algorithm show satisfactory adaptability and practical execution effectiveness for complex marine engineering operations. Based on all experimental results, the ACWMA can efficiently obtain the global optimal solution for UUV 3D path planning in typical marine operating conditions. Its advantage in dynamically balancing global exploration and local exploitation is sufficiently verified.

Future research will be systematically conducted from two perspectives: algorithmic model optimization and application extension. At the model optimization level, configuration spaces will be constructed based on high-fidelity UUV dynamic models, incorporating refined constraint conditions to achieve deep coupling between the algorithm and the intrinsic dynamic characteristics of the vehicle, thereby further enhancing the engineering executability of planned trajectories. At the application extension level, the ACWMA will be generalized to optimization problems involving diverse standard benchmark functions; through systematic performance validation, its optimization mechanisms and adaptive characteristics will be analyzed, and its universality for other high-dimensional non-convex multi-constrained optimization problems will be explored, thereby establishing theoretical and experimental foundations for cross-domain applications of this algorithm.

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