Journal of Physics:
Conference Series

PAPER • OPEN ACCESS

Further enhancing the sharpness of an acoustic focus employing split processing

To cite this article: Ting Li *et al* 2026 *J. Phys.: Conf. Ser.* **3178** 012050

Further enhancing the sharpness of an acoustic focus employing split processing

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Abstract. A low-frequency source always has a large focus even if it is focused by artificial iterative phase-conjugated processing (AIPCP), which can break the half-wavelength limit at a wavelength in a homogenous medium. Combining AIPCP with split beamforming, we propose split artificial iterative phase-conjugated processing (SAIPCP), which is proven to enhance the focus of a point-like source in a homogenous medium. Theoretical analyses, numerical simulations and experiments in an audible sound field are employed to compare the focal spots focused by AIPCP and SAIPCP using a uniform linear array. The results consistently show that in the same conditions, SAIPCP achieves a smaller focal spot than AIPCP in both the near and far fields with the same iteration loop. In addition, the focal patterns have little side lobe disturbance by SAIPCP. Meanwhile, the focus by SAIPCP is smaller than half a wavelength, along with having a sharper focus than AIPCP at a wavelength away from the source.

1. Introduction

Phase-conjugated mirror (PCM) has a wide ground in under water acoustic field such as acoustic communication, acoustic focusing and imaging of a localized source[1,2]. Although the random inhomogeneities of the medium can in some circumstances enhance the focal spot by PCM[3,4], sharpening the focus by algorithm is still an attractive direction, especially for low-frequency case[5]. Meanwhile in a homogeneous medium, because of the subwavelength limit, obtaining a sharp focal spot is always a challenge. Although, based on a passive PCM, artificial iterative phase-conjugated processing (AIPCP) has been proven to achieve a sharp focal spot as the iteration number increases in a homogeneous medium[6], its focus cannot be smaller than at a distance from the point-like source equal to the wavelength because the iteration number is restricted to the transducers. Therefore, an accurate location scope of a low-frequency source is also expected.

In this paper, split artificial iterative phase-conjugated processing (SAIPCP) is proposed by introducing the split beamforming idea [7] into AIPCP. Split beamforming is an optimal technique for bearing estimation and is always combined with conventional beamforming to provide a better bearing resolution than conventional beamforming. In split beamforming, a linear array is separated into two parts; conventional beamforming steered to each, and then their outputs are cross-correlated. Because the distance between center of each subarray is larger than element space, split beamforming can improve bearing estimation performance[7]. In this paper, the linear array has two subarrays. At each subarray, AIPCP is steered, and their outputs are cross-correlated. Then, the outputs of SAIPCP are obtained. Theoretical analysis compares the focus calculated by AIPCP and SAIPCP in the same situation. The result comes out that SAIPCP generates narrower focal spot than AIPCP, which is confirmed by numerical simulations and experiments in an audible sound field. Furthermore, increasing iteration number, the focal spot size decreases.

In the following sections, we propose SAIPCP after introducing AIPCP at each subarray (Section 2.1). Foci are compared in the far field (Section 2.2). Simulation (Section 3) and experiment (Section 4) are employed to compare the performance of SAIPCP and AIPCP. Conclusions come out that SAIPCP can enhance the sharpness of foci than AIPCP in both near and far fields.

2. Theory

2.1. Split artificial iterative phase-conjugated processing (SAIPCP)

In this section, SAIPCP is proposed. There are two subarrays, at each of which signals are independently processed by AIPCP before their outputs are cross-correlated.

Based on passive PCM, AIPCP iterates the phase-conjugated process for each transducer by computer. According to PCM, AIPCP has two calculation formulas, the monopole form (AIPC/M) and the dipole form (AIPC/D)[6]. At each subarray, AIPC/M is shown as follows. Assume that the element number L is even.

At field point $\boldsymbol{r}$, AIPC/M[6] of each subarray after iterating N loops is:

$$A_{lPC/M}^{N}(\mathrm{R},\mathrm{r},\omega)=\sum_{j=1}^{L/2}\left[P_{r_s r_j}^{*}G_{r_j r}\right]^{N+1} \tag{1}$$

$$A_{rPC/M}^{N}(\mathrm{R},\mathrm{r},\omega)=\sum_{j=L/2+1}^{L}\left[P_{r_s r_j}^{*}G_{r_j r}\right]^{N+1} \tag{2}$$

The subscripts 'l' expresses processing at the left subarray. 'r' expresses processing at the right subarray. ω is angular frequency, $P_{r_s r_j}^{*}$ is complex conjugation of the sound pressure received at $\boldsymbol{r_j}$ from source $\boldsymbol{r_s}$. $G_{r_j r}$ denotes the Green function from receiver $\boldsymbol{r_j}$ to field point $\boldsymbol{r}$.

At field point $\boldsymbol{r}$, AIPC/D[6] of each subarray after iterating N loops is:

$$A_{lPC/D}^{N}(\mathrm{R},\mathrm{r},\omega)=\sum_{j=1}^{L/2}\left[\frac{\partial P_{r_s r_j}^{*}}{\partial \mathrm{n}}\frac{\partial G_{r_j r}}{\partial \mathrm{n}}\right]^{N+1} \tag{3}$$

$$A_{rPC/D}^{N}(\mathrm{R},\mathrm{r},\omega)=\sum_{j=L/2+1}^{L}\left[\frac{\partial P_{r_s r_j}^{*}}{\partial \mathrm{n}}\frac{\partial G_{r_j r}}{\partial \mathrm{n}}\right]^{N+1} \tag{4}$$

$\frac{\partial}{\partial \mathbf{n}}$ is subarray's normal gradient .

After each subarray finishes its AIPCP independently, outputs are cross-correlated as follows[8]:

$$SAIPCP=\frac{A_l\times A_r^{*}+A_l^{*}\times A_r}{2} \tag{5}$$

Substituting equation (1) as A_l and equation (2) as A_r into equation (5), after normalization, the focal pattern of the source is obtained by the monopole form (SAIPC/M). Substituting equation (3) as A_l and equation (4) as A_r into equation (5), after normalization, the focal pattern of the source is obtained by the dipole form (SAIPC/D).

2.2. Theorical analysis

A point-like source and a uniform linear receiving array are located in infinite, homogeneous, and isotropic medium. Focal equations are deduced and discussed basing on the Fraunhofer far-field approximation. We use the normalized beam pattern[9] to describe the focusing field. Its maximum appears at the source location.

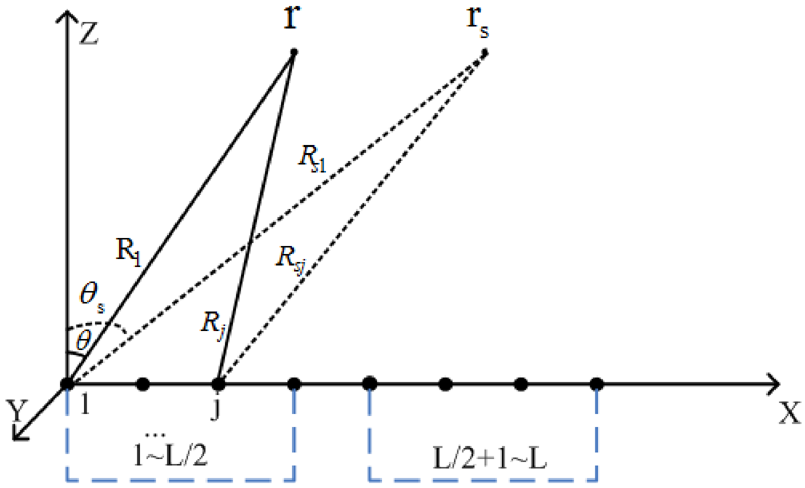

Figure 1. Diagram of the split array.

In Cartesian coordinates, the source $\mathbf{r_s}$, the field point $\mathbf{r}$ and the linear array are assumed locating at XOZ plane (Figure 1). The incident angle of the source is marked θ_s and the exit angle of the first element is marked θ . $\mathrm{R_{sj}}$ presents the distance from $\mathbf{r_s}$ to element j . R_j presents the distance between the field point $\mathbf{r}$ and transducer j . The uniformly spaced linear array spaced by d located along the X-axis is separated into two halves, with each called a subarray. Using Fraunhofer far-field approximation, the complex conjugation of the sound pressure at receiver $\boldsymbol{r_j}$ and green function are presents as follows:

$$P^*(\mathbf{r}_s,\mathbf{r}_j,\omega)=\frac{\exp(-ikR_{sj})}{4\pi R_{s1}} \tag{6}$$

$$\mathrm{G}(\mathbf{r}_j,\mathbf{r},\omega)=\frac{\exp(ik\mathrm{R}_j)}{4\pi R_1} \tag{7}$$

where $\mathrm{R}_j = R_1-(j-1)d\sin\theta$, $R_{sj} = R_{s1}-(j-1)d\sin\theta_s$, and $\theta\subset\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Substituting equations (6) and (7) into the corresponding equations of SAIPCP, after normalization, the same normalized beam patterns of SAIPC/M and SAIPC/D are obtained as those of equation (5). The main lobe widths is $\frac{4\pi R_1}{\mathrm{L(N+1)kd}}$ by AIPCP and $\frac{2\pi R_1}{\mathrm{L(N+1)kd}}$ by SAIPCP on X-axis. That means the focus is sharpen by SAIPCP. Then, graphs of these focal pattern functions are presented in Figure 2 to express the search field determined by the PCM. The main lobe peak corresponding to the maximum value points to the source orientation for both AIPCP and SAIPCP.

Increasing the iteration number, the grating lobes appearing at $R_1\,sin\,\theta=R_1\left(\frac{2\pi}{(N+1)kd}+sin\,\theta_s\right)$ move near the main lobe, potentially causing a misjudgement of the source location. Employing the focal spot of PCM as the search area, the problem can be solved. Grating lobe disturbance disappears in the search field calculated with the PCM as N=14. The iteration number of AIPCP and SAIPCP obeys the same restraint. Moreover, the amplitudes of the side lobes of SAIPCP are smaller than those of AIPCP (Figure 2).

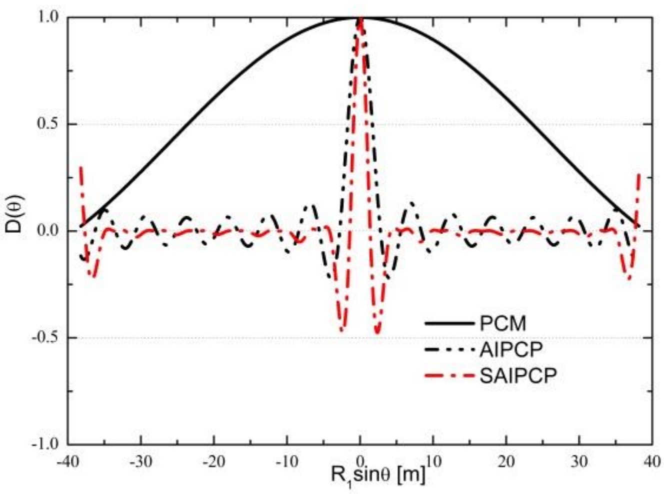

Figure 2. Focal patterns along the X-axis, $N = 14$ and $\theta_s = 0$. The receiver linear array has 16 transducers spaced 0.08 m at 50 m.

3. Numerical analyses

To prove results in Section 2.2, focal patterns of point-like source are painted by SAIPCP in free space in numerical simulation. SAIPCP calculates smaller foci and lower side lobe than AIPCP in both the near and far fields.

The source at (0,0,0) in coordinates produces sound (344 Hz, $\lambda = 1\mathrm{m}$) in the infinite homogeneous medium. The positions of the sound source, field point and search range are drawn in Figure 3. The receiver array is a linear array. It is parallel to X-axis and about Z -symmetry. It has 1.2 metres with 16 transducers that are uniformly spaced apart by 0.08 m. Sounds are received, processed by SAIPCP and AIPCP, and painted on the searching area as shown in Figure 3.

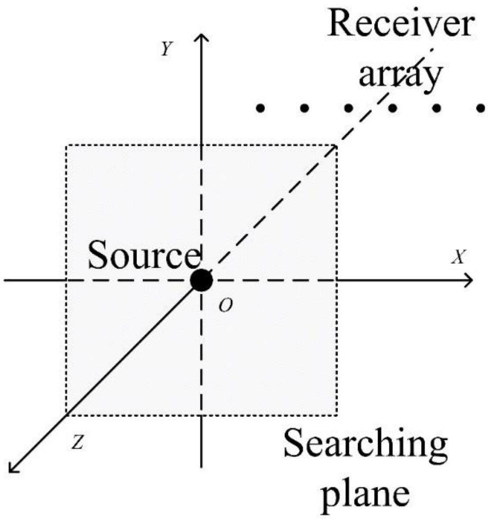

Figure 3. Sketch of the search plane.

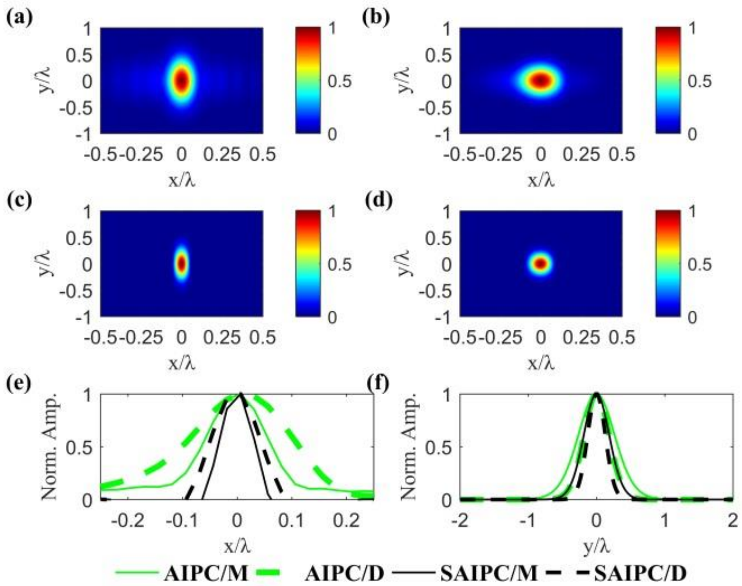

Figure 4. Focal spots calculated at λ, $N = 14$:
a Focused by AIPC/M,
b Focused by AIPC/D,
c Focused by SAIPC/M,
d Focused by SAIPC/D,
e Focus on *X*-axis,
f Focus on *Y*-axis
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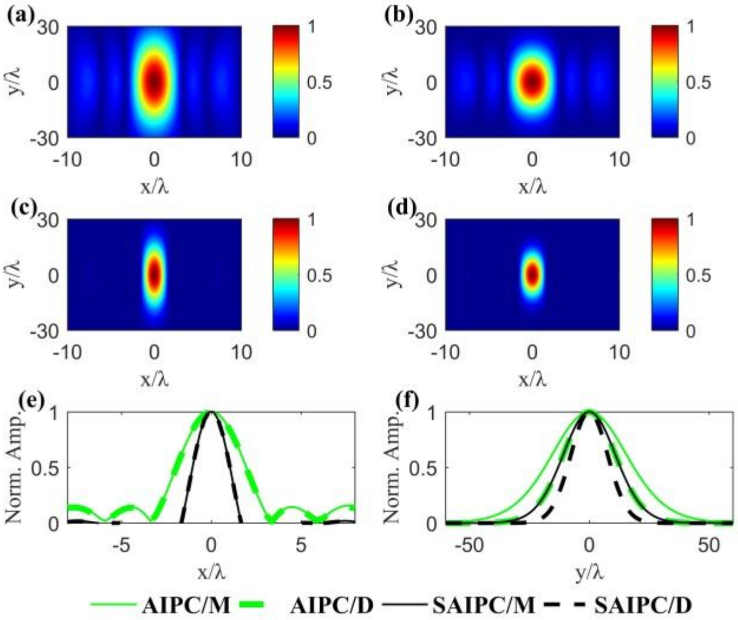

Figure 5. Focal spots calculated at 60λ, $N = 14$:
a Focused by AIPC/M,
b Focused by AIPC/D,
c Focused by SAIPC/M,
d Focused by SAIPC/D,
e Focus on *X*-axis,
f Focus on *Y*-axis.

The legends "AIPC/M" and "AIPC/D" mean that figures are focused by AIPC/M and AIPC/D, respectively. "SAIPC/M" and "SAIPC/D" mean that figures are focused by SAIPC/M and SAIPC/D. "focus along *X*-axis" denotes foci along the X-axis by AIPC/M, AIPC/D, SAIPC/M and SAIPC/D.

Table 1. Comparison between foci calculated by AIPCP and SAIPCP.

Range between the array and source	14 loops	Focus on X-axis/m	Focus on Y-axis/m
λ (Figure 4)	AIPC/M	0.28	1.40
	AIPC/D	0.32	0.90
	SAIPC/M	0.14	1.20
	SAIPC/D	0.20	0.80
60λ (Figure 5)	AIPC/M	6.74	120.00
	AIPC/D	6.74	71.20
	SAIPC/M	3.20	71.20
	SAIPC/D	3.20	48.80

In this section, foci calculated by SAIPCP are located at the source position in both the near and far fields (Figs. 4 and 5). SAIPCP can sharpen foci comparing with AIPCP. For instance, focus by SAIPCP (0.14 m) is 50% of AIPCP (0.28 m) on the X-axis at λ. Foci by SAIPCP are 47% of AIPCP in size on the X-axis and 59~69% along the Y-axis (Figure 5 and Table1). Moreover, the side lobes are lower than AIPCP (Figure 4 and Figure 5). These conclusions are consistent with the analyses in Section 2.2.

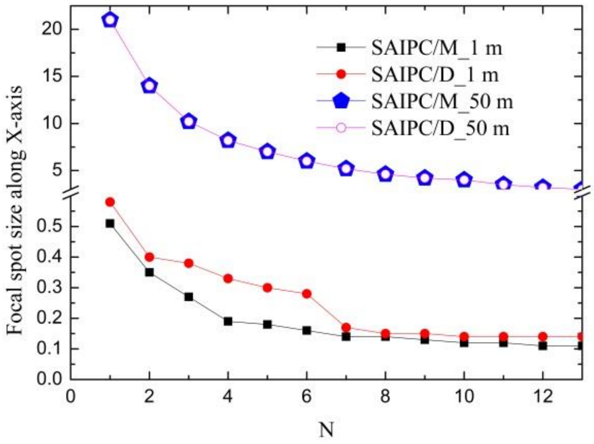

Figure 6. Iterative history of SAIPCP along the *X*-axis.

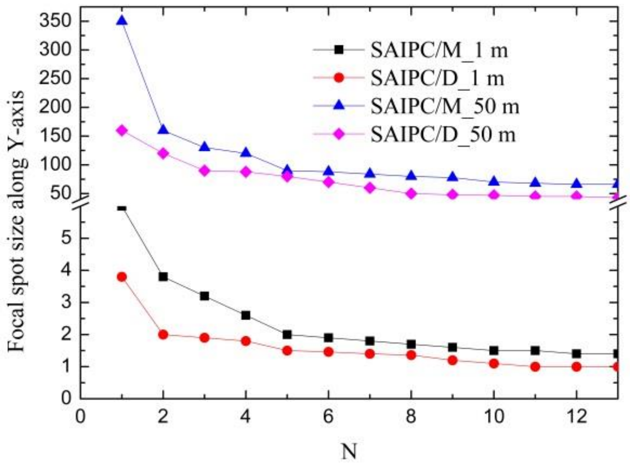

Figure 7. Iterative history of SAIPCP along the *Y*-axis.

In Figures. 6 and 7, legend “SAIPC/M_1m” means pattern processed by SAIPC/M at 1metre. Legend “SAIPC/D_50m” means processed by SAIPC/D at 50 metres away from the source.

Focus become sharp quickly as iteration number increases when it is little (in Figures 6 and 7, iteration number is less than 7). As iteration number N is big, focus becomes stable about iteration number.

4.Semi-anechoic chamber experiments

To support these conclusions, an air acoustic experiments were conducted in a semi-silent chamber. Sound source is a loudspeaker (EDIFIER R26T) at (0,0,0) sending sound at 344 Hz. The signals are recorded by a linear microphone array. A referential transducer (PCB130E20) is used to calculate phase information [10]. Parameters of microphone array are same with that in section 3 shown in Figure 8. After signals are collected, the focal patterns are drawn in computer at the search plane by SAIPCP and AIPCP. Conclusions show that SAIPCP can focus to a sharper spot than AIPCP at a wavelength, as confirmed with Section 3.

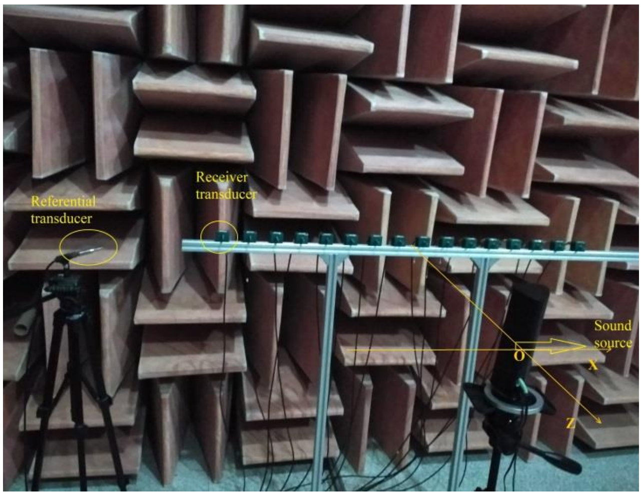

Figure 8. Photo of experiment.

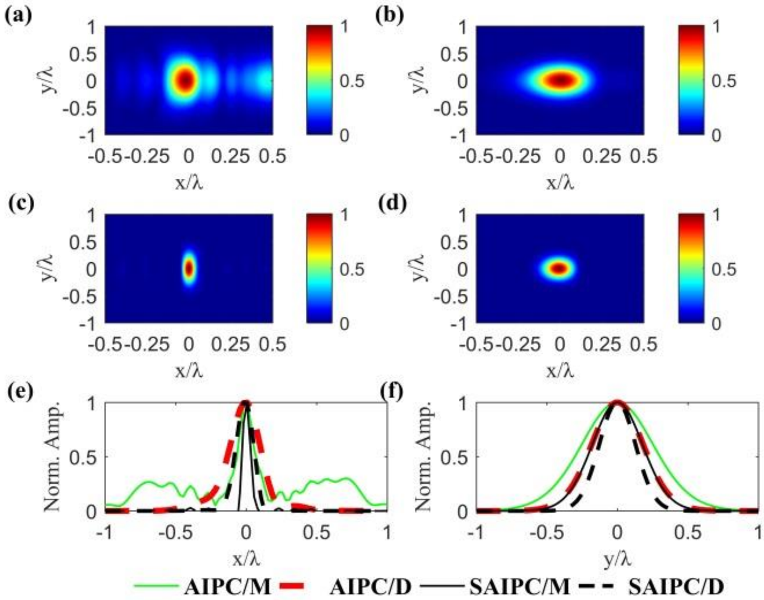

Figure 9. Experimental results at λ, $N=14$: **a** Focused by AIPC/M, **b** Focused by AIPC/D, **c** Focused by SAIPC/M, **d** Focused by SAIPC/D, **e** Focus on *X*-axis, **f** Focus on *Y*-axis.

Table 2. Foci calculated by AIPC/M, AIPC/D, SAIPC/M and SAIPC/D in experiments.

Distance from source	14 iteration loops	Focus on X-axis/m	Focus on Y-axis/m
1 m (Figure 9)	AIPC/M	0.40	1.60
	AIPC/D	0.70	1.20
	SAIPC/M	0.13	1.20
	SAIPC/D	0.36	0.80

SAIPCP can focus a sharper spot than AIPCP at a wavelength(Figure 9 , Table 2). In experiments, foci by SAIPCP are 33~51% of those by AIPCP along the X-axis and 67~75% along the Y-axis at 14 loops. Moreover, SAIPCP has smaller sidelobe than AIPCP, which is consistent with the conclusions in Section 3.

Along the Y-axis, focus by SAIPC/D is smaller (0.4 m smaller than SAIPC/M) . Meanwhile, a large focus along the X-axis (0.23 m larger than SAIPC/M), which is consistent with the conclusions of AIPCP[6]. Moreover, the focus by SAIPC/M is nearly $\lambda/7$ along the X-axis at a wavelength, which breaks the half-wavelength limit.

5. Conclusion

Based on AIPCP, we propose an improved algorithm called SAIPCP to sharpen the focus of acoustic source. It has two forms: the monopole one called SAIPC/M and the dipole one called SAIPC/D to calculate the focal patterns. Conclusions come out by theoretical analysis and numerical analysis, and then proved by air acoustic experiments.

Comparing with AIPCP, it is proven that SAIPCP can further enhance the focus and has a small sidelobe in the same iteration loops. Increasing the iteration number, focus by SAIPCP becomes sharper and grating disturbance gradually appears. The iteration number should be less than element number minus one, which agreed with AIPCP.

6. References

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Acknowledgments

We would like to gratefully acknowledge the support of the College Development Found of Dalian Naval Academy (Grant No.DJYKYKT).