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A USBL/DR underwater integrated positioning method based on the cubature particle filter

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Abstract. This paper proposes a USBL/DR underwater integrated navigation method based on the cubature particle filter (CPF) to address the challenges of navigation in complex underwater environments. The proposed algorithm utilizes an autonomous underwater vehicle (AUV) equipped with a Doppler velocity log (DVL) and a heading reference system for dead reckoning (DR). A hybrid positioning system is developed by integrating position and velocity data from the ultra-short baseline (USBL) and DVL. To overcome the issue of particle degradation commonly observed in particle filters during integrated navigation data fusion, CPF is employed for enhanced filtering performance. The proposed algorithm was successfully tested through both simulation experiments and lake trials. The approach successfully combines the advantages of both navigation systems, demonstrating superior underwater positioning accuracy over short durations compared to conventional particle filter (PF) and unscented particle filter (UPF) methods.

1. Introduction

AUV is an underwater robot that does not require cable control, carries energy on its own, and communicates wirelessly with the mother ship or shore-based unit to perform highly complex and dangerous tasks. Due to its own advantages, AUVs have great prospects in both military and civilian applications. High-precision positioning is the key to autonomous navigation of AUVs, but a single navigation technology can cause serious errors in navigation results. A more accurate integrated navigation system is the key to solving these problems.

Common underwater sensors include fiber optic compasses, pressure sensors, DVL, and USBL, which can provide attitude and orientation information of the carrier. DR based on DVL and compass has the characteristics of high autonomy and strong concealment, with smooth data output and high short-term accuracy. It is a commonly used navigation system for underwater vehicles. Utilizing the advantages of ultra-short baseline positioning and dead reckoning systems, and integrating the two information, is an effective underwater integrated navigation method.Currently AUV integrated positioning relies on GPS, AHRS, and DVL navigation methods to ensure both high-frequency output and robust performance[1].

However, GPS has underwater interruption issues, and AUV positioning requires frequent signal reception on the water surface. Using deep learning to estimate AUV speed eliminates the need to standardize DVL measurements in the dead reckoning (DR) navigation method [2]. However, this method has certain limitations in both training samples and practical use. The USBL system, as one of the most effective underwater positioning systems, has advantages such as stability and accuracy. Utilizing a nonlinear extended Kalman filter model to fuse USBL data with dead reckoning trajectory helps mitigate positioning errors of AUVs in complex nonlinear underwater environments[3].

The above system relies on an accurate system model and is unreliable when dealing with strongly nonlinear systems.

An adaptive fading unscented Kalman filter-based SINS /DVL/USBL fault-tolerant integrated navigation system is designed to enhance the robustness of underwater navigation and address system model uncertainties[4].The design of SINS based on the error model was completed by adaptively adjusting the state prediction covariance matrix [5]. I Considering the variability of the underwater environment, a two-stage filtering method incorporating DVL is proposed. First, a particle filter selects candidate particles by minimizing the local backward DR error. Then, the Weighted Extended Kalman Filter (WEKF) further smooths and improves the results from previous step [6]. However, the above methods do not solve the problem of complex noise interference in non-Gaussian environments, and particle filter suffers from particle degradation.

In response to the above research and existing problems, this article takes the USBL/DR integrated navigation system in complex underwater environments as the research object. ,Meanwhile, to improve the algorithm's adaptability to noise and mitigate particle degradation, the volume particle filter is introduced to realize the robust processing of external non-Gaussian measurement information, so as to improve the navigation accuracy.

2. Cubature particle filter based on nonlinear motion state

2.1. AUV state equation and measurement equation

Dead reckoning uses the starting position, speed, and direction of the vehicle to calculate its current coordinate position information. AUVs mainly use depth determination to reach the target area, incorporating dead reckoning into AUV navigation. The compass and DVL provide the attitude and ground speed of the AUV.

We utilized the North-East-Sky (NES) system as the navigation frame and applied the right-handed body-fixed frame for the carrier[7]. The AUV's velocity components in the eastward and northward directions can be represented using the velocity and heading angle within the carrier coordinate system:

$$\begin{bmatrix} V_E \\ V_N \end{bmatrix} = \begin{bmatrix} V_{dx}\cos\psi_c - V_{dy}\sin\psi_c \\ V_{dx}\sin\psi_c + V_{dy}\cos\psi_c \end{bmatrix} \tag{1}$$

Where V_E and V_N represent the velocities of the carrier in the navigation system towards the east and north directions, respectively. V_{dx} and V_{dy} denote the lateral and longitudinal velocities of the load system as measured by the DVL, respectively. ψ_c represents the heading angle as measured by the compass.

The state vector of AUV consists of its position, attitude, velocity, and angular velocity, while the navigation system's state vector is:

$$\boldsymbol{X}_k = [x, y, \psi, u, v, r]^{\mathrm{T}} \tag{2}$$

where x is the eastward position, y is the northward position, ψ is the yaw, (u, v) is the velocity in the carrier coordinate system, and r is the angular velocity of the yaw. navigation systems' state equation is given below:

$$\boldsymbol{X}_k = f(\boldsymbol{X}_{k-1}, \boldsymbol{W}_{k-1}) \tag{3}$$

$$\begin{bmatrix} x \\ y \\ \psi \\ u \\ v \\ r \end{bmatrix}_k = \begin{bmatrix} x + x\cos(\psi)t - y\sin(\psi)t + W_x \\ y + x\sin(\psi)t + y\cos(\psi)t + W_y \\ \psi + rt + W_\psi \\ u + W_u \\ v + W_v \\ r + W_r \end{bmatrix}_{k-1} \tag{4}$$

Among them, W represents process noise and t is the sampling time.

$\boldsymbol{Z}_k$ represents the observation vector :

$$\boldsymbol{Z}_k = [x_{usbl}, y_{usbl}, x_{dvl}, y_{dvl}]^{\mathrm{T}} \tag{5}$$

In the equation, x_{usbl} represents the eastward position of the AUV observed by UBSL, y_{usbl} represents the northward position of the AUV observed by UBSL, and u_{dvl} represents the AUV's rightward velocity measured by DVL. v_{dvl} represents the forward velocity of DVL[8].The measurement model is:

$$\boldsymbol{Z}_k = \boldsymbol{H}_k \boldsymbol{X}_k + \boldsymbol{V}_k \tag{6}$$

where $\boldsymbol{V}_k$ is the observation noise, and the measurement matrix $\boldsymbol{H}_k$ is as follows:

$$\boldsymbol{H}_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{7}$$

2.2. Cubature particle filter algorithm

We redesign the importance density function of the particle filter using the Cubature Kalman Filter (CKF) algorithm, which results in the CPF algorithm. We outline the specific process of the cubature particle filter algorithm as follows:

(1) We initialize the number of particles N, extract the sampled particles $\boldsymbol{x}_0^i$ from the prior distribution $p(x_0)$, then the initial particles are $\hat{\boldsymbol{x}}_0^i$, each particle has a weight of $\frac{1}{N}$, the state mean is $\hat{\boldsymbol{x}}_0^i = E(\boldsymbol{x}_0^i)$, and the variance is $\hat{\boldsymbol{p}}_0^i = E\left(\boldsymbol{x}_0^i - \hat{\boldsymbol{x}}_0^i\right)\left(\boldsymbol{x}_0^i - \hat{\boldsymbol{x}}_0^i\right)^{\mathrm{T}}$.

(2) When $k = 2, 3, \cdots,$ CKF is used to calculate the mean and covariance of each particle at the next moment, time and measurement updates are performed separately.

We calculate the cubature point:

$$\boldsymbol{p}_{k-1} = \sqrt{\boldsymbol{p}_{k-1}}(\sqrt{\boldsymbol{p}_{k-1}})^{\mathrm{T}} \tag{8}$$

$$\boldsymbol{x}_k^i = \sqrt{\boldsymbol{p}_{k-1}}\boldsymbol{\xi}_j + \hat{\boldsymbol{x}} \quad j = 1, 2, \cdots, 2n \tag{9}$$

The cubature point is spread as:

$$\boldsymbol{x}_{k|k-1}^i = f(\boldsymbol{x}_k^i) \tag{10}$$

The square root of state prediction and variance prediction are calculated as:

$$\hat{\boldsymbol{x}}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} \boldsymbol{x}_{k|k-1}^j (\boldsymbol{x}_{k|k-1}^j)^{\mathrm{T}} \tag{11}$$

$$\boldsymbol{P}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} \boldsymbol{x}_{k|k-1}^j \left(\boldsymbol{x}_{k|k-1}^j\right)^{\mathrm{T}} - \hat{\boldsymbol{x}}_{k|k-1}\left(\hat{\boldsymbol{x}}_{k|k-1}\right)^{\mathrm{T}} + \boldsymbol{Q} \tag{12}$$

The cubature point for the next state prediction value is calculated as:

$$\boldsymbol{P}_{k|k-1} = \sqrt{\boldsymbol{P}_{k|k-1}}(\sqrt{\boldsymbol{P}_{k|k-1}})^{\mathrm{T}} \tag{13}$$

$$\boldsymbol{x}_{k|k-1}^j = \sqrt{\boldsymbol{P}_{k|k-1}}\boldsymbol{\xi}_j + \hat{\boldsymbol{x}}_{k|k-1} \tag{14}$$

The cubature point is spread as:

$$\boldsymbol{z}_k^j = h\left(\boldsymbol{x}_{k|k+1}^i\right) \tag{15}$$

The measured predictive value is calculated as:

$$\hat{\boldsymbol{z}}_k = \frac{1}{m}\sum_{j=1}^{m} \boldsymbol{z}_k^j \tag{16}$$

We calculate the covariance of measurement error and the cross variance of measurement error and state error:

$$\boldsymbol{P}_k^z = \frac{1}{m}\sum_{i=1}^{m}\boldsymbol{z}_k^j(\boldsymbol{z}_k^j)^{\mathrm{T}} - \hat{\boldsymbol{z}}_k(\boldsymbol{z}_k^j)^{\mathrm{T}} + \boldsymbol{R} \tag{17}$$

$$\boldsymbol{P}_k^{xz} = \frac{1}{m}\sum_{t=1}^{2n}\boldsymbol{x}_k^j(\boldsymbol{z}_k^j)^{\mathrm{T}} - \hat{\boldsymbol{x}}_k(\hat{\boldsymbol{z}}_k)^{\mathrm{T}} \tag{18}$$

Predicted state values and measurement covariance integrated with current observations are expressed as:

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k(\boldsymbol{z}_k - \hat{\boldsymbol{z}}_k) \tag{19}$$

$$\boldsymbol{P}_k = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k\boldsymbol{P}_k^z\boldsymbol{K}_k^{\mathrm{T}} \tag{20}$$

where the filter gain matrix is $\boldsymbol{K}_k = \boldsymbol{P}_k^{xz}(\boldsymbol{P}_k^z)^{-1}$.

(3) The importance density function $q(\boldsymbol{x}_k^i \mid \boldsymbol{x}_{k-1}^i, \boldsymbol{z}_k) = N(\boldsymbol{x}_k^i, \hat{\boldsymbol{p}}_k^i)$ is obtained by importance sampling according to the predicted state value and measurement covariance, and the particle weight is calculated and normalized.

(4) Resampling, copying, and eliminating normalized particles according to Markov Chain Monte-Carlo method.

(5) The state estimation is expressed as follows:

$$\hat{\boldsymbol{x}}_k = \frac{1}{N}\sum_{i=1}^{N}(\hat{\boldsymbol{x}}_k^i) \tag{21}$$

$$\hat{\boldsymbol{P}}_k = \frac{1}{N}\sum_{i=1}^{N}(\hat{\boldsymbol{x}}_k^i(\hat{\boldsymbol{x}}_k^i)^{\mathrm{T}} - \hat{\boldsymbol{x}}_k(\hat{\boldsymbol{x}}_k)^{\mathrm{T}}) \tag{22}$$

3. Analysis of Experimental Findings

3.1. Simulation test

We design and conduct simulations to examine the effectiveness of the algorithm introduced in this paper.

Initial conditions of simulation: initial position (origin of navigation coordinate system), attitude (yaw 5 °).

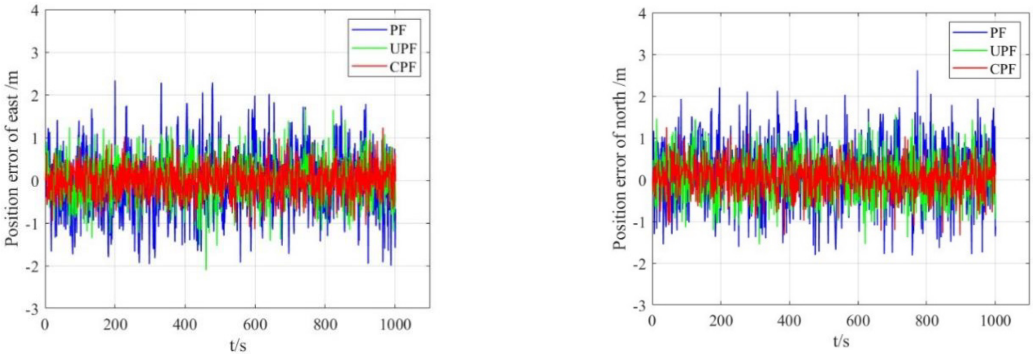
Simulation track: the track is set to the carrier with a constant heading, the right speed and forward speed are both 1m/s and the vehicle travels at a constant speed for 1000s. The DVL scale factor error is 1%, and the frequency of the output signal is set to 5 Hz. Compass error is 0.1° and the output frequency is 5Hz. The USBL error is 2%, and the output frequency is 1Hz.

In order to simulate the interference of an underwater complex environment on measurement, noise is added on the basis of simulation parameters:

$$\boldsymbol{V}_N^b \sim \begin{cases} N\left(0, \boldsymbol{R}_D\right) & \text{w.p.0.70} \\ N\left(0, 10\boldsymbol{R}_D\right) & \text{w.p.0.30} \end{cases} \tag{23}$$

where $\boldsymbol{R}_D$ represents the variance matrix of the measurement noise.; $\boldsymbol{V}_N^b$ represents denotes the noise output by the equipment; w.p.0.30 represents the probability, that is, the measurement noise with variance 10 times of the original appears in the original Gaussian distribution with 30% probability.

From Figure 1, the average errors of the PF in the east and north directions are 0.61m and 0.59m, respectively. For the UPF, the average errors are 0.37m and 0.41m, while the CPF algorithm shows even smaller average errors of 0.31m and 0.30m. Overall, the mean errors of the UPF and CPF algorithms are significantly smaller compared to PF.



(a) (b)

Figure 1. Position error of simulation test.

3.2. Lake experiment

To validate the effectiveness of proposed algorithm, a lake test platform was built. Experimental equipment includes IMU, DVL, USBL, GPS, mobile power supply, data acquisition equipment, computers, etc. The DVL is fixed at the bottom of the ship, the USBL is fixed to the ship through the steel pole at the tail of the ship, and the USBL beacon is 1m underwater. Figure 2 is the site map of the lake test and the schematic diagram of the construction and installation of the experimental platform.

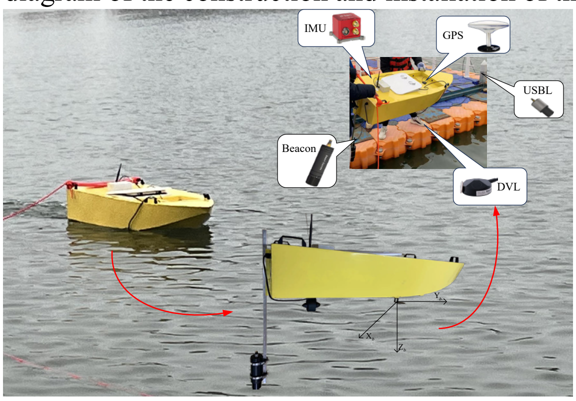
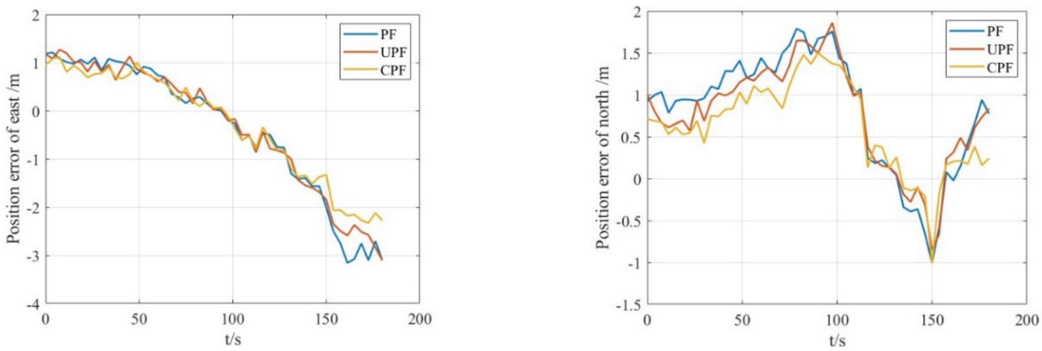


Figure 2. Lake test site photos.

Figure 3 illustrates that the proposed method achieves superior positioning accuracy with minimal error. The average eastward error is 0.96 m, which is lower than the 1.07 m and 1.26 m recorded for UPF and PF, respectively. In the northward direction, the method maintains an average error of 0.75 m, outperforming UPF (0.88 m) and PF (0.96 m). Overall, the horizontal positioning accuracy of this method demonstrates a clear advantage over the other two.



(a) (b)

Figure 3. Position error of lake test.

4. Conclusion

Aiming at the navigation problem in complex underwater environments, this paper presents a USBL/DR-based integrated navigation method utilizing the cubature particle filter. Then the importance density function is redesigned by using the CKF algorithm to obtain the CPF algorithm; A simulation experiment is conducted to evaluate the CPF algorithm in an underwater environment, comparing its performance with the PF and UPF algorithms. The results demonstrate that the CPF algorithm outperforms both PF and UPF in terms of tracking accuracy and stability in underwater conditions.

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