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Advances in muometric navigation system and its use in underwater localization

Mailun CHEN[1,2], Guo WEI[1,2], Chunfeng GAO[1,2*], Hongliang TANG[3], Ye TIAN[1,2], Jing WANG[1,2], Chengzhi HOU[1,2], Xu ZHU[1,2] and Jiayi CHENG[1,2]

[1] College of Advanced Interdisciplinary Studies, National University of Defense Technology, Changsha, Hunan 410073, China

[2] Nanhu Laser Laboratory, National University of Defense Technology, Changsha, Hunan 410073, China

[3] People's Liberation Army of China 92199 Unit

[*] Corresponding author

Email: neil1989@126.com

Abstract. As a kind of widely existed cosmic ray particles, muons have strong penetrativity to water layer and rock layer, which is suitable for anti-interference navigation and positioning at underwater depth of one hundred meters, and its positioning ability in the environments such as strong electromagnetic suppression and electromagnetic silence can become a new type of auxiliary means for underwater positioning. This paper summarizes the latest progress of muometric navigation technology, introduces its generation mechanism and basic physical properties, reviews the research results of domestic and foreign research in muon detection and localization, analyzes the principle and key technology of its underwater navigation and localization, looks forward to the future direction of the research and technological breakthroughs, and discusses the prospects for its application in underwater navigation and the challenges it faces.

1. Introduction

After six decades of development, global navigation satellite system (GNSS) has now become the mode of application for most navigation platforms or are utilized to correct positioning errors using information from GNSS, owing to their high accuracy and their ability to be integrated into small hand-held devices and their low cost. However, satellite navigation signals in the form of radio wave propagation communication, attenuation is particularly serious in denser substances, and it is difficult to obtain practical signals in the underground and underwater environments; especially in the vicinity of the polar regions, due to the small number of satellites, the lower degree of coverage, and the interference of ionospheric disturbances, and other factors, which lead to weaker and more unstable satellite signals. Current solutions for navigation in underwater environments include acoustic navigation [6], which is mediated by sound waves, geomagnetic navigation [22], which utilizes the relatively stable distribution characteristics of the Earth's magnetic field for measurement, and inertial navigation, which relies on the measurement of angular velocity and acceleration. Positioning errors and modeling delays can be caused by acoustic navigation, because the propagation speed of sound waves in water is affected by water temperature and density; geomagnetic navigation relies on an accurate geomagnetic field model and is susceptible to interference from external magnetic fields; and inertial navigation generates cumulative errors during operation and needs to be corrected in integration with other navigation methods. Deep water, underground, under ice and other environments, electromagnetic signal attenuation is serious, and the transmission distance of optical signals is also very limited, thus making satellite navigation, radio navigation, astronomical navigation and other methods unusable. In response to the above problems, the study of muons, which are produced in nature and have strong penetrativity, as propagation carriers provides a new way to locate and navigate in environments where satellite signals are weak.

Cosmic ray muons are strongly relativistic particles produced by the decay of secondary mesons produced by the collisional reaction of primary cosmic rays (mainly galactic cosmic rays) with atomic nuclei in the atmosphere. Muons are very energetic, with an average energy of up to 4 GeV, and can pass through almost any type of material at the speed of light. Muons have a static mass of about 207 times that of an electron, and a half-life period of 2.197 μs, and are a natural source of widely distributed and relatively stable high-energy particles. A natural, widely distributed and relatively stable source of energetic particles. At sea level, an average of about 10,000 muons per square meter per minute in the vertical direction of the zenith uniformly impact and penetrate the Earth's surface; however, due to the presence of a large number of non-relativistic muons (velocity$<$ 0.9c), which are still absorbed, this value decreases to 1 at a water depth of 1,000 meters. Energetic muons are affected by the relativistic clock-slow effect, which extends their travel time and distance before decay, and the average decay length of airborne muons is approximately proportional to the muons energy, with a decay length of 6,200 km per TeV of energy.

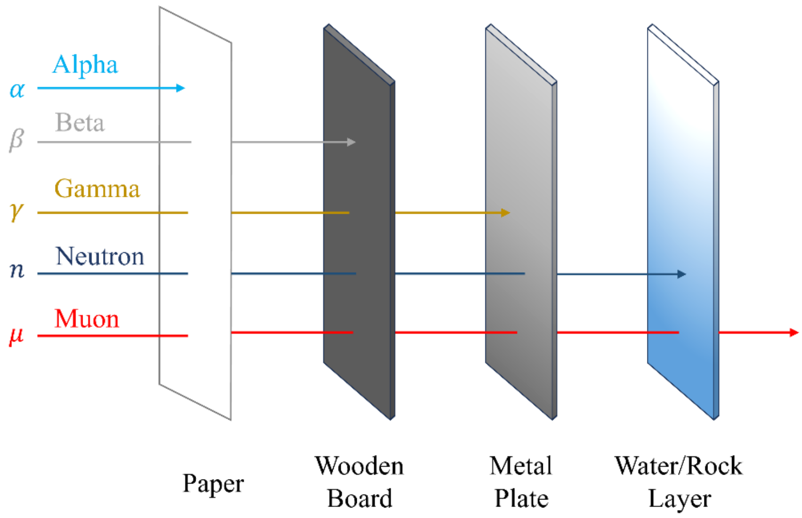

Figure 1. Muons are highly penetrating and can penetrate a wide range of substances.

Muons have a strong penetrativity, and can penetrate through many kinds of low-density materials such as rocks, buildings, ice, seawater and so on almost without any damage and at a uniform speed. Even in the underground dark matter laboratory in Jinping, China, which has the thickest rock cover in the world, traces of muons can still be found [21].

However, when it penetrates dense materials such as metals, it collides with atomic nuclei and scatters, and its intensity increases with density. Based on this characteristic, there are many technologies applied to the detection of inner core materials in containers and vehicles, as well as in the fields of archaeology, prospecting, and exploration of geological structures. The related detection technology, also known as "muon tomography", is now relatively mature in the field of geological research, and has been widely used to observe the internal structure of huge objects, and research in nuclear material monitoring is also being vigorously carried out.

Also based on the easy detectability of muons, the use of muon detectors consisting of plastic scintillators has potential applications for localization and navigation in underwater and underground environments. Since muons always pass down from above the horizontal plane, multiple reference detectors can be set up above the receiver detector to be localized, and the function of localization can be realized by measuring the time of flight (TOF) of the muon between the reference detector and the receiving detector, and multiplying it by the speed of light in vacuum to get the straight line distance between them.

2. Domestic and international research status

Currently, muons are widely used to detect the internal structure of large objects, such as containers and vehicles, as well as in archaeology, prospecting, geological structure exploration, etc., and research in nuclear material monitoring is also being vigorously carried out. The relevant detection technology is called "muon tomography".

Compared to muon detection, muon positioning and navigation started much later. 2021, as the winning solution of the Global-X Challenge organized by the U.S. Office of Naval Research and Army Development Command, Muon positioning was proposed by a four-country team spanning the U.S., the U.K., Japan, and Finland, led by the University of Tokyo, Japan, and GEOPTIC, Inc. with the aim of achieving high-precision navigation in areas that cannot be located using GPS. The results of the muometric navigation system are mainly focused on the team of Hiroyuki K.M. Tanaka at the University of Tokyo and the team of GEOPTIC Corporation funded by the U.S. Office of Naval Research, and there are various ways of muon positioning and navigation.

2.1. Current status of muon tomography research

When the muon penetrates a material, it is subject to Coulomb scattering by a strong electric field near the nucleus and deviates from its original trajectory, the degree of which depends on the density of the material and the weight of the nucleus. The average angle of deflection of the muon varies considerably when it passes through different materials with different atomic numbers and densities, such as plastics, iron and steel as well as materials with large differences such as lead. At the same time, since ionization and atomic excitation are the main energy loss pathways for muons passing through materials, the energy loss rate for the same energy of muons passing through a material is linearly related to the density of the material, which can be utilized to perform transmission imaging with muons.

Both muon transmission and scattering imaging rely on statistical distributions over large samples, and the results are subsequently analyzed and processed to create an internal image of the object being measured. Generally, because scattering imaging requires the reconstruction of muon trajectories, muon transmission imaging is more effective for larger target bodies, whereas muon scattering imaging is better suited for small and medium sized target bodies.

In 2001, the Los Alamos National Laboratory (LANL) of the United States first proposed the use of natural muons for nuclear material detection to prevent nuclear material smuggling, and confirmed the feasibility of muon radiation imaging by means of a small-scale simulation device. Since then, LANL has developed a large muon track detection system with a detector size of $3.65\,m \times 3.65\,m$, with a positional resolution of up to 400 μm, can clearly detect special nuclear materials on trucks within 4 minutes. since 2007, countries in Europe, America and Japan have used cosmic muon imaging methods for the detection of small scale fine density anomalous results in volcanoes, tunnels, pyramid chambers, etc., as well as the monitoring of active faults and volcanic magma movements, etc, which have shown high-resolution and high-precision characteristics. In 2008, the team of Hiroyuki Tanaka conducted perspective imaging of a volcano on Satsuma Sulphur Island based on the fact that some non-relativistic muons are absorbed in high density objects, showing the internal structure of the volcano as well as the convection of magma.

China has carried out fruitful work in inertial navigation, acoustic navigation and optical navigation for underwater environments, and has achieved a series of research results in the areas of muon detection and research on navigation methods for hidden environments. Institute of High Energy Physics of Chinese Academy of Sciences has hosted the development of Large High Altitude Air Cluster Observatory (LHAASO) detector array; Tsinghua University has carried out a research on the identification and imaging of high atomic number substances by using muons; Institute of Control Engineering in Beijing has carried out an exploratory research on the directional navigation method of the hidden environment carrier based on neutrino signals; and China Academy of Space Technology (CAST) and other organizations have completed the development of the muon imaging instrument, algorithm and software code, and completed the development of the muon imaging instrument and algorithm software code. The China Academy of Space Technology and other organizations have completed the development of muon-imaging instruments and algorithm software codes, completed imaging experiments on volcano/tunnel/subway/large-scale equipment, and developed high resolution and precision muon imaging instruments for use in multiple scenarios. Lanzhou University has developed a small cosmic ray muon imaging prototype, LUMIS, based on a three-prism plastic scintillator. The angular resolution is $8.73\,\mathrm{mrad}$ and the position resolution is 2.5 mm. The high resolution measurement of a city wall in Xi'an shows the density anomalous region inside the wall. The University of Science and Technology of China (USTC) built the muon Imaging System μSTC based on Micromegas, with the effective detective area of $100\,mm \times 100\,mm$ with a positional resolution of $200\,\mu m$. The system is based on Micromegas, and has been used for transmission imaging of Dashushan Mountain in Hefei for about 100 days to obtain the density distribution inside the mountain.

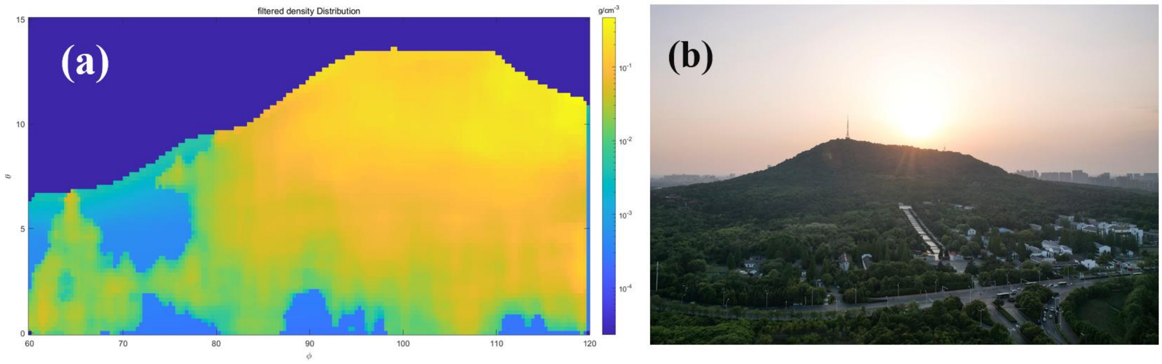

Figure 2. Transmission imaging of Mt. Dashu by μSTC. (a) shows the internal structure of the mountain after the processed.

Compared with the traditional radiation imaging technology, muon tomography uses natural muons, which do not require external sources, will not jeopardize human health, and can be used for non-destructive detection of the object to be measured; the strong penetration of muons is very suitable for detecting high atomic number materials, even shielded high atomic number materials. However, this technique may require a long scanning time to obtain the imaging results, and the differentiation ability of the material is very limited. In the development of muon tomography, the key is to shorten the scanning time, reduce the cost, improve the ability to differentiate the material, and seek for appropriate algorithms to derive the image of the object to be viewed according to the measurement data.

2.2. Fundamentals of muometric navigation and localization and current research status

In 2020, Hiroyuki Tanaka of the University of Tokyo proposed the muometric positioning system (μPS) [1]. As a passive navigation system, μPS has the characteristics of strong penetration, uniformly distributed over the surface, non-radiation, and high concealment, and it can be used in areas where the signal of satellite navigation systems is poor, such as underwater, underground, and high-latitude areas. As shown in the Figure 3 below, multiple reference detectors located above the target detectors provide the TOF of particles traveling through the reference/receiver detectors, which is converted to the distance between these detectors by multiplying it by the speed of light in vacuum. These muons can be visualized as probes for large objects, and when these naturally occurring particles are detected, they can be navigated and located in a defined coordinate system with centimeter-level accuracy.

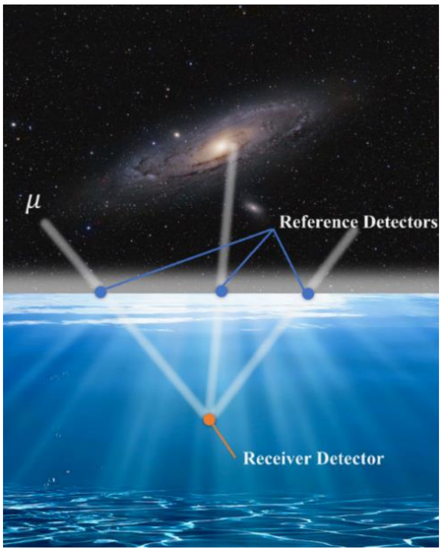

Figure 3. Conceptual diagram of the muometric positioning system (μPS). Multiple reference detectors are positioned at sea level, while the receiving detector is placed underwater, awaiting muons that pass through both, and measuring the TOF.

Specifically, the μPS consists of multiple reference detectors and a receiver detector that together define the coordinates of the system within a spatial coordinate system. The TOF of the muon crossing between the reference and receiver detectors is simultaneously measured to obtain its linear distance L_i , and thus the relationship (1) defining the relative position within this coordinate system:

$$L_i^2 = (x_i - x_p)^2 + (y_i - y_p)^2 + (z_i - z_p)^2 \quad (1)$$

Here, L_i represents the geometric distance between the $i-th$ reference detector located at (x_i, y_i, z_i) and the receiver detector located at (x_p, y_p, z_p). The position of the reference detectors can be determined by satellite navigation signals.

μPS operates similarly to GNSS, but instead of satellites, at least three reference detectors are set up to establish the reference coordinate system, with a wired connection between the detectors to maintain precise time synchronization. By measuring the distance to the receiver detectors, the position of the receiver detectors can be determined.

The setup of the μPS structure is illustrated in Figure 4. Based on the muon propagation direction, the detectors are labeled A, B, and C from top to bottom. Detector A serves as the reference detector, B as the receiver detector, and C, the checking detector, is used to select relativistic muons. A lead plate, 15 cm thick with a density equivalent to 1.8 meters of water, is placed between detectors A and B to filter out non-relativistic muons. When a muon passes through the detector, a photomultiplier tube (PMT) generates an electrical signal. This signal is transmitted via wired connections to ensure precise time synchronization and is sent to a time digitizer and terminal, where coincidence measurements are performed to calculate the distance between detectors A and B.

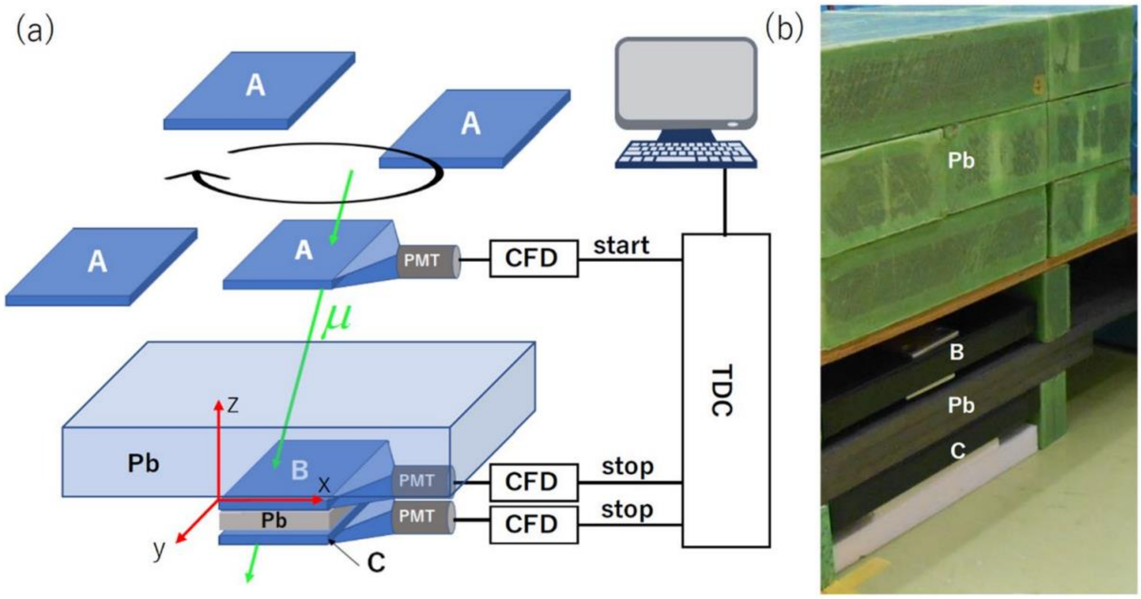

Figure 4. Experimental setup. Figure (a) shows the symbols A, B, C, Pb, PMT, CFD, and TDC denoting the reference detector, the receiver detector, the checking detector, the lead block, the photomultiplier tube, the constant fraction discriminator, and the time-to-digital converter, respectively. A close up view of the receiver detector and checking detector is shown in Figure (b).

Referring to the positioning solution idea of satellite navigation, the team demonstrated three-dimensional positioning on a kilometer scale within 5 microseconds (equivalent to a relativistic muon travel distance of 1.5 km) by muon TOF simulation under laboratory conditions and tested the system performance under laboratory conditions. Detector localization experiments were performed by simulating the deep earth situation through a lead block, and the measured time spectrum between the reference and receiver detectors is shown in Figure 5, where in the ideal geometric configuration of the reference detector, through approximately 1,000 muon events, the detector localization accuracy under the lead block can reach 1 cm.

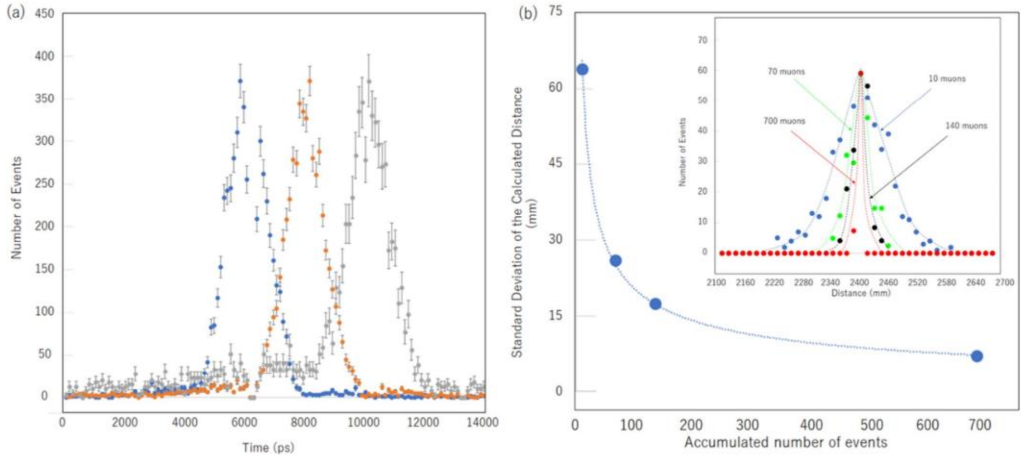

Figure 5. Time spectra measured between the reference and receiver detectors of the μPS at the University of Tokyo, Japan. Figure (a) shows that they are measured at different given distances of 1.8 m (blue circles), 2.4 m (orange circles) and 3.0 m (gray circles). The error bars indicate the standard deviation as a function of the number of events for the accuracy of the distance calculation. Figure (b) shows the TOF spectra for different sub-total numbers. The horizontal axis is converted to distance by multiplying the speed of light in vacuum.

According to a study by the University of Tokyo, μPS can effectively overcome the shortcomings of traditional positioning technology that cannot penetrate high-density media, and can be used for underwater positioning to achieve centimeter-level positioning accuracy. On the basis of laboratory tests, the team designed a scheme of μPS mounted on Hitachi-Zosen GPS floats to realize seabed displacement detection with the help of 18 Hitachi-Zosen GPS buoys in Japan, as shown in Figure 6.

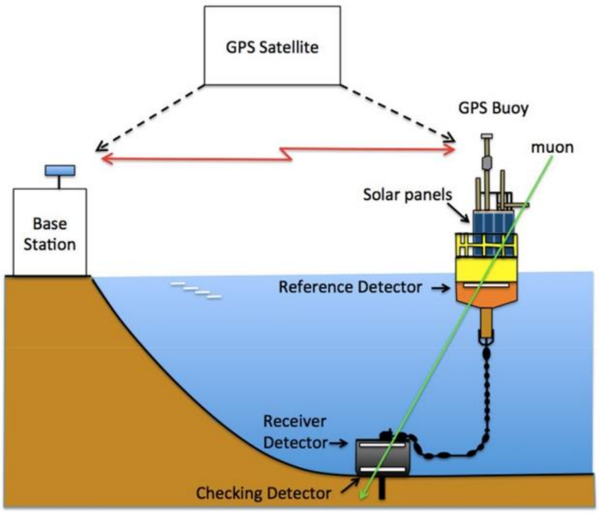

Figure 6. Deployment of MuPS installed on Hitachi-Zosen GPS buoys.

A steel cable is used to connect the buoy to a 27 tons of anchor embedded in the seafloor, and the diameter of the reference detector placed inside this buoy is 4 meters (with an area of $16\ \mathrm{m}^2$). The receiver detector and the checking detector are embedded in the top and bottom of the anchor, respectively, and are powered via cables attached to the chain. Muons detected by the receiver detector are transmitted to the buoy through signal cables connected to the chain, allowing for the determination of the particle crossing time between the reference and receiver detectors. This enables centimeter-level relative positioning between the anchor and the buoy. The μPS for underwater positioning is unaffected by changes in seawater temperature, salinity, or other environmental conditions that could impact relative position measurements. However, μPS relies on a wired connection between the reference and receiver detectors and requires the use of GPS buoys to function in tandem, which limits the system's range of movement and flexibility, despite its high positioning accuracy.

2.3. Fundamentals and research status of wireless muometric navigation and localization technology

In 2022, Hiroyuki Tanaka announced the development of a next-generation "Wireless Muometric Navigation System (MuWNS)" for muon measurements in the Nature journal Scientific Reports [2-3]. The system uses a high-precision grand master clock (GMC) and a crystal oscillator OCXO to synchronize the clocks between the reference detectors and the receiver detector, eliminating the need for a wired connection. When the GPS signal is unavailable (due to certain reasons such as multipath reception, GPS interference and spoofing, system failure, underwater diving, subsurface movement, etc.), the GMC automatically switches from the GPS mode to the holdover mode and outputs an independent OCXO signal. Both the reference and receive detectors are equipped with an Event Counter (EC) and Time-to-Digital Converter (TDC) for time synchronization and counting.

Although OCXO can provide reliable standard clock signals, in environments where synchronized satellite signals are not available, there is still some frequency drift that produces time errors Δt between the reference detector and the receiver detector. For this error, equation (1) is modified:

$$L_i^2 = (x_i - x_p)^2 + (y_i - y_p)^2 + (z_i - z_p)^2 + (c\Delta t)^2 \quad (2)$$

Here the time error Δt of each detector is considered as the same.

Experimental and simulation results show that navigation and localization of approximately less than 3m, 10m and 1m can be achieved in shallow underground, deep underground and underwater environments. The article gives the arrangement scheme and examples of MuWNS, including in the shallow underground, using the Toranomon Hillside Station building on the Tokyo Metro Hibiya Line and Akiyoshido in Japan as examples, as shown in Figure 7.

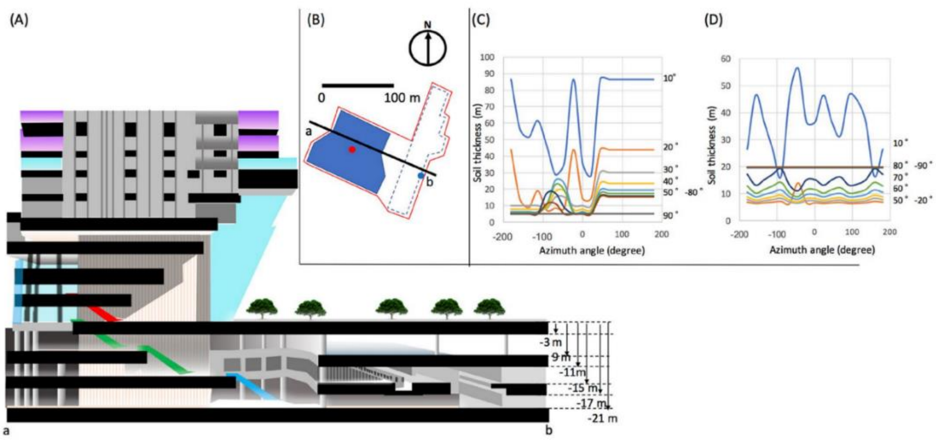

Figure 7. Schematic diagram of the Huzhimen New Town station complex and MuWNS arrangement

scheme. The internal structure and top view of the complex are shown in (A) and (B), respectively. (C) and (D) show the azimuthal distribution of soil/concrete thickness at locations A and B.

In June 2023, the team published an article in the journal iScience, announcing the successful completion of the world's first underground navigation and localization experiment using the "Muon Wireless Navigation System (MuWNS)," as illustrated in Figure 8 below..

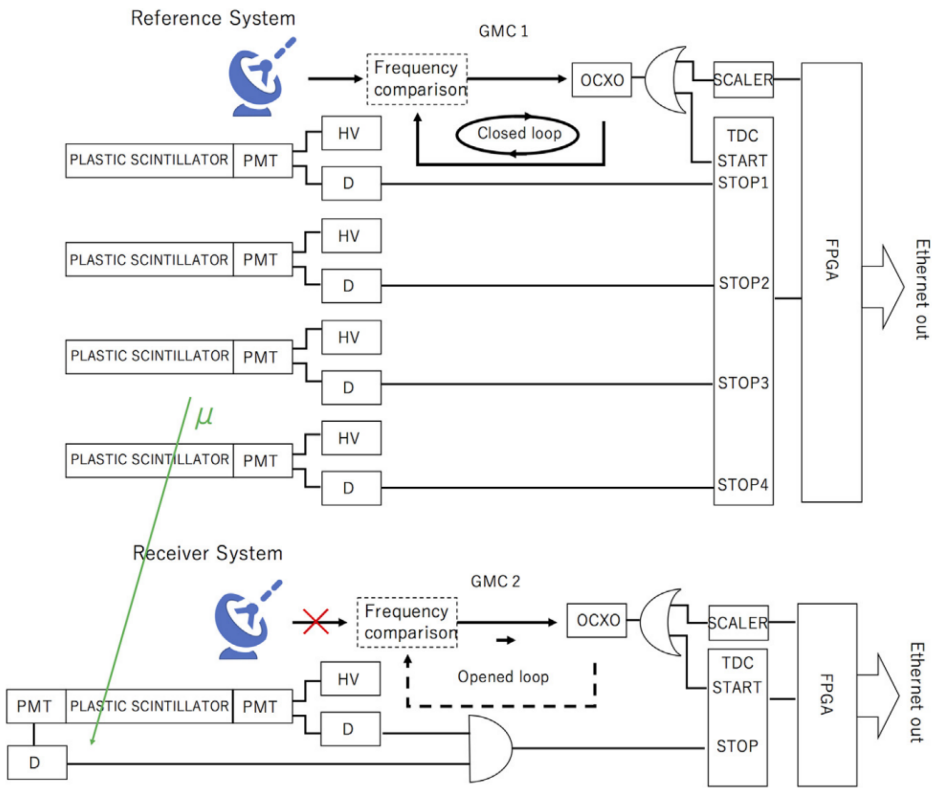

Figure 8. Block diagram of the wireless measurement and navigation system (the reference system block diagram (top) and the receiver system block diagram (bottom) are shown in the figure).

The labels PMT, HV, D, OCXO, GMC, TDC, and FPGA denote photomultiplier tubes, high-voltage power supplies, discriminators, crystal oscillators, grand master clocks, time-to-digital converters, and field-programmable gate arrays, respectively, while the red cross markers indicate that the system is disconnected from the GPS antenna, and the green line denotes the direction of muon propagation. The experiment penetrated 7 floors and accurately located the tester's route of travel in the underground corridor of the building with a positioning accuracy of 2-25 meters, which can be further improved to about 1 meter with the upgrading of the synchronization clock accuracy.

2.4. Current status of research on vector muometric navigation system

In 2024, Hiroyuki Tanaka proposed in Scientific Reports the vector muometric navigation system (MuWNS-V), which is localized by measuring the angle of incidence of muons [10]. This localization method, compared to the one described in Section 2.2, combines inter-detector height information and requires only a minimum of one muon track to obtain the position of the receiver detector. And two unrelated muons in Section 2.2 may hit the detector within the same time window, leading to mismatching, which can be reduced by chance by reducing the size of the time window; the use of angular information in MuWNS-V can greatly minimize this situation, as muons within the same time window tend to have different angles of incidence.

Since muon rates are naturally limited to the order of about 100 Hz/m², this results in intervals of more than 1 ms between successive muon events. muons capable of traveling through two detectors can do so without precise clocks or fast wireless communication (t_w practically on the order of 0.1 ms) can be tagged without a precise clock or fast wireless communication to ensure and continuously maintain time synchronization between the reference and the receiver. This concept is called "Cosmic Timing Calibration" (CTC) and eliminates the need for wired connections and the use of expensive clocks in μPS. In this work the two detectors are "CTC locked" to each other.

The team showed results for two similar configurations in their experiments. The "LowRes" configuration uses a smaller reference detector with an angular resolution of about $15\,\mathrm{mrad}$ The "LowRes" configuration uses a smaller reference detector with an angular resolution of about 3.9 cm (RMS), and measures the center position of the receiver detector with an accuracy of 3.9 cm (RMS) when the reference detector is positioned 2.4 m above the receiver detector. Another version, called the "HiRes" configuration, uses a larger reference detector with an improved angular resolution of $6-8\,\mathrm{mrad}$ (using the same receiver detector). In addition, the detectors are located on two different floors of the building. At a height difference of 3.3 m between the detectors, the accuracy of positioning the center of the receiver detector was improved to less than 3cm (RMS).

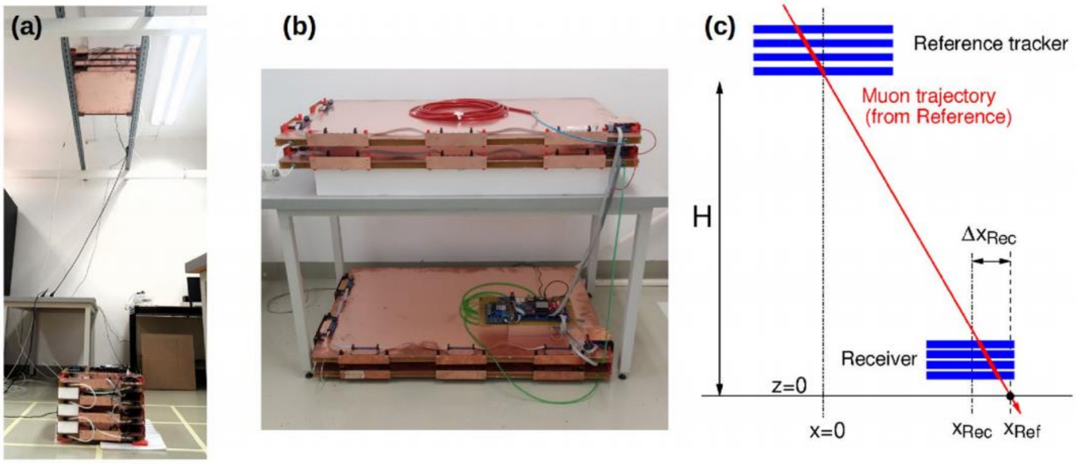

Figure 9. Experimental setup. (a) Photograph of the LowRes system with the reference detector attached to the track under the ceiling and the receiver detector on the floor; (b) Reference detector for the HiRes system; (c) shows the geometry of the experiment.

The time synchronization between the detectors is "updated" every time a muon passes through the reference detector and the receiver detector. The key point of the MuWNS-V is that, by using CTC locks, the reference detectors and the receiver detector can be strongly synchronized over a relatively slow wireless connection. With this technique, even if the internal clocks of the reference and receiver detectors are not synchronized immediately after turn-on, the two clocks are eventually synchronized during the first few compliance events by using muons that pass through both the reference and receiver detectors at the same time. This approach is similar to the standard GPS system, which requires several minutes to provide accurate and continuous position information.

3. Key technologies and related issues

In muon detection techniques, coincidence measurements are a key technique to ensure that multiple detectors can accurately identify and track the same muon. However, this method is highly dependent on the time synchronization accuracy between detectors, which poses challenges to system stability and cost. In addition, the presence of non-relativistic muons may generate anomalous TOF information, which affects the positioning accuracy; muometric navigation and inertial navigation systems can also be effectively integrated to compensate for their respective shortcomings.

3.1. Coincidence measurement

A key issue in muon detection is coincidence measurement, where two or more different detectors are able to detect and recognize the same muon. Since they do not carry information themselves, it is not possible to determine directly whether they are the same muon or not. However, based on its uniform distribution, a "time window" can be used to determine whether it is the same muon or not.

Specifically, the approximate distance between detectors, underwater depth, and other environmental factors are combined to set the appropriate time window size t_w. As shown in Figure 10, ideally, when a certain muon passes through two detectors one after another, the detectors will successively detect two muon events at two different moments, and the two signals are considered to be generated by a single muon only when both muon events are in the same t_w. The case where a muon passes through two detectors is called an "update". The time difference between the two events is the TOF of the muon, and the distance between the detectors can be calculated.

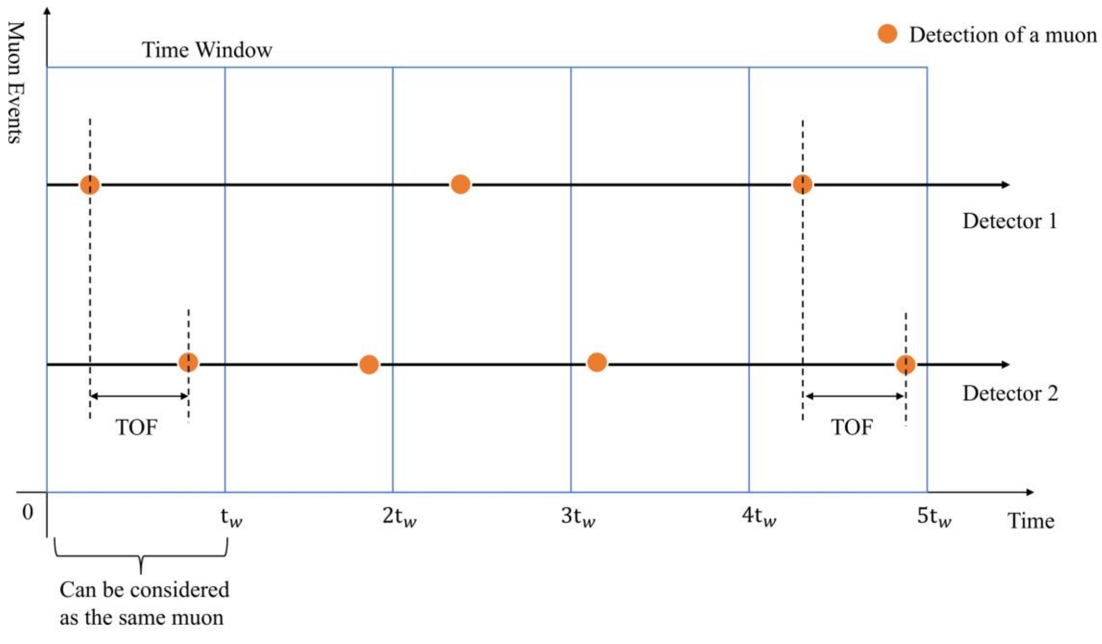

Figure 10. Schematic diagram of coincidence measurements. When muons are both detected in the same time window, it can be assumed that both are produced by the same muon.

Since muons traveling at the speed of light move approximately 30 cm per nanosecond, highly precise time synchronization between detectors is essential. OCXO exhibits time frequency fluctuations of tens of nanoseconds, which can result in ranging uncertainties of several meters. Additionally, mismatching issues may occur. To address potential mismatches of muons at different times, the fundamental approach is to record the time-of-arrival (TOA) sequence of incident muons within the reception angle at both the reference and receiver detectors. Considering that muons typically enter from the water’s surface toward the submerged area, the signal recorded by the reference detector should precede that of the receiver detector. The time interval between the TOAs recorded by the two detectors should be within a specific time window, selected based on the distance range between the reference and receiver detectors. When these conditions are satisfied, the TOAs recorded by the reference and receiver detectors can be considered valid signals for TOF calculation.

For example, for a detector with an effective area of 1 m^2 , it receives about 10,000 muons per minute, and the time interval between every two muons is 6 ms on average. For a detection distance of the order of 200 m, the TOF of the same batch of muons is 0.67 μs, so that if the time threshold is set to 1 μs, both detectors receive muon events within its range, which can be regarded as the same muon.

3.2. Coincidence measurement of MuWNS-V

Combined with information on the direction vector of muon incidence, i.e., the angle of incidence, MuWNS-V will not need to rely on high-precision time synchronization to make coincidence measurements. Similar to time windows, two detectors can be labeled as the same muon if they match two muon events within a narrow angular window. The RMS scattering angle of a muon with a GeV energy is only 1.8 mrad, and this technique is possible.

The most attractive advantage of MuWNS-V is that it does not require expensive clocks to maintain extremely accurate time synchronization, and requires fewer detectors and achieves centimeter-level positioning accuracy. This makes the whole system more stable, less expensive and more widely applicable. However, MuWNS-V can still suffer from the problem of mismatching, which requires that the angular relationship between detectors can be obtained frequently and accurately.

3.3. Non-Real-Time Problems with μPS

As mentioned in Sections 2.2 and 2.3, μPS needs to perform coincidence measurements before it can perform localization and navigation, which means that it needs a certain amount of time to wait for each update with a low frequency, and relying solely on μPS does not satisfy the needs of navigating high-speed moving objects.

To solve this problem, inertial navigation INS is an ideal solution.INS is able to monitor the angular velocity and speed of the motion in real time without the need for information interaction with the outside world. Although the update frequency of μPS is much less than that of INS, the positioning error of INS drifts with time, and μPS can correct the cumulative error generated by INS. Therefore, in voyages where INS is the main method of navigation, the position can be corrected by μPS. Compared with satellite navigation, μPS is unaffected by underwater and subsurface environments and will not be interrupted. the integrationof INS and μPS solves both the problem of accumulated error of the former and the constraint of the latter's inability to measure in real time.

Xiong Kai et al. from the Institute of Control Engineering, Beijing, gave a mathematical model of the μPS and INS fusion navigation system and conducted simulation experiments [17]. The conditions are set as follows: the initial error of the position of the underwater vehicle is 10 m, the standard deviation of the measurement noise of the distance between the receiver detector and the reference detector is 0.2 m, the standard deviation of the measurement noise of the line-of-sight direction of the reference detector is 0.2°, the data updating rate is 1Hz, the measurement accuracy of the gyro angular rate is 0.2°/h, the measurement accuracy of the accelerometer is 5×10^{-5} g, and three reference detectors are deployed on the sea surface in the range of about 200 m above the underwater vehicle, assuming that only one reference detector establishes association with the receiver detector at the same moment. For the above conditions, the positioning and attitude accuracy of the underwater vehicle is 0.307 m and 0.025° , respectively.

3.4. Detection and suppression of anomalous TOF information due to nonrelativistic muons

The smaller speed of the non-relativistic muon, a longer TOF would be obtained for the same distance traveled, so the calculated position obtained at the speed of light in vacuum would incorrectly point to a position farther away from reality. In order to $L_i = 100\ m$ case to obtain a positional accuracy of 1 m, the detection of muons will be limited to velocities greater than 0.99c of the muon, which corresponds to a muon with an energy of 700 Mev. If a 50 cm lead plate shield is inserted above the receiver detector, muons with energies less than 700 Mev may be absorbed and shielded, but at the same time the weight of the receiver detector will increase significantly. Therefore, this solution is not ideal for practical use.

At sea level, however, only about 10% of the muons have energies below 700 Mev. Thus if TOF information is collected from multiple events and operations such as averaging these events and compensating the TOF based on the non-relativistic muon occupancy are performed, there is hope that the impact of non-relativistic muons can be reduced.

3.5. High-precision synchronized clock solution design and fluctuation error compensation

Fluctuations in the synchronization clock of about ±50 ns increase the uncertainty in positioning accuracy to 30 m. The degradation in positioning accuracy can be mitigated by using a clock that is more stable than OCXO, such as a Cs oscillator.

Small electronic chip-scale Cs oscillators are being developed, such as the CPT atomic clock developed by the Wuhan Institute of Physics and Mathematics of the Chinese Academy of Sciences, with an overall volume of about19cm^3 The stability is about $1.04 \times 10^{-10}\tau^{-1/2}$ The stabilization is about . If applied to a muon detector, the jitter and drift levels will be substantially suppressed and more stable, which is conducive to more accurate localization as well as a narrower time window.

4. Summary and outlook

Muometric navigation technology, as an independent and unconstrained navigation method, offers strong penetrability and high concealment, presenting broad application prospects in detection and navigation, particularly in underwater environments where satellite signals are difficult to access, providing a unique navigation solution. In recent years, significant progress has been made both at home and abroad in muon tomography, as well as wireless and vector muon localization and navigation. However, the technology still faces some technical challenges, and the future development directions and focuses on the following aspects:

(1) Improvement of Time Synchronization Accuracy: Coincidence measurements in the muometric navigation system place extremely high demands on the time synchronization accuracy between detectors. Enhancing the time synchronization performance of the detectors and ensuring the overall stability of the system remain critical factors for achieving technological breakthroughs.

(2) Fusion with other navigation technologies: The integration of muometric navigation with other sensor technologies, such as the fusion with inertial navigation systems, can effectively enhance the accuracy and applicability of navigation. By increasing the number of reference detectors, improving detector measurement accuracy, and raising data update frequency, muometric navigation systems are

expected to achieve higher precision and robustness in complex environments.

(3) Response to the effects of mismatching and non-relativistic muons: Since mismatching and non-relativistic muons can lead to anomalous TOF data, resulting in positioning errors, the development of more advanced detection and suppression techniques to minimize the impact of such errors on the navigation system's accuracy is a core focus of future research.

It is worth emphasizing that muometric navigation offers unique advantages in environments with poor radio signals, such as underground and underwater. In these environments, conventional navigation methods, such as acoustic, inertial, and geomagnetic navigation, face limitations such as vulnerability to ambient noise, transmission delays, and accuracy constraints. In contrast, muometric navigation provides stable and efficient navigation solutions in these complex scenarios due to its low environmental dependence and superior penetration capability. Therefore, with the ongoing advancement of muometric navigation technology, its potential applications in underground, underwater, and other challenging environments will continue to expand, particularly in detection accuracy, real-time performance, and cost control. This is expected to lead to significant breakthroughs, promoting the widespread application of this technology in scientific research, military, and civilian fields.

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